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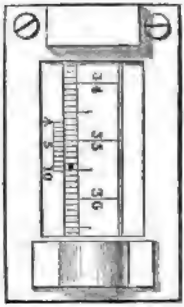

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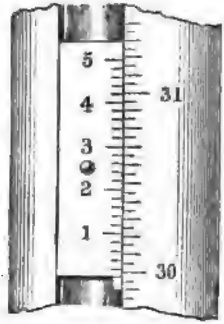

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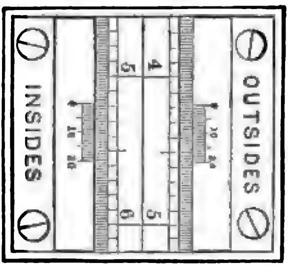

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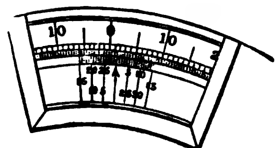

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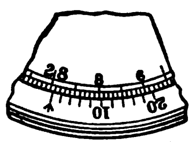

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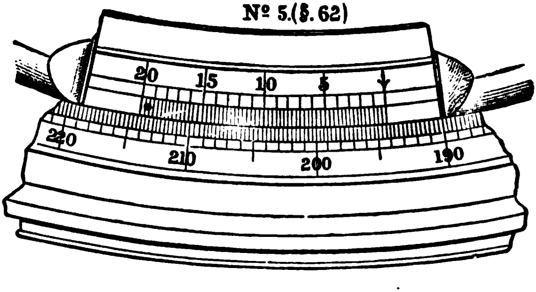

VERNIERS FOR READING LENGTHS AND ANGLES

PRACTICAL PHYSICS

BY

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PREFACE

TO

THE FOURTH EDITION.

THE issue of a new edition affords us the opportunity of making some alterations and additions which the experience of ourselves or our successors at the Cavendish Laboratory has shewn to be desirable.

The development of physical science on the lines indicated by the principle of the conservation of energy has made more conspicuous the importance of experimental Dynamics as the basis of experimental physics, so that some considerable space has been given to that branch of the subject, and a good deal of attention has been devoted to the geometrical representation of rates of variation, especially as illustrating the determination of the velocity and acceleration of a body the position of which is known for successive instants of time. Geometrical representation has, indeed, been kept in view throughout.

The advances that have been made in the sciences of magnetism and electro-magnetism have also necessitated some considerable additions. The chapter on magnetism

has been enlarged, and a chapter on electro-magnetic induction has been added.

It has been thought better not to disturb the numbering of the sections, and the new sections have therefore been separately numbered A–Z and Γ to Θ.

In the preparation of this edition we are greatly indebted both to Mr. H. F. Newall, who was demonstrator when the apparatus for many of the new sections was first set up, and also especially to Mr. G. F. C. Searle, of Peterhouse, upon whose version of the Laboratory MSS. the text of many of the new sections depends. Mr. Searle, besides contributing the section on the dynamical equivalent of heat, has also been good enough to revise the whole of the proof sheets and to give us the advantage of his experience in the Laboratory by making numerous valuable suggestions.

Many of the original drawings for the figures were made for us by Mr. Hayles, the Lecture Assistant at the Laboratory.

R. T. GLAZEBROOK.
W. N. SHAW.

January 6, 1893.

PREFACE.

THIS book is intended for the assistance of Students and Teachers in Physical Laboratories. The absence of any book covering the same ground made it necessary for us, in conducting the large elementary classes in Practical Physics at the Cavendish Laboratory, to write out in MS. books the practical details of the different experiments. The increase in the number of well-equipped Physical Laboratories has doubtless placed many teachers in the same position as we ourselves were in before these books were compiled; we have therefore collected together the manuscript notes in the present volume, and have added such general explanations as seemed necessary.

In offering these descriptions of experiments for publication we are met at the outset by a difficulty which may prove serious. The descriptions, in order to be precise, must refer to particular forms of instruments, and may therefore be to a certain extent inapplicable to other instruments of the same kind but with some difference, perhaps in the arrangement for adjustment, perhaps in the method of graduation. Spherometers, spectrometers, and kathetometers are instruments with which this difficulty is particularly likely to occur. With considerable diffidence we have thought it best to adhere to the precise descriptions referring

to instruments in use in our own Laboratory, trusting that the necessity for adaptation to corresponding instruments used elsewhere will not seriously impair the usefulness of the book. Many of the experiments, however, which we have selected for description require only very simple apparatus, a good deal of which has in our case been constructed in the Laboratory itself. We owe much to Mr. G. Gordon, the Mechanical Assistant at the Cavendish Laboratory, for his ingenuity and skill in this respect.

Our general aim in the book has been to place before the reader a description of a course of experiments which shall not only enable him to obtain a practical acquaintance with methods of measurement, but also as far as possible illustrate the more important principles of the various subjects. We have not as a rule attempted verbal explanations of the principles, but have trusted to the ordinary physical text-books to supply the theoretical parts necessary for understanding the subject; but whenever we have not been able to call to mind passages in the text-books sufficiently explicit to serve as introductions to the actual measurements, we have either given references to standard works or have endeavoured to supply the necessary information, so that a student might not be asked to attempt an experiment without at least being in a position to find a satisfactory explanation of its method and principles. In following out this plan we have found it necessary to interpolate a considerable amount of more theoretical information. The theory of the balance has been given in a more complete form than is usual in mechanical text-books; the introductions to the measurement of fluid pressure, thermometry, and calorimetry have been inserted in order to accentuate certain important practical points which, as a rule, are only briefly touched upon;

while the chapter on hygrometry is intended as a complete elementary account of the subject. We have, moreover, found it necessary to adopt an entirely different style in those chapters which treat of magnetism and electricity. These subjects, regarded from the point of view of the practical measurement of magnetic and electric quantities, present a somewhat different aspect from that generally taken. We have accordingly given an outline of the general theory of these subjects as developed on the lines indicated by the electro-magnetic system of measurement, and the arrangement of the experiments is intended, as far as possible, to illustrate the successive steps in the development. The limits of the space at our disposal have compelled us to be as concise as possible; we have, therefore, been unable to illustrate the theory as amply as we could have wished. We hope, however, that we have been successful in the endeavour to avoid sacrificing clearness to brevity.

We have made no attempt to give anything like a complete list of the experiments that may be performed with the apparatus that is at the present day regarded as the ordinary equipment of a Physical Laboratory. We have selected a few—in our judgment the most typical—experiments in each subject, and our aim has been to enable the student to make use of his practical work to obtain a clearer and more real insight into the principles of the subjects. With but few exceptions, the experiments selected are of an elementary character; they include those which have formed for the past three years our course of practical physics for the students preparing for the first part of the Natural Sciences Tripos; to these we have now added some experiments on acoustics, on the measurement of wave-lengths,

and on polarisation and colours. Most of the students have found it possible to acquire familiarity with the contents of such a course during a period of instruction lasting over two academical terms.

The manner in which the subjects are divided requires perhaps a word of explanation. In conducting a class including a large number of students, it is essential that a teacher should know how many different students he can accommodate at once. This is evidently determined by the number of independent groups of apparatus which the Laboratory can furnish. It is, of course, not unusual for an instrument, such as a spectrometer, an optical bench, or Wheatstone bridge, to be capable of arrangement for working a considerable number of different experiments; but this is evidently of no assistance when the simultaneous accommodation of a number of students is aimed at. For practical teaching purposes, therefore, it is an obvious advantage to divide the subject with direct reference to the apparatus required for performing the different experiments. We have endeavoured to carry out this idea by dividing the chapters into what, for want of a more suitable name, we have called 'sections,' which are numbered continuously throughout the book, and are indicated by black type headings. Each section requires a certain group of apparatus, and the teacher knows that that apparatus is not further available when he has assigned the section to a particular student. The different experiments for which the same apparatus can be employed are grouped together in the same section, and indicated by italic headings.

The proof-sheets of the book have been in use during the past year, in the place of the original MS. books, in the following manner:—The sheets, divided into the sections

above mentioned, have been pasted into MS. books, the remaining pages being available for entering the results obtained by the students. The apparatus referred to in each book is grouped together on one of the several tables in one large room. The students are generally arranged in pairs, and before each day's work the demonstrator in charge assigns to each pair of students one experiment—that is, one section of the book. A list shewing the names of the students and the experiment assigned to each is hung up in the Laboratory, so that each member of the class can know the section at which he is to work. He is then set before the necessary apparatus with the MS. book to assist him; if he meets with any difficulty it is explained by the demonstrator in charge. The results are entered in the books in the form indicated for the several experiments. After the class is over the books are collected and the entries examined by the demonstrators. If the results and working are correct a new section is assigned to the student for the next time; if they are not so, a note of the fact is made in the class list, and the student's attention called to it, and, if necessary, he repeats the experiment. The list of sections assigned to the different students is now completed early in the day before that on which the class meets, and it is hoped that the publication of the description of the experiment will enable the student to make himself acquainted beforehand with the details of his day's work.

Adopting this plan, we have found that two demonstrators can efficiently manage two classes on the same day, one in the morning, the other in the afternoon, each containing from twenty-five to thirty students. The students have hitherto been usually grouped in pairs, in consequence of the want of space and apparatus. Although this plan

has some advantages, it is, we think, on the whole, undesirable.

We have given a form for entering results at the end of each section, as we have found it an extremely convenient, if not indispensable, arrangement in our own case. The numerical results appended as examples are taken, with very few exceptions, from the MS. books referred to above. They may be found useful, as indicating the degree of accuracy that is to be expected from the various experimental methods by which they are obtained.

In compiling a book which is mainly the result of Laboratory experience, the authors are indebted to friends and fellow-workers even to an extent beyond their own knowledge. We would gladly acknowledge a large number of valuable hints and suggestions. Many of the useful contrivances that facilitate the general success of a Laboratory in which a large class works, we owe to the Physical Laboratory of Berlin; some of them we have described in the pages that follow.

For a number of valuable suggestions and ideas we are especially indebted to the kindness of Lord Rayleigh, who has also in many other ways afforded us facilities for the development of the plans and methods of teaching explained above. Mr. J. H. Randell, of Pembroke College, and Mr. H. M. Elder, of Trinity College, have placed us under an obligation, which we are glad to acknowledge, by reading the proof-sheets while the work was passing through the press. Mr. Elder has also kindly assisted us by photographing the verniers which are represented in the frontispiece.

R. T. GLAZEBROOK.
W. N. SHAW.

CAVENDISH LABORATORY:
December 1, 1884.

CONTENTS.

CHAPTER I.

PHYSICAL MEASUREMENTS.

PAGE

Direct and indirect Method of Measurement 1
Indirect Measurements reducible to Determinations of Length and Mass 4
Origin of the Similarity of Observations of Different Quantities . 7

CHAPTER II.

UNITS OF MEASUREMENT.

Method of expressing a Physical Quantity 9
Arbitrary and Absolute Units 10
Absolute Units 13
Fundamental Units and Derived Units 17
Absolute Systems of Units 17
The C.G.S. System 21
Arbitrary Units at present employed 22
Changes from one Absolute System of Units to another. Dimensional Equations 24
Conversion of Quantities expressed in Arbitrary Units . . . 28

CHAPTER III.

PHYSICAL ARITHMETIC.

Approximate Measurements 30
Errors and Corrections 31
Mean of Observations 32
Possible Accuracy of Measurement of different Quantities . . 35
Arithmetical Manipulation of Approximate Values 36
Facilitation of Arithmetical Calculation by means of Tables. Interpolation 40

PAGE

Algebraical Approximation—Approximate Formulæ—Introduction of small Corrections 41
Application of Approximate Formulæ to the Calculation of the Effect of Errors of Observation 44
Graphical Methods 49
The Slide Rule 51

CHAPTER IV.

MEASUREMENT OF THE MORE SIMPLE QUANTITIES.

SECTION

LENGTH MEASUREMENTS 59
1. The Calipers 59
2. The Beam-Compass 63
3. The Screw-Guage 66
4. The Spherometer 68
5. The Reading Microscope—Measurement of a Base-Line . 73
A. The Kathetometer Microscope 77
6. The Kathetometer 78
Adjustments 79
Method of Observation 83
MEASUREMENT OF AREAS 85
7. Simpler Methods of measuring Areas of Plane Figures . . 85
Orthogonal Projection 87
8. Determination of the Area of the Cross-section of a Cylindrical Tube—Calibration of a Tube 89
MEASUREMENT OF VOLUMES 92
9. Determination of Volumes by Weighing 92
10. Testing the Accuracy of the Graduation of a Burette . . 93
MEASUREMENT OF ANGLES 94
MEASUREMENT OF SOLID ANGLES 94
MEASUREMENTS OF TIME 96
11. Rating a Watch by means of a Seconds-Clock . . . 97

CHAPTER V.

MEASUREMENT OF MASS AND DETERMINATION OF SPECIFIC GRAVITIES.

12. The Balance 99
General Considerations 99
The Sensitiveness of a Balance 100

SECTION PAGE

The Adjustment of a Balance 103

Practical Details of Manipulation—Method of Oscillations 107

13. Testing the Adjustments of a Balance 114

Determination of the Ratio of the Arms of a Balance and of the true Mass of a Body when the Arms of the Balance are unequal 116

Comparison of the Masses of the Scale Pans . . 117

14. Correction of Weighings for the Buoyancy of the Air . . 119

DENSITIES AND SPECIFIC GRAVITIES—*Definitions* . 121

15. The Hydrostatic Balance 123

Determination of the Specific Gravity of a Solid heavier than Water 123

Determination of the Specific Gravity of a Solid lighter than Water 125

Determination of the Specific Gravity of a Liquid . 127

16. The Specific Gravity Bottle 128

Determination of the Specific Gravity of small Fragments of a Solid 128

Determination of the Specific Gravity of a Powder . 132

Determination of the Specific Gravity of a Liquid . 132

17. Nicholson's Hydrometer 133

Determination of the Specific Gravity of a Solid . 133

Determination of the Specific Gravity of a Liquid . 135

18. Jolly's Balance 136

Determination of the Mass and Specific Gravity of a small Solid Body 137

Determination of the Specific Gravity of a Liquid . 138

19. The Common Hydrometer 139

Method of comparing the Densities of two Liquids by the Aid of the Kathetometer 141

CHAPTER V*.

MEASUREMENT OF VELOCITY AND ACCELERATION.

B. Measurement of the Velocity of a Pendulum 144

C. Tracing the Curve described by a Falling Body and the Character of its Downward Acceleration 148

CHAPTER VI.

MECHANICS OF SOLIDS.

SECTION		PAGE
20.	The Pendulum	152
	Determination of the Acceleration of Gravity by Pendulum Observations	152
	Comparison of the Times of Vibration of two Pendulums—Method of Coincidences—Kater's Pendulum	155
21.	Atwood's Machine	160
D.	The Fly-wheel	166
	Moment of Inertia	166
E.	Pendulum of any shape	171
F.	Ballistic Pendulum—Measurement of Moment of Momentum and of Momentum	174
	Measurement of Times of Oscillation	180
	Measurement of First Swing	180
G.	Funicular Polygon—Graphic Method of Comparing Forces	182
	SUMMARY OF THE GENERAL THEORY OF ELASTICITY	185
22.	Young's Modulus	187
	Modulus of Torsion	190
23.	Maxwell's Vibration Needle	191
	Observation of the Time of Vibration	193
	Calculation of the Alteration of Moment of Inertia	195

CHAPTER VII.

MECHANICS OF LIQUIDS AND GASES.

	Measurement of Fluid Pressure	197
24.	The Mercu Barometer	198
	Se. o and reading the Barometer	199
	Correction of the Observed Height for Temperature, &c.	200
25.	The Aneroid Barometer	202
	Measurement of Heights	203
26.	The Volumenometer	205
	Verification of Boyle's Law	206
	Determination of the Density of a Solid	209
H.	Capillarity	210
	Measurement of the Surface Tension of a Liquid	210
I.	Worthington's Capillary Multiplier	215

CHAPTER VIII.

ACOUSTICS.

SECTION		PAGE
	Definitions, &c.	218
27.	Comparison of the Pitch of Tuning-forks—Adjustment of two Forks to Unison	219
28.	The Siren	222
29.	Determination of the Velocity of Sound in Air by Measurement of the Length of a Resonance Tube corresponding to a given Fork	226
30.	Verification of the Laws of Vibration of Strings—Determination of the Absolute Pitch of a Note by the Monochord	229
31	Determination of the Wave-Length of a high Note in Air by means of a Sensitive Flame	234

CHAPTER IX.

THERMOMETRY AND EXPANSION.

	Measurement of Temperature	237
32.	Construction of a Water Thermometer	244
33.	Thermometer Testing	247
34.	Determination of the Boiling Point of a Liquid	250
35.	Determination of the Fusing Point of a Solid	251
K.	Effects of Dissolved Salts on the Freezing Point	253
	COEFFICIENTS OF EXPANSION	255
36.	Determination of the Coefficient of Linear Expansion of a Rod	257
37.	The Weight Thermometer	259
38.	The Constant Volume Air Thermometer	265
L.	The Constant Pressure Air Thermometer	268

CHAPTER X.

CALORIMETRY.

39.	The Method of Mixture	272
	Determination of the Specific Heat of a Solid	272
	Determination of the Specific Heat of a Liquid	278
	Determination of the Latent Heat of Water	279
	Determination of the Latent Heat of Steam	281

SECTION PAGE

40. The Method of Cooling 285
M. Method of Cooling—Graphic Method of Calculation . 288
N. Determination of the Mechanical Equivalent of Heat . . 290

CHAPTER XI.

TENSION OF VAPOUR AND HYGROMETRY.

41. Dalton's Experiment on the Pressure of Mixed Gases and Vapours 294
HYGROMETRY 299
42. The Chemical Method of determining the Density of Aqueous Vapour in the Air 301
43. Dines's Hygrometer—The Wet and Dry Bulb Thermometers 306
44. Regnault's Hygrometer 309

CHAPTER XII.

PHOTOMETRY.

45. Bunsen's Photometer 312
46. Rumford's Photometer 316

CHAPTER XIII.

REFLEXION AND REFRACTION—MIRRORS AND LENSES.

47. Verification of the Law of Reflexion of Light . . . 318
48. The Sextant 322
O. Refraction of Light through a Plate and through a Prism . 328
Verification of the Law of Refraction . . . 329
Caustic Curve by Refraction 330
To find the Refractive Index 331
Verification of the Law of Reflexion . . . 332
Refraction through a Prism 333
OPTICAL MEASUREMENTS 335
49. Measurement of the Focal Length of a Concave Mirror . . 337
50. Measurement of the Radius of Curvature of a Reflecting Surface by Reflexion 339
Measurement of Focal Lengths of Lenses 343
51. Measurement of the Focal Length of a Convex Lens (First Method) 343
52. Measurement of the Focal Length of a Convex Lens (Second Method) 344

SECTION PAGE

53. Measurement of the Focal Length of a Convex Lens (Third Method) 345
54. Measurement of the Focal Length of a Concave Lens . . 350
P. Focal Lengths—Additional Methods of Measurement . 352
55. Focal Lines 354
Q. Focal Lines formed by a Prism 356
Magnifying Powers of Optical Instruments 358
56. Measurement of the Magnifying Power of a Telescope (First Method) 359
57. Measurement of the Magnifying Power of a Telescope (Second Method) 361
58. Measurement of the Magnifying Power of a Lens or of a Microscope 363
59. The Testing of Plane Surfaces 367

CHAPTER XIV.

SPECTRA, REFRACTIVE INDICES AND WAVE-LENGTHS.

Pure Spectra 375
60. The Spectroscope 377
Mapping a Spectrum 377
Comparison of Spectra 381
Refractive Indices 382
61. Measurement of the Index of Refraction of a Plate by means of a Microscope 383
62. The Spectrometer 385
The Adjustment of a Spectrometer 386
Measurements with the Spectrometer 388
(1) *Verification of the Law of Reflexion* . . . 388
(2) *Measurement of the Angle of a Prism* . . 388
(3) *Measurement of the Refractive Index of a Prism (First Method)* 389
Measurement of the Refractive Index of a Prism (Second Method) 393
(4) *Measurement of the Wave-Length of Light by means of a Diffraction Grating* 395
63. The Optical Bench 398
Measurement of the Wave-Length of Light by means of Fresnel's Bi-prism 399
Diffraction Experiments 404

CHAPTER XV.

POLARISED LIGHT.

SECTION		PAGE
	On the Determination of the Position of the Plane of Polarisation	405
64.	The Bi-quartz	407
65.	Shadow Polarimeters	412

CHAPTER XVI.

COLOUR VISION.

66.	The Colour Top	417
67.	The Spectro-Photometer	421
68.	The Colour Box	425
R.	Colour Photometry	427

CHAPTER XVII.

MAGNETISM.

	Properties of Magnets	433
	Definitions	434
	Magnetic Potential	439
	Forces on a Magnet in a Uniform Field	441
	Magnetic Moment of a Magnet	442
	Potential due to a Solenoidal Magnet	444
	Force due to a Solenoidal Magnet	445
	Action of one Solenoidal Magnet on another	447
	Measurement of Magnetic Force	450
	Magnetic Induction	452
69.	Experiments with Magnets	453
	(*a*) *Magnetisation of a Steel Bar*	453
	(*b*) *Comparison of the Magnetic Moment of the same Magnet after different Methods of Treatment, or of two different Magnets*	456
	(*c*) *Comparison of the Strengths of different Magnetic Fields of approximately Uniform Intensity*	459
	(*d*) *Measurement of the Magnetic Moment of a Magnet and of the Strength of the Field in which it hangs*	459
	(*e*) *Determination of the Magnetic Moment of a Magnet of any shape*	461

SECTION PAGE

(*f*) *Determination of the Direction of the Earth's Horizontal Force* 461

(*g*) *Experiments on two Magnets—Comparison of Magnetic Moments* 464

S. Comparison of Gravitational and Magnetic Forces . . 467

T. Gauss' Verification of the Law of Magnetic Force . . 470

U. Magnetic Induction due to the Earth 474

70. Exploration of the Magnetic Field due to a given Magnetic Distribution 476

V. Magnetic Induction due to Iron 479

Determination of the Magnetic Moment of a Soft Iron Rod 481

Determination of the Magnetic Susceptibility of a Soft Iron Rod 482

Magnetic Cycles—Hysterisis 484

CHAPTER XVIII.

ELECTRICITY—DEFINITIONS AND EXPLANATIONS OF ELECTRICAL TERMS.

Conductors and Non-conductors 488

Resultant Electrical Force 488

Electromotive Force 489

Electrical Potential. 489

Current of Electricity. 492

C.G.S. Absolute Unit of Current 494

Sine and Tangent Galvanometers 496

CHAPTER XIX.

EXPERIMENTS IN THE FUNDAMENTAL PROPERTIES OF ELECTRIC CURRENTS—MEASUREMENT OF ELECTRIC CURRENT AND ELECTROMOTIVE FORCE.

71. Absolute Measure of the Current in a Wire . . . 497

GALVANOMETERS 501

Galvanometer Constant 503

Reduction Factor of a Galvanometer 507

Sensitiveness of a Galvanometer 508

The Adjustment of a Reflecting Galvanometer . . . 510

72. Determination of the Reduction Factor of a Galvanometer 511

SECTION PAGE

SECTION		PAGE
	Electrolysis	512
	Definition of Electro-chemical Equivalent	512
73.	Faraday's Law—Comparison of Electro-chemical Equivalents	517
74.	Joule's Law—Measurement of Electromotive Force	522

CHAPTER XX.

OHM'S LAW—COMPARISON OF ELECTRICAL RESISTANCES AND ELECTROMOTIVE FORCES.

	Definition of Electrical Resistance	527
	Series and Multiple Arc	528
	Shunts	530
	Absolute Unit of Resistance	531
	Standards of Resistance	532
	Resistance Boxes	533
	Relation between the Resistance and Dimensions of a Wire of given Material	534
	Specific Resistance	535
75.	Comparison of Electrical Resistances	536
76.	Comparison of Electromotive Forces	541
77.	Wheatstone's Bridge	543
	Measurement of Resistance	549
	Measurement of a Galvanometer Resistance—Thomson's Method	551
	Measurement of a Battery Resistance—Mance's Method	553
78.	The British Association Wire Bridge	557
	Measurement of Electrical Resistance	557
79.	Carey Foster's Method of Comparing Resistances	561
	Calibration of a Bridge-Wire	566
80.	Poggendorff's Method for the Comparison of Electromotive Forces—Latimer Clark's Potentiometer	567
W.	The Clark Cell	572
	To Set up a Clark Cell	572
	The Clark Cell as a Standard of Electromotive Force	577
	Measuring a Current	578
X.	The Silver Voltameter	579
	Method of making a Measurement	580

CHAPTER XXI.

GALVANOMETRIC MEASUREMENT OF A QUANTITY OF ELECTRICITY.

SECTION PAGE

Theory of the Method 582

Relation between the Quantity of Electricity which passes through a Galvanometer and the initial Angular Velocity produced in the Needle . . 582

Work done in turning the Magnetic Needle through a given Angle 583

Electrical Accumulators or Condensers 586

Definition of the Capacity of a Condenser . . . 587

The Unit of Capacity. 587

On the Form of Galvanometer suitable for the Comparison of Capacities 588

81. Comparison of the Capacities of two Condensers . . . 589

(1) *Approximate Method* 589

(2) *Null Method* 592

82. Measurement in Absolute Measure of the Capacity of a Condenser 595

CHAPTER XXII.

ELECTROMAGNETIC INDUCTION.

Propositions for determining the Electromagnetic Induction 598

Y. Experimental Laws of Electromagnetic Induction . . 604

(1) *The quantity of Electricity traversing the Secondary is directly proportional to the Primary Current.* 605

(2) *The quantity of Electricity traversing the Secondary is inversely proportional to the whole resistance of the Secondary* . . . 608

(3) *The quantity of Electricity traversing the Secondary depends on the mutual position of the two Circuits* 609

(4) *Examination of the effect of the Medium near the Coils*. 610

(5) *Induction due to the Motion of a Magnet* . . 611

Z. Comparison of a Coefficient of Mutual Induction and the Product of a Resistance and a Time 613

SECTION PAGE

Γ. Comparison of a Coefficient of Mutual Induction with the Capacity of a Condenser and the Product of two Resistances 616

Δ. Comparison of two Coefficients of Mutual Induction . . 618

Θ. The Earth Inductor 624

Determination of the Dip by the Earth Inductor . 626

Measurement of the Magnetic Induction at any point of a Magnetic Field by means of an Induced Current 626

INDEX 629

PRACTICAL PHYSICS.

CHAPTER I.

PHYSICAL MEASUREMENTS.

THE greater number of the physical experiments of the present day and the whole of those described in this book consist in, or involve, measurement in some form or other. Now a physical measurement—a measurement, that is to say, of a physical *quantity*—consists essentially in the comparison of the quantity to be measured with a unit quantity of the same kind. By comparison we mean here the determination of the number of times that the unit is contained in the quantity measured, and the number in question may be an integer or a fraction, or be composed of an integral part and a fractional part. In one sense the unit quantity must remain from the nature of the case perfectly arbitrary, although by general agreement of scientific men the choice of the unit quantities may be determined in accordance with certain general principles which, once accepted for a series of units, establish certain relations between the units thus chosen, so that they form members of a system known as an absolute system of units. For example, to measure energy we must take as our unit the energy of some body under certain conditions, but when we agree that it shall always be the energy of a body on which a unit force has acted through unit space, our choice has been exercised, and the unit of energy is no longer arbitrary, but

B

defined, as soon as the units of force and space are agreed upon; we have thus substituted the right of selection of the general principle for the right of selection of the particular unit.

We see, then, that the number of physical units is at least as great as the number of physical quantities to be measured, and indeed under different circumstances several different units may be used for the measurement of the same quantity. The physical quantities may be suggested by or related to phenomena grouped under the different headings of Mechanics, Hydro-mechanics, Heat, Acoustics, Light, Electricity or Magnetism, some being related to phenomena on the common ground of two or more such subjects. We must expect, therefore, to have to deal with a very large number of physical quantities and a correspondingly large number of units.

The process of comparing a quantity with its unit—the measurement of the quantity—may be either direct or indirect, although the direct method is available perhaps in one class of measurements only, namely, in that of length measurements. This, however, occurs so frequently in the different physical experiments, as scale readings for lengths and heights, circle readings for angles, scale readings for galvanometer deflections, and so on, that it will be well to consider it carefully.

The process consists in laying off standards against the length to be measured. The unit, or standard length, in this case is the distance under certain conditions of temperature between two marks on a bar kept in the Standards Office of the Board of Trade. This, of course, cannot be moved from place to place, but a portable bar may be obtained and compared with the standard, the difference between the two being expressed as a fraction of the standard. Then we may apply the portable bar to the length to be measured, determining the number of times the length of the bar is contained in the given length, with due allowance for temperature, and

thus express the given length in terms of the standard by means of successive direct applications of the fundamental method of measurement. Such a bar is known as a scale or rule. In case the given length does not contain the length of the bar an exact number of times, we must be able to determine the excess as a fraction of the length of the bar; for this purpose the length of the bar is divided by transverse marks into a number of equal parts—say 10—each of these again into 10 equal parts, and perhaps each of these still further into 10 equal parts. Each of these smallest parts will then be $\frac{1}{1000}$ of the bar, and we can thus determine the number of tenths, hundredths, and thousandths of the bar contained in the excess. But the end of the length to be measured may still lie between two consecutive thousandths, and we may wish to carry the comparison to a still greater accuracy, although the divisions may be now so small that we cannot further subdivide by marks. We must adopt some different plan of estimating the fraction of the thousandth. The one most usually employed is that of the 'vernier.' An account of this method of increasing the accuracy of length measurements is given in § 1.

This is, as already stated, the only instance usually occurring in practice of a direct comparison of a quantity with its unit. The method of determining the mass of a body by double weighing (see § 13), in which we determine the number of units and fractions of a unit of mass, which together produce the same effect as was previously produced by the mass to be measured, approaches very nearly to a direct comparison. And the strictly analogous method of substitution of units and fractions of a unit of electrical resistance, until their effect is equal to that previously produced by the resistance to be measured, may also be mentioned, as well as the measurement of time by the method of coincidences (§ 20).

But in the great majority of cases the comparison is far from direct. The usual method of proceeding is as follows:—

An experiment is made the result of which depends upon the relative magnitude of the quantity and its unit, and the numerical relation is then deduced by a train of reasoning which may, indeed, be strictly or only approximately accurate. In the measurement, for instance, of a resistance by Wheatstone's Bridge, the method consists in arranging the unknown resistance with three standard resistances so chosen that under certain conditions no disturbance of a galvanometer is produced. We can then determine the resistance by reasoning based on Ohm's law and certain properties of electric currents. These indirect methods of comparison do not always afford perfectly satisfactory methods of measurement, though they are sometimes the only ones available. It is with these indirect methods of comparing quantities with their units that we shall be mostly concerned in the experiments detailed in the present work.

We may mention in passing that the consideration of the experimental basis of the reasoning on which the various methods depend forms a very valuable exercise for the student. As an example, let us consider the determination of a quantity of heat by the method of mixture (§ 39). It is usual in the rougher experiments to assume (1) that the heat absorbed by water is proportional to the rise of temperature ; (2) that no heat is lost from the vessel or calorimeter; (3) that in case two thermometers are used, their indications are identical for the same temperature. All these three points may be considered with advantage by those who wish to get clear ideas about the measurement of heat.

Let us now turn our attention to the *actual* process in which the measurement of the various physical quantities consists. A little consideration will show that, whether the quantity be mechanical, optical, acoustical, magnetic or electric, the process really and truly resolves itself into measuring certain lengths, or masses.[1] Some examples will

[1] *See* articles by Clifford and Maxwell: *Scientific Apparatus. Handbook to the Special Loan Collection*, 1876, p. 55.

make this sufficiently clear. Angles are measured by readings of *length* along certain arcs; the ordinary measurement of time is the reading of an *angle* on a clock face or the *space* described by a revolving drum; force is measured by *longitudinal extension* of an elastic body or by weighing; pressure by reading the *height* of a column of fluid supported by it; differences of temperature by the *lengths* of a thermometer scale passed over by a mercury thread; heat by measuring a *mass* and a difference of temperature; luminous intensity by the *distances* of certain screens and sources of light; electric currents by the *angular deflection* of a galvanometer needle; coefficients of electro-magnetic induction also by the *angular throw* of a galvanometer needle.

Again, a consideration of the definitions of the various physical quantities leads in the same direction. Each physical quantity has been defined in some way for the purpose of its measurement, and the definition is insufficient and practically useless unless it indicates the basis upon which the measurement of the quantity depends. A definition of force, for instance, is for the physicist a mere arrangement of words unless it states that a force is measured by the quantity of momentum it generates in the unit of time; and in the same way, while it may be interesting to know that 'electrical resistance of a body is the opposition it offers to the passage of an electric current,' yet we have not made much progress towards understanding the precise meaning intended to be conveyed by the words 'a resistance of 10 ohms,' until we have acknowledged that the ratio of the electromotive force between two points of a conductor to the current passing between those points is a quantity which is constant for the same conductor in the same physical state, and is called and is the 'resistance' of the conductor; and, further, this only conveys a definite meaning to our minds when we understand the bases of measurement suggested by the definitions of electromotive force and electric current.

When the quantity is once defined, we may possibly be able to choose a unit and make a direct comparison; but such a method is very seldom, if ever, adopted, and the measurements really made in any experiment are often suggested by the definitions of the quantities measured.

The following table gives some instances of indirect methods of measurement suggested by the definitions of the quantities to be measured. The student may consult the descriptions of the actual processes of measurement detailed in subsequent chapters:—

Name of quantity measured	Measurement actually made
MECHANICS.	
Area	Length (§ 1–6).
Volume	Length.
Velocity	Length and time.
Acceleration	Velocity and time.
Force	Mass and acceleration, or extension of spring.
Work	Force and length.
Energy	Work, or mass and velocity.
Fluid pressure (in absolute units)	Force and area (§ 24–26).
Coefficients of elasticity	Stress and strain, *i.e.* force, and length or angle (§§ 22, 23).
SOUND.	
Velocity	Length and time (§ 29).
Pitch	Time (§ 28).
HEAT.	
Temperature	Length (§ 32).
Quantity of heat	Temperature and mass (§ 39).
Conductivity	Temperature, heat, length, and time.
LIGHT.	
Index of refraction	Angles (§ 62).
Intensity	Length (§ 45).
MAGNETISM.	
Quantity of magnetism	Force and length (§ 69).
Intensity of field	Force and quantity of magnetism (§ 69).
Magnetic moment	Quantity of magnetism and length (§ 69).

Name of quantity measured	Measurements actually made
ELECTRICITY.	
Electric current . .	Quantity of magnetism, force, and length (§ 71).
Quantity of Electricity .	Current and time (§ 72).
Electromotive force .	Quantity of electricity and work (§ 74).
Resistance . . .	Electric current and E. M. F. (§ 75).
Electro-chemical equivalent.	Mass and quantity of electricity (§ 72).

The quantities given in the second column of the table are often such as are not measured directly, but the basis of measurement has, in each case, already been given higher up in the table. If the measurement of any quantity be reduced to its ultimate form it will be found to consist always in measurements of length or mass.[1] The measurement of time by counting 'ticks' may seem at first sight an exception to this statement, but further consideration will shew that it, also, depends ultimately upon length measurement.

As far as the apparatus for making the actual observations is concerned, many experiments, belonging to different subjects, often bear a striking similarity. The observing apparatus used in a determination of a coefficient of torsion, the earth's horizontal magnetic intensity, and a coefficient of electro-magnetic induction, are practically identical in each case, namely, a heavy swinging needle and a telescope and scale; the difference between the experiments consists in the difference in the origin of the forces which set the moving needle in motion. Many similar instances might be quoted. Maxwell, in the work already referred to ('Scientific Apparatus,' p. 15), has laid down the grounds on which this analogy between the experiments in different branches of the subject is based. 'All the physical sciences relate to the passage of energy under its various forms from one body to another,' and, accordingly,

[1] The measurement of mass may frequently be resolved into that of length. The method of double weighing, however, is a fundamental measurement *sui generis.*

all instruments, or arrangements of apparatus, possess the following functions :—

'1. The Source of energy. The energy involved in the phenomenon we are studying is not, of course, produced from nothing, but enters the apparatus at a particular place which we may call the Source.

'2. The channels or distributors of energy, which carry it to the places where it is required to do work.

'3. The restraints which prevent it from doing work when it is not required.

'4. The reservoirs in which energy is stored up when it is not required.

'5. Apparatus for allowing superfluous energy to escape.

'6. Regulators for equalising the rate at which work is done.

'7. Indicators or movable pieces which are acted upon by the forces under investigation.

'8. Fixed scales on which the position of the indicator is read off.'

The various experiments differ in respect of the functions included under the first six headings, while those under the headings numbered 7 and 8 will be much the same for all instruments, and these are the parts with which the actual observations for measurement are made. In some experiments, as in optical measurements, the observations are simply those of length and angles, and we do not compare forces at all, the whole of the measurements being ultimately length measurements. In others we are concerned with forces either mechanical, hydrostatic, electric or magnetic, and an experiment consists in observations of the magnitude of these forces under certain conditions; while, again, the ultimate measurements will be measurements of length and of mass. In all these experiments, then, we find a foundation in the fundamental principles of the measurement of length and of the measurements of force and mass. The knowledge of the first involves an acquaintance with

some of the elementary properties of space, and to understand the latter we must have some acquaintance with the properties of matter, the medium by which we are able to realise the existence of force and energy, and with the properties of motion, since all energy is more or less connected with the motion of matter. We cannot, then, do better than urge those who intend making physical experiments to begin by obtaining a sound knowledge of those principles of dynamics, which are included in an elementary account of the science of matter and motion. The opportunity has been laid before them by one—to whom, indeed, many other debts of gratitude are owed by the authors of this work—who was well known as being foremost in scientific book-writing, as well as a great master of the subject. For us it will be sufficient to refer to Maxwell's work on 'Matter and Motion' as the model of what an introduction to the study of physics should be.

CHAPTER II.

UNITS OF MEASUREMENT.

Method of Expressing a Physical Quantity.

IN considering how to express the result of a physical experiment undertaken with a view to measurement, two cases essentially different in character present themselves. In the first the result which we wish to express is a *concrete physical quantity*, and in the second it is merely the *ratio* of two physical quantities of the same kind, and is accordingly a *number*. It will be easier to fix our ideas on this point if we consider a particular example of each of these cases, instead of discussing the question in general terms. Consider, therefore, the difference in the expression of the result of two experiments, one to measure a quantity of heat and the second to measure a specific heat—the measurements

of a mass and a specific gravity might be contrasted in a perfectly similar manner—in the former the numerical value will be different for every different method employed to express quantities of heat; while in the latter the result, being a pure number, will be the same whatever plan of measuring quantities of heat may have been adopted in the course of the experiment, provided only that we have adhered throughout to the same plan, when once adopted. In the latter case, therefore, the number obtained is a complete expression of the result, while in the former the numerical value alone conveys no definite information. We can form no estimate of the magnitude of the quantity unless we know also the unit which has been employed. The complete expression, therefore, of a *physical quantity* as distinguished from a mere *ratio* consists of two parts: (1) the unit quantity employed, and (2) the numerical part expressing the number of times, whole or fractional, which the unit quantity is contained in the quantity measured. *The unit is a concrete quantity of the same kind as that in the expression of which it is used.*

If we represent a quantity by a symbol, that must likewise consist of two parts, one representing the numerical part and the other representing the concrete unit. A general form for the complete expression of a quantity may therefore be taken to be q [Q], where q represents the numerical part and [Q] the concrete unit. For instance, in representing a certain length we may say it is 5 [feet], when the numerical part of the expression is 5 and the unit [foot]. The number q is called the numerical measure of the quantity for the unit [Q].

Arbitrary and Absolute Units.

The method of measuring a quantity, q [Q], is thus resolved into two parts: (1) the selection of a suitable unit [Q], and (2) the determination of q, the number of times which this unit is contained in the quantity to be measured. The second part is a matter for experimental determination, and

has been considered in the preceding chapter. We proceed to consider the first part more closely.

The selection of [Q] is, and must be, entirely arbitrary—that is, at the discretion of the particular observer who is making the measurement. It is, however, generally wished by an observer that his numerical results should be understood and capable of verification by others who have not the advantage of using his apparatus, and to secure this he must be able so to define the unit he selects that it can be reproduced in other places and at other times, or compared with the units used by other observers. This tends to the general adoption on the part of scientific men of common standards of length, mass, and time, although agreement on this point is not quite so general as could be wished. There are, however, two well-recognised standards of length[1]: viz. (1) the British standard yard, which is the length at 62° F. between two marks on the gold plugs of a bronze bar in the Standards Office; and (2) the standard metre as kept in the French Archives, which is equivalent to 39·37079 British inches. Any observer in measuring a length adopts the one or the other as he pleases. All graduated instruments for measuring lengths have been compared either directly or indirectly with one of these standards. If great accuracy in length measurement is required a direct comparison must be obtained between the scale used and the standard. This can be done by sending the instrument to be used to the Standards Office of the Board of Trade.

There are likewise two well-recognised standards of mass, viz. (1) the British standard pound, a certain mass of platinum kept in the Standards Office; and (2) the kilogramme des Archives, a mass of platinum kept in the French Archives, originally selected as the mass of one thousandth part of a cubic metre of pure water at 4° C. One

[1] See Maxwell's *Heat*, chap. iv. The British Standards are now kept at the Standards Office at the Board of Trade, Westminster, in accordance with the 'Weights and Measures Act,' 1878.

or other of these standards, or a simple fraction or multiple of one of them, is generally selected as a unit in which to measure masses by any observer making mass measurements. The kilogramme and the pound were carefully compared by the late Professor W. H. Miller; one pound is equivalent to ·453593 kilogramme.

With respect to the unit of time there is no such divergence, as the second is generally adopted as the unit of time for scientific measurement. The second is $\frac{1}{86400}$ of the mean solar day, and is therefore easily reproducible as long as the mean solar day remains of its present length.

These units of length, mass, and time are perfectly arbitrary. We might in the same way, in order to measure any other physical quantity whatever, select arbitrarily a unit quantity of the same kind, and make use of it just as we select the standard pound as a unit of mass and use it. Thus to measure a force we might select a unit of force, say the force of gravity upon a particular body at a particular place, and express forces in terms of it. This is the gravitation method of measuring forces which is often adopted in practice. It is not quite so arbitrary as it might have been, for the body generally selected as being the body upon which, at Lat. 45°, gravity exerts the unit force is either the standard pound or the standard gramme, whereas some other body quite unrelated to the mass standards might have been chosen. In this respect the gallon, as a unit of measurement of volume, is a better example of arbitrariness. It contains ten pounds of water at a certain temperature.

We may mention here, as additional examples of arbitrary units, the degree as a unit of angular measurement, the thermometric degree as the unit of measurement of temperature, the calorie as a unit of quantity of heat, the standard atmosphere, or atmo, as a unit of measurement of fluid pressure, Snow Harris's unit jar for quantities of electricity, and the B.A. unit of electrical resistance

Absolute Units.

The difficulty, however, of obtaining an arbitrary standard which is sufficiently permanent to be reproducible makes this arbitrary method not always applicable. A fair example of this is in the case of measurement of electro-motive force,[1] for which no generally accepted arbitrary standard has yet been found, although it has been sought for very diligently. There are also other reasons which tend to make physicists select the units for a large number of quantities with a view to simplifying many of the numerical calculations in which the quantities occur, and thus the arbitrary choice of a unit for a particular quantity is directed by a principle of selection which makes it depend upon the units already selected for the measurement of other quantities. We thus get systems of units, such that when a certain number of fundamental units are selected, the choice of the rest follows from fixed principles. Such a system is called an 'absolute' system of units, and the units themselves are often called 'absolute,' although the term does not strictly apply to the individual units. We have still to explain the principles upon which absolute systems are founded.

Nearly all the quantitative physical laws express relations between the numerical measures of quantities, and the general form of relation is that the numerical measure of some quantity, Q, is proportional (either directly or inversely) to certain powers of the numerical measures of the quantities X, Y, Z . . . If $q, x, y, z, \ldots$ be the numerical measures of these quantities, then we may generalise the physical law, and express it algebraically thus: q is proportional to $x^\alpha, y^\beta, z^\gamma, \ldots$, or by the variation equation

$$q \propto x^\alpha . y^\beta . z^\gamma \ldots .$$

where α, β, γ may be either positive or negative, whole or fractional. The following instances will make our meaning clear:

[1] Since this was written, it has been shewn that the E.M.F. of a Latimer-Clark's cell is very nearly constant, and equal to 1·434 volt at 15° C. (See p. 572.)

(1.) The volumes of bodies of similar shape are proportional to the third power of their linear dimensions, or

$$v \propto l^3.$$

(2.) The rate of change of momentum is proportional to the impressed force, and takes place in the direction in which the force is impressed (Second Law of Motion), or

$$f \propto m\,a.$$

(3.) The pressure at any point of a heavy fluid is proportional to the depth of the point, the density of the fluid, and the intensity of gravity, or

$$p \propto h\,\rho\,g.$$

(4.) When work produces heat, the quantity of heat produced is directly proportional to the quantity of work expended (First Law of Thermo-dynamics), or

$$h \propto w.$$

(5.) The force acting upon a magnetic pole at the centre of a circular arc of wire in which a current is flowing, is directly proportional to the strength of the pole, the length of the wire, and the strength of the current, and inversely proportional to the square of the radius of the circle, or

$$f \propto \frac{\mu\,l\,c}{r^2},$$

and so on for all the experimental physical laws.

We may thus take the relation between the numerical measures—

$$q \propto x^\alpha y^\beta z^\gamma \ldots$$

to be the general form of the expression of an experimental law relating to physical quantities. This may be written in the form

$$q = k\,x^\alpha y^\beta z^\gamma \ldots \qquad . \qquad . \qquad (1)$$

when k is a 'constant.'

This equation, as we have already stated, expresses a

relation between the *numerical measures* of the quantities involved, and hence if one of the units of measurement is changed, the numerical measure of the *same actual quantity* will be changed in the inverse ratio, and the value of k will be thereby changed.

We may always determine the numerical value of k if we can substitute actual numbers for $q, x, y, z, \ldots$ in the equation (1).

For example, the gaseous laws may be expressed in words thus:—

'The pressure of a given mass of gas is directly proportional to the temperature measured from $-273°$ C., and inversely proportional to the volume,' or as a variation equation—

$$p \propto \frac{\theta}{v}$$

or

$$p = k\frac{\theta}{v}.$$

We may determine k for 1 gramme of a given gas, say hydrogen, from the consideration that 1 gramme of hydrogen, at a pressure of 760 mm. of mercury and at 0° C., occupies 11200 cc.

Substituting $p = 760$, $\theta = 273$, $v = 11200$, we get

$$k = \frac{760 \times 11200}{273} = 31180,$$

and hence

$$p = 31180\frac{\theta}{v} \quad . \quad . \quad . \quad (2).$$

Here p has been expressed in terms of the length of an equivalent column of mercury; and thus, if for v and θ we substitute in equation (2) the numerical measures of any volume and temperature respectively, we shall obtain the corresponding *pressure* of 1 gramme of hydrogen expressed in *millimetres of mercury*.

This, however, is not the standard method of expressing

a pressure; its standard expression is the force per unit of area. If we adopt the standard method we must substitute for p not 760, but $76 \times 13{\cdot}6 \times 981$, this being the number of units of force[1] in the weight of the above column of mercury of one square-centimetre section. We should then get for k a different value, viz.:—

$$k = \frac{1{,}014{,}000 \times 11200}{273} = 41500000,$$

so that

$$p = 41500000 \frac{\theta}{v} \quad . \quad . \quad . \quad (3),$$

and now substituting any values for the temperature and volume, we have the corresponding *pressure* of 1 gramme of hydrogen *expressed in units of force per square centimetre.*

Thus, in the general equation (1), the numerical value of k depends upon the units in which the related quantities are measured; or, in other words, we may assign any value we please to k by properly selecting the units in which the related quantities are measured.

It should be noticed that in the equation

$$q = k\, x^a y^b z^c \quad . \quad .$$

we only require to be able to select one of the units in order to make k what we please; thus x, y, z, . . . may be beyond our control, yet if we may give q any numerical value we wish, by selecting its unit, then k may be made to assume any value required. It need hardly be mentioned that it would be a very great convenience if k were made equal to unity. This can be done if we choose the proper unit in which to measure Q. Now, it very frequently happens that there is no other countervailing reason for selecting a different unit in which to measure Q, and our power of arbitrary selection of a unit for Q is thus exercised, not by selecting a particular quantity of the same kind as Q as unit,

[1] The units of force here used are dynes or C.G.S. units of force.

and holding to it however other quantities may be measured, but by agreeing that the choice of a unit for Q shall be determined by the previous selections of units for X, Y, Z, . . . together with the consideration that the quantity k shall be equal to unity.

Fundamental Units and Derived Units.

It is found that this principle, when fully carried out, leaves us free to choose arbitrarily *three* units, which are therefore called *fundamental units*, and that most of the other units employed in physical measurement can be defined with reference to the fundamental units by the consideration that the factor k in the equations connecting them shall be equal to unity. Units obtained in this way are called *derived units*, and all the derived units belong to an absolute system based on the three fundamental units.

Absolute Systems of Units.

Any three units (of which no one is derivable from the other two) may be selected as fundamental units. In those systems, however, at present in use, the units of length, mass, and time have been set aside as arbitrary fundamental units, and the various systems of absolute units differ only in regard to the particular units selected for the measurement of length, mass, and time. In the absolute system adopted by the British Association, the fundamental units selected are the centimetre, the gramme, and the second respectively, and the system is, for this reason, known as the C.G.S. system.

For magnetic surveying the British Government uses an absolute system based on the foot, grain, and second; and scientific men on the Continent frequently use a system based on the millimetre, milligramme, and second, as fundamental units. An attempt was also made, with partial success, to introduce into England a system of absolute units, based upon the foot, pound, and second as fundamental units.

DERIVATION OF UNITS ON THE C.G.S. SYSTEM.

Quantity	Physical law	Variation equation	Definition of derived unit	Derived unit	Dimensional equation [1]
Length, *l*. Mass, *m*. Time, *t*.	FUNDAMENTAL ARBITRARY	UNITS . .	. . . . .	Centimetre. cm. Gramme. gm. Second. sec.	
Area, A.	The areas of similar figures are proportional to the second power of their linear dimensions.	$A \propto l^2$	The area of a square the length of a side of which is 1 centimetre.	Sq. cm.	$[A]=[L]^2$.
Volume, V.	The volumes of similar solids are proportional to the third power of their linear dimensions.	$V \propto l^3$	The volume of a cube the length of a side of which is 1 centimetre.	C.c.	$[V]=[L]^3$.
Density, *d*.	The mass of a body is proportional to its volume and its density conjointly (Definition of Density).	$m \propto V d$	The density of a body of which the mass of 1 cubic centimetre is 1 gramme.	Gm. per c.c.	$[D]=[M][L]^{-3}$.
Velocity, *v*.	The space passed over by a body moving uniformly is proportional to the velocity and the time of passage.	$s \propto v t$	The velocity of a body moving so that it passes over 1 centimetre in a second.	Cm. per sec.	$[v]=[L][T]^{-1}$.
Acceleration, *a*.	The increase in the velocity of a body moving with a constant acceleration is proportional to the acceleration and the time during which the motion has been accelerated.	$v \propto a t$	The acceleration of a body whose velocity is increasing every second by 1 cm. per sec.	Cm. per sec. per sec.	$[a]=[L][T]^{-2}$.
Force, *f*.	The force which produces change of motion in a body is proportional to the rate of change of momentum produced (the rate of change of momentum being measured numerically by the product of the numerical measure of the mass of the body, and the numerical measure of the acceleration produced). (Second Law of Motion.)	$f \propto m a$	The force which acting upon 1 gramme produces per sec. an acceleration of 1 cm. per sec.	Dyne	$[F]=[M][L][T]^{-2}$.

[1] See page 24.

Work or energy, w.	The work done by a force is proportional to the force and to the distance through which it moves its point of application. The energy of a body is its capacity for doing work, measured by the work to which it is equivalent.	$w \propto fl$	The work done by a dyne when it has moved its point of application through 1 centimetre.	Erg.	$[W]=[M][L]^2[T]^{-2}$.
Pressure, (tension), p.	The force exerted by a fluid upon a given area is proportional to the area and to the pressure of the fluid at any point of the area, this pressure being supposed uniform over the area.	$f \propto p A$	The pressure exerted when the force on every sq. cm. of area is 1 dyne.	Dyne per sq. cm.	$[P]=[M][L]^{-1}[T]^{-2}$.
Volume elasticity, e.	The fractional diminution in volume of a fluid under increased pressure is proportional directly to the increase of pressure and inversely to the elasticity of the fluid.	$\frac{v_1-v_2}{v_1}=\frac{p_2-p_1}{e}$	The elasticity of a body such that an unit diminution of pressure would double the volume.		$[E]=[P]=[M][L]^{-1}[T]^{-2}$.
	MAGNETIC UNITS.				
Strength of magnetic pole, μ.	The force between two magnetic poles is proportional to the product of the strengths of the poles and inversely proportional to the square of the distance between them.	$f \propto \frac{\mu\mu'}{r^2}$	That pole which repels an equal and similar pole at a distance of 1 cm. with a force of 1 dyne.	C.G.S. unit magnetic pole.	$[\mu]=[M]^{\frac{1}{2}}[L]^{\frac{3}{2}}[T]^{-1}$.
Strength of a magnetic field, H.	A magnetic pole, when placed in a magnetic field, is acted upon by a force proportional to the strength of the pole and the strength of the field.	$f \propto H\mu$	That field in which a C.G.S. unit magnetic pole is acted upon with a force of 1 dyne.	C.G.S. unit magnetic field.	$[H]=[M]^{\frac{1}{2}}[L]^{-\frac{1}{2}}[T]^{-1}$.
Magnetic moment of a magnet, M.	The magnetic moment of a solenoidal magnet is proportional to the strength of each pole and the distance between them.	$M \propto \mu l$	The moment of a solenoidal magnet with poles of 1 C.G.S. unit strength and length 1 centimetre.	C.G.S. unit of magnetic moment.	$[m]=[M]^{\frac{1}{2}}[L]^{\frac{5}{2}}[T]^{-1}$.

DERIVATION OF UNITS ON THE C.G.S. SYSTEM—*continued.*

Quantity.	Physical law	Variation equation	Definition of derived unit	Derived unit	Dimensional equation
	ELECTRO-MAGNETIC UNITS.[1]				
Current c.	The force acting upon a magnetic pole at the centre of a circular arc of wire carrying a current, is proportional to the strength of the current, the length of the wire, and the strength of the pole, and inversely proportional to the square of the radius of the arc.	$f \propto \frac{\mu l c}{r^2}$	That current which, flowing along unit length of wire in the form of a circular arc of unit radius acts with a force of 1 dyne upon a magnetic pole of C.G.S. unit strength placed at the centre.	C.G.S. electro-magnetic unit current.	$[c]=[M]^{\frac{1}{2}}[L]^{\frac{1}{2}}[T]^{-1}$.
Electric quantity, q.	The quantity of electricity which passes across any section of a wire is proportional to the current and to the time of passage.	$q \propto ct$	The quantity which in one second crosses any section of a wire in which a C.G.S. unit current is flowing.	C.G.S. electro-magnetic unit of quantity.	$[Q]=[M]^{\frac{1}{2}}[L]^{\frac{1}{2}}$.
Electromotive force or difference of potential, E.	The work done by a quantity of electricity in passing between two points is proportional to the quantity and to the electromotive force or difference of potential between the points.	$w \propto Eq$	The E.M.F. between two points, such that the C.G.S. unit quantity of electricity in passing between the points does an erg of work.	C.G.S. electro-magnetic unit of E.M.F.	$[E]=[M]^{\frac{1}{2}}[L]^{\frac{3}{2}}[T]^{-2}$.
Resistance, r.	The current passing between two points of a conductor is directly proportional to the electromotive force, and inversely proportional to the resistance of the conductor between the points (Definition of Resistance).	$c \propto \frac{E}{r}$	The resistance of a conductor in which the C.G.S. unit E.M.F. produces the C.G.S. unit current.	C.G.S. electro-magnetic unit of resistance.	$[R]=[L][T]^{-1}=[V]$.
Capacity of a conductor, C.	The increase in the potential of a conductor produced by the addition of a given quantity to its charge is inversely proportional to the capacity of the conductor.	$E \propto \frac{q}{C}$	The capacity of a conductor which is charged to 1 C.G.S. unit potential by 1 C.G.S. unit of electricity.	C.G.S. electro-magnetic unit of capacity.	$[C]=[L]^{-1}[T]^2$.

[1] See chaps. xvii. and xviii. The electrostatic system is not employed in the experiments described below.

The C.G.S. System.

The table, p. 18, shows the method of derivation of such absolute units on the C.G.S. system as we shall have occasion to make use of in this book. The first column contains the denominations of the quantities measured; the second contains the verbal expression of the physical law on which the derivation is based, while the third gives the expression of the law as a variation equation; the fourth and fifth columns give the definition of the C.G.S. unit obtained and the name assigned to it respectively, while the last gives the dimensional equation. This will be explained later (p. 24).

The equations given in the third column are reduced to ordinary equalities by the adoption of the unit defined in the next column, or of another unit belonging to an absolute system based on the same principles.

Some physical laws express relations between quantities whose units have already been provided for on the absolute system, and hence we cannot reduce the variation equations to ordinary equalities. This is the case with the formula for the gaseous laws already mentioned (p. 15).

A complete system of units has thus been formed on the C.G.S. absolute system, many of which are now in practical use. Some of the electrical units are, however, proved to be not of a suitable magnitude for the electrical measurements most frequently occurring. For this reason practical units have been adopted which are not identical with the C.G.S. units given in the table (p. 20), but are immediately derived from them by multiplication by some power of 10. The names of the units in use, and the factors of derivation from the corresponding C.G.S. units are given in the following table :—

TABLE OF PRACTICAL UNITS FOR ELECTRICAL MEASUREMENT RELATED TO THE C.G.S. ELECTRO-MAGNETIC SYSTEM.

Quantity	Unit	Equivalent in C.G.S. units
Electric current . .	Ampère	10^{-1}
Electromotive force .	Volt	10^{8}
Resistance . . .	Ohm	10^{9}
Capacity . . .	Farad	10^{-9}
Rate of working . .	Watt	10^{7}
Quantity of Electricity .	Coulomb	10^{-1}

To shorten the notation when a very small fraction or a very large multiple of a unit occurs, the prefixes *micro-* and *mega-* have been introduced to represent respectively division and multiplication by 10^6. Thus:—

$$\text{A mega-dyne} = 10^6 \text{ dynes.}$$

$$\text{A micro-farad} = \frac{1}{10^6} \text{ farad.}$$

Arbitrary Units at present employed.

For many of the quantities referred to in the table (p. 18) no arbitrary unit has ever been used. Velocity, for instance, has always been measured by the space passed over in a unit of time. And for many of them the physical law given in the second column is practically the definition of the quantity; for instance, in the case of resistance, Ohm's law is the only definition that can be given of resistance as a measurable quantity.

For the measurement of some of these quantities, however, arbitrary units have been used, especially for quantities which have long been measured in an ordinary way as volumes, forces, &c.

Arbitrary units are still in use for the measurement of temperature and quantities of heat; also for light intensity, and some other magnitudes.

We have collected in the following table some of the arbitrary units employed, and given the results of experimental determinations of their equivalents in the absolute

units for the measurement of the same quantity when such exist :—

TABLE OF ARBITRARY UNITS.

Quantity	Arbitrary unit employed	Equivalent in absolute units
Angle	Degree ($\frac{1}{180}$ part of two right angles) Radian (unit of circular measure)	
Force	Pound weight Gramme weight	32·2 poundals (British absolute units) 981 dynes
Work	Foot-pound Kilogramme-metre	32·2 foot-poundals $9·81 \times 10^7$ ergs
Temperature	Degree Centigrade, corresponding to $\frac{1}{100}$ of the expansion of mercury in glass between the freezing and boiling points; degree Fahrenheit, corresponding to $\frac{1}{180}$ of the same quantity	
Quantity of heat	Amount of heat required to raise the temperature of unit mass of water one degree	The gramme-centigrade unit is equivalent to $4·214 \times 10^7$ ergs
Intensity of light	Standard candle. Sperm candles of six to the pound, each burning 120 grains an hour The Paris Conference standard. The light emitted by 1 sq. cm. of platinum at its melting point	
Electrical resistance	The B.A. unit (originally intended to represent the ohm) The 'ohm' adopted by the Board of Trade. The resistance at 0° C. of a column of mercury 106·3 cm. long, of uniform cross-section, 14·4521 grms. in mass.	·9866 true ohm[1]

[1] Cavendish Laboratory determinations.

Changes from one Absolute System of Units to another. Dimensional equations.

We have already pointed out that there are more than one absolute system of units in use by physicists. They are deduced in accordance with the same principles, but are based on different values assigned to the fundamental units. It becomes, therefore, of importance to determine the factor by which a quantity measured in terms of a unit belonging to one system must be multiplied, in order to express it in terms of the unit belonging to another system. Since the systems are absolute systems, certain variation equations become actual equalities; and since the two systems adopt the same principles, the corresponding equations will have the constant k equal to unity for each system. Thus, if we take the equation (1) (p. 14) as a type of one of these equations, we have the relation between the numerical measures

$$q = x^{\alpha} y^{\beta} z^{\gamma}$$

holding simultaneously for both systems.

Or, if q, x, y, z, be the numerical measures of any quantities on the one absolute system; q', x', y', z', the numerical measures of the *same actual quantities* on the other system, then

$$q = x^{\alpha} y^{\beta} z^{\gamma} \quad . \quad . \quad . \quad . \ (1)$$

and

$$q' = x'^{\alpha} y^{\beta} z'^{\gamma} \quad . \quad . \quad . \quad . \ (2).$$

Now, following the usual notation, let [Q], [X], [Y], [Z] be the concrete units for the measurement of the quantities on the former, which we will call the old, system, [Q′], [X′], [Y′], [Z′] the concrete units for their measurement on the new system.

Then, since we are measuring *the same actual quantities*,

$$\left.\begin{aligned} q\,[\mathrm{Q}] &\equiv q'\,[\mathrm{Q}'] \\ x\,[\mathrm{X}] &\equiv x'\,[\mathrm{X}'] \\ y\,[\mathrm{Y}] &\equiv y'\,[\mathrm{Y}'] \\ z\,[\mathrm{Z}] &\equiv z'\,[\mathrm{Z}'] \end{aligned}\right\}^{1} \quad . \quad . \quad . \ (3).$$

[1] The symbol $\equiv$ is used to denote absolute identity, as distinguished from numerical equality.

In these we may see clearly the expression of the well-known law, that if the unit in which a quantity is measured be changed, the ratio of the numerical measures of the same quantity for the two units is the inverse ratio of the units.

From equations (1) and (2) we get

$$\frac{q}{q'}=\left(\frac{x}{x'}\right)^{\alpha}\left(\frac{y}{y'}\right)^{\beta}\left(\frac{z}{z'}\right)^{\gamma};$$

and substituting from (3).

$$\frac{q}{q'}=\frac{[Q']}{[Q]}=\left(\frac{[X']}{[X]}\right)^{\alpha}\left(\frac{[Y']}{[Y]}\right)^{\beta}\left(\frac{[Z']}{[Z]}\right)^{\gamma}.$$

Thus, if ξ, η, ζ be the ratio of the new units $[X']$, $[Y']$, $[Z']$ to the old units $[X]$, $[Y]$, $[Z]$ respectively, then the ratio ρ of the new unit $[Q']$ to the old unit $[Q]$ is equal to $\xi^{\alpha}\eta^{\beta}\zeta^{\gamma}$, and the ratio of the new numerical measure to the old is the reciprocal of this.

Thus

$$\rho=\xi^{\alpha}\eta^{\beta}\zeta^{\gamma} \quad . \quad . \quad . \quad . \quad . \quad (4).$$

The equation (4), which expresses the relation between the ratios in which the units are changed, is of the same form as (1), the original expression of the physical law. So that whenever we have a physical law thus expressed, we get at once a relation between the ratios in which the units are changed. We may, to avoid multiplying notations, write it, if we please, in the following form :—

$$[Q]=[X]^{\alpha}[Y]^{\beta}[Z]^{\gamma} \quad . \quad . \quad (5),$$

where now $[Q]$, $[X]$, $[Y]$, $[Z]$ *no longer stand for concrete units, but for the ratios in which the concrete units are changed.* It should be unnecessary to call attention to this, as it is, of course, impossible even to imagine the multiplication of one concrete quantity by another, but the constant use of the identical form may sometimes lead the student to infer that the actual multiplication or division of concrete quantities

takes place. If we quite clearly understand that the sentence has no meaning except as an abbreviation, we may express equation (5) in words by saying that the unit of Q is the product of the α power of the unit of X, the β power of the unit of Y, and the γ power of the unit of Z; but if there is the least danger of our being taken at our word in expressing ourselves thus, it would be better to say that the ratio in which the unit of Q is changed when the units of X, Y, Z are changed in the ratios of [X] : 1, [Y] : 1 and [Z] : 1 respectively, is equal to the product of the α power of [X], the β power of [Y], and the γ power of [Z].

We thus see that if [X], [Y], [Z] be the ratios of the new units to the old, then equation (5) gives the ratio of the new unit of Q to the old, and the reciprocal is the ratio of the new numerical measure to the old numerical measure.

We may express this concisely, thus:—If in the equation (5) we substitute for [X], [Y], [Z] the new units in terms of the old, the result is the factor by which the old unit of Q must be multiplied to give the new unit; if, on the other hand, we substitute for [X], [Y], [Z] the old units in terms of the new, then the result is the factor by which the old numerical measure must be multiplied to give the new numerical measure.

If the units [X], [Y], [Z] be derived units, analogous equations may be obtained, connecting the ratios in which they are changed with those in which the fundamental units are changed, and thus the ratio in which [Q] is changed can be ultimately expressed in terms of the ratios in which the fundamental units are changed.

We thus obtain for every derived unit

$$[Q]=[L]^{\alpha}[M]^{\beta}[T]^{\gamma} \quad . \quad . \quad . \ (6).$$

[L], [M], [T] representing the ratios in which the fundamental units of length, mass, and time, respectively, are changed.

The equation (6) is called the dimensional equation for

[Q], and the indices α, β, γ are called the dimensions of Q with respect to length, mass, and time respectively.

The dimensional equation for any derived unit may thus be deduced from the physical laws by which the unit is defined, namely, those whose expressions are converted from variation equations to equalities by the selection of the unit.

We may thus obtain the dimensional equations which are given in the last column of the table (p. 18). We give here one or two examples.

(1) *To find the Dimensional Equation for Velocity.*

Physical law

$$s = v\,t,$$

or

$$v = \frac{s}{t}.$$

Hence

$$[v] = \frac{[L]}{[T]} = [L][T]^{-1}.$$

(2) *To find the Dimensional Equation for Force.*

Physical law

$$f = m\,a.$$

Hence

$$[F] = [M]\,[a];$$

but

$$[a] = [L]\,[T]^{-2}$$

$$\therefore [F] = [M]\,[L]\,[T]^{-2}.$$

(3) *To find the Dimensional Equation for Strength of Magnetic Pole.*

Physical law

$$\frac{\mu^2}{l^2} = f.$$

$$\therefore \mu^2 = l^2 f.$$

Hence

$$[\mu]^2 = [L]^2\,[F].$$

But

$$F = [M][L][T]^{-2}.$$

$$\therefore\ [\mu]^2 = [M][L]^3[T]^{-2},$$

or

$$[\mu] = [M]^{\frac{1}{2}}[L]^{\frac{3}{2}}[T]^{-1}.$$

When the dimensional equations for the different units have been obtained, the calculation of the factor for conversion is a very simple matter, following the law given on p. 26. We may recapitulate the law here.

To find the Factor by which to multiply the Numerical Measure of a Quantity to convert it from the old *System of Units to the* new, *substitute for* [L] [M] *and* [T] *in the Dimensional Equation the* old *Units of Length, Mass, and Time respectively, expressed in terms of the* new.

We may shew this by an example.

To find the Factor for converting the Strength of a Magnetic Pole from C.G.S. to Foot-grain-second Units—

1 C.G.S. unit of magnetic pole

$$\begin{aligned}
&= 1 \times [M]^{\frac{1}{2}}[L]^{\frac{3}{2}}[T]^{-1} \\
&= 1 \times [\text{gm.}]^{\frac{1}{2}}[\text{cm.}]^{\frac{3}{2}}[\text{sec.}]^{-1} \\
&= 1 \times [15{\cdot}4\ \text{gr.}]^{\frac{1}{2}}[0{\cdot}0328\ \text{ft.}]^{\frac{3}{2}}[\text{sec.}]^{-1} \\
&= 1 \times (15{\cdot}4)^{\frac{1}{2}}(0{\cdot}0328)^{\frac{3}{2}}[\text{gr.}]^{\frac{1}{2}}[\text{ft.}]^{\frac{3}{2}}[\text{sec.}]^{-1} \\
&= {\cdot}0233 \text{ foot-grain-second unit.}
\end{aligned}$$

That is, a pole whose strength is 5 in C.G.S. units has a strength of ·1165 foot-grain-second units.

Conversion of Quantities expressed in Arbitrary Units.

We have shewn above how to change from one system of units to another when both systems are absolute and based on the same laws. If a quantity is expressed in

arbitrary units, it must first be expressed in a unit belonging to some absolute system, and then the conversion factor can be calculated as above. For example:—

To express 15 *Foot-pounds in Ergs.*

The foot-pound is not an absolute unit. We must first obtain the amount of work expressed in absolute units. Now, since $g = 32{\cdot}2$ in British absolute units, 1 foot-pound $= 32{\cdot}2$ foot-poundals (British absolute units).

$$\therefore\ 15 \text{ foot-pounds} = 15 \times 32{\cdot}2 \text{ foot-poundals.}$$

We can now convert from foot-poundals to ergs.

The dimensional equation is

$$[\text{W}] = [\text{M}]\,[\text{L}]^2\,[\text{T}]^{-2}.$$

Since

$$1 \text{ foot} = 30{\cdot}5 \text{ cm.}$$
$$1 \text{ lb.} = 454 \text{ gm.}$$

Substituting

$$[\text{M}] = 454,\ [\text{L}] = 30{\cdot}5$$

we get

$$[\text{W}] = 454 \times (30{\cdot}5)^2.$$

Hence

$$15 \text{ foot-pounds} = 15 \times 32{\cdot}2 \times 454 \times (30{\cdot}5)^2 \text{ ergs.}$$
$$= 2{\cdot}04 \times 10^8 \text{ ergs.}$$

Sometimes neither of the units belongs strictly to an absolute system, although a change of the fundamental units alters the unit in question. For example:—

To find the Mechanical Equivalent of Heat in C.G.S. Centigrade Units, knowing that its Value for a Pound Fahrenheit Unit of Heat is 772 *Foot-pounds.*

The mechanical equivalent of heat is the amount of work equivalent to one unit of heat. For the C.G.S. Centigrade unit of heat, it is, therefore,

$$\frac{9}{5} \times \frac{1}{454} \times 772 \text{ foot-pounds.}$$

This amount of heat is equivalent to

$$\frac{9}{5} \times \frac{1}{454} \times 772 \times 1{\cdot}36 \times 10^7 \text{ ergs},$$

or the mechanical equivalent of one C.G.S. Centigrade unit of heat

$$= 4{\cdot}14 \times 10^7 \text{ ergs}.$$

If the agreement between scientific men as to the selection of fundamental units had been universal, a great deal of arithmetical calculation which is now necessary would have been avoided. There is some hope that in future one uniform system may be adopted, but even then it will be necessary for the student to be familiar with the methods of changing from one system to another in order to be able to avail himself of the results already published. To form a basis of calculation, tables showing the equivalents of the different fundamental units for the measurement of the same quantity are necessary. Want of space prevents our giving them here; we refer instead to Nos. 9–12 of the tables by Mr. S. Lupton (Macmillan & Co.). We take this opportunity of mentioning that we shall refer to the same work[1] whenever we have occasion to notice the necessity for a table of constants for use in the experiments described.

CHAPTER III.

PHYSICAL ARITHMETIC

Approximate Measurements.

ONE of the first lessons which is learned by an experimenter making measurements on scientific methods is that the number obtained as a result is not a perfectly exact expression of the quantity measured, but represents it only within

[1] *Numerical Tables and Constants in Elementary Science*, by S. Lupton

certain limits of error. If the distance between two towns be given as fifteen miles, we do not understand that the distance has been measured and found to be exactly fifteen miles, without any yards, feet, inches, or fractions of an inch, but that the distance is nearer to fifteen miles than it is to sixteen or fourteen. If we wished to state the distance more accurately we should have to begin by defining two points, one in each town—marks, for instance, on the door-steps of the respective parish churches—between which the distance had been taken, and we should also have to specify the route taken, and so on. To determine the distance with the greatest possible accuracy would be to go through the laborious process of measuring a base line, a rough idea of which is given in § 5. We might then, perhaps, obtain the distance to the nearest inch and still be uncertain whether there should not be a fraction of an inch more or less, and if so, what fraction it should be. If the number is expressed in the decimal notation, the increase in the accuracy of measurement is shewn by filling up more decimal places. Thus, if we set down the mechanical equivalent of heat at $4{\cdot}2 \times 10^7$ ergs, it is not because the figures in the decimal places beyond the 2 are all zero, but because we do not know what their values really are, or it may be, for the purpose for which we are using the value, it is immaterial what they are. It is known, as a matter of fact, that a more accurate value is $4{\cdot}214 \times 10^7$, but at present no one has been able to determine what figure should be put in the decimal place after the second 4.

Errors and Corrections.

The determination of an additional figure in a number representing the magnitude of a physical quantity generally involves a very great increase in the care and labour which must be bestowed on the determination. To obtain some idea of the reason for this, let us take, as an example, the case of determining the mass of a body of about 100

grammes. By an ordinary commercial balance the mass of a body can be easily and rapidly determined to 1 gramme, say 103 grammes. With a better arranged balance we may shew that 103·25 is a more accurate representation of the mass. We may then use a very sensitive chemical balance which shews a difference of mass of 0·1 mgm., but which requires a good deal of time and care in its use, and get a value 103·2537 grammes as the mass. But, if now we make another similar determination with another balance, or even with the same balance, at a different time, we may find the result is not the same, but, say, 103·2546 grammes. We have thus, by the sensitive balance, carried the measurement two decimal places further, but have got from two observations two different results, and have, therefore, to decide whether either of these represents the mass of the body, and, if so, which. Experience has shewn that some, at any rate, of the difference may be due to the balance not being in adjustment, and another part to the fact that the body is weighed in air and not in vacuo. The observed weighings may contain errors due to these causes. The effects of these causes on the weighings can be calculated when the ratio of the lengths of the arms and other facts about the balance have been determined, and when the state of the air as to pressure, temperature, and moisture is known (see §§ 13, 14).

We may thus, by a series of auxiliary observations, determine a correction to the observed weighing corresponding to each known possible error. When the observations are thus corrected they will probably be very much closer. Suppose them to be 103·2543 and 103·2542.

Mean of Observations.

When all precautions have been taken, and all known errors corrected, there may still be some difference between different observations which can only arise from causes beyond the knowledge and control of the observer. We

must, therefore, distinguish between errors due to known causes, which can be allowed for as corrections, or eliminated by repeating the observations under different conditions, and errors due to unknown causes, which are called 'accidental' errors. Thus, in the instance quoted, we know of no reason for taking 103·2543 as the mass of the body in preference to 103·2542. It is usual in such cases to take the arithmetic mean of the two observations, i.e. the number obtained by adding the two values together, and dividing by 2, as the nearest approximation to the true value.

Similarly if any number, n, of observations be taken, *each one of which has been corrected for constant errors*, and is, therefore, so far as the observer can tell, as worthy of confidence as any of the others, the arithmetic mean of the values is taken as that most nearly representing the true value of the quantity. Thus, if $q_1, q_2, q_3 \ldots \ldots q_n$ be the results of the n observations, the value of q is taken to be

$$q = \frac{q_1 + q_2 + q_3 + \ldots\ldots + q_n}{n}.$$

It is fair to suppose that, if we take a sufficient number of observations, some of them give results that are too large, others again results that are too small; and thus, by taking the mean of the observations as the true value, we approach more nearly than we can be *sure* of doing by adopting any single one of the observations.

We have already mentioned that allowance must be made by means of a suitable correction for each constant error, that is for each known error whose effect upon the result may be calculated or eliminated by some suitable arrangement. It is, of course, possible that the observer may have overlooked some source of constant error which will affect the final result. This must be very carefully guarded against, for taking the mean of a number of obser-

vations affords, in general, no assistance in the elimination of an error of that kind.

The difference between the mean value and one of the observations is generally known technically as the 'error' of that observation. The theory of probabilities has been applied to the discussion of errors of observations[1], and it has been shewn that by taking the mean of n observations instead of a single observation, the so-called 'probable error' is reduced in the ratio of $1/\sqrt{n}$.

On this account alone it would be advisable to take several observations of each quantity measured in a physical experiment. By doing so, moreover, we not only get a result which is probably more accurate, but we find out to what extent the observations differ from each other, and thus obtain valuable information as to the degree of accuracy of which the method of observation is capable. Thus we have, on p. 72, four observations of a length, viz.—

	3·333	in.
	3·332	"
	3·334	"
	3·334	"
Mean =	3·3332	"

Taking the mean we are justified in assuming that the true length is accurately represented by 3·333 to the third decimal place, and we see that the different observations differ only by two units at most in that place.

In performing the arithmetic for finding the mean of a number of observations, it is only necessary to add those columns in which differences occur—the last column of the example given above. Performing the addition on the other columns would be simply multiplying by 4, by which number we should have subsequently to divide.

An example will make this clear.

[1] See Airy's tract on the *Theory of Errors of Observations.*

Find the mean of the following eight observations:—

$$\begin{array}{rr} & 56{\cdot}231 \\ & 56{\cdot}275 \\ & 56{\cdot}243 \\ & 56{\cdot}255 \\ & 56{\cdot}256 \\ & 56{\cdot}267 \\ & 56{\cdot}273 \\ & 56{\cdot}266 \\ \hline \text{Adding} & (8 \times 56{\cdot}2 +){\cdot}466 \\ \hline \text{Mean} \quad . & .\ 56{\cdot}2582 \end{array}$$

The figures introduced in the bracket would not appear in ordinary working.

The separate observations of a measurement should be made quite independently, as actual mistakes in reading are always to be regarded as being within the bounds of possibility. Thus, for example, mistakes of a whole degree are sometimes made in reading a thermometer, and again in weighing, a beginner is not unlikely to mis-count the weights. Mistakes of this kind, which are to be very carefully distinguished from the 'errors of observation,' would probably be detected by an independent repetition of the observation. If there be good reason for thinking that an observation has been affected by an unknown error of this kind, the observation must be rejected altogether.

Possible Accuracy of Measurement of different Quantities.

The degree of accuracy to which measurements can be carried varies very much with different experiments. It is usual to estimate the limit of accuracy as a fractional part or percentage of the quantity measured.

Thus by a good balance a weighing can be carried out to a tenth of a milligramme; this, for a body weighing about 100 grammes, is as far as one part in a million, or ·0001 per cent.—an accuracy of very high order The measurement

of a large angle by the spectrometer (§ 62) is likewise very accurate; thus with a vernier reading to 20″, an angle of 45° can be read to one part in four thousand, or 0·025 per cent. On the other hand, measurements of temperature cannot, without great care, be carried to a greater degree of accuracy than one part in a hundred, or 1 per cent., and sometimes do not reach that. A length measurement often reaches about one part in ten thousand. For most of the experiments which are described in this work an accuracy of one part in a thousand is ample, indeed generally more than sufficient.

It is further to be remarked that, if several quantities have to be observed for one experiment, some of them may be capable of much more accurate determination than others. It is, as a general rule, useless to carry the accuracy of the former beyond the possible degree of accuracy of the latter. Thus, in determining specific heats, we make some weighings and measure some temperatures. It is useless to determine the weights to a greater degree of accuracy than one part in a thousand, as the accuracy of the result will not reach that limit in consequence of the inaccuracy of the temperature measurements. In some cases it is necessary that one measurement should be carried out more accurately than others in order that the errors in the result may be all of the same order. The reason for this will be seen on p. 48.

Arithmetical Manipulation of Approximate Values.

In order to represent a quantity to the degree of accuracy of one part in a thousand, we require a number with four digits at most, exclusive of the zeros which serve to mark the position of the number in the decimal scale.[1] It frequently

[1] It is now usual, when a very large number has to be expressed, to write down the digits with a decimal point after the first, and indicate its position in the scale by the power of 10, by which it must be multiplied: thus, instead of 42140000 we write $4{\cdot}214 \times 10^7$. A corresponding notation is used for a very small decimal fraction: thus, instead of ·00000588 we write $5{\cdot}88 \times 10^{-6}$.

happens that some arithmetical process, employed to deduce the required result from the observations, gives a number containing more than the four necessary digits. Thus, if we take seven observations of a quantity, each to three figures, and take the mean, we shall usually get any number of digits we please when we divide by the 7. But we know that the observations are only accurate to three figures; hence, in the mean obtained, all the figures after the fourth, at any rate, have no meaning. They are introduced simply by the arithmetical manipulation, and it is, therefore, better to discard them. It is, indeed, not only useless to retain them, but it may be misleading to do so, for it may give the reader of the account of the experiment an impression that the measurements have been carried to a greater degree of accuracy than is really the case. Only those figures, therefore, which really represent results obtained by the measurements should be included in the final number. In discarding the superfluous digits we must increase the last digit retained by unity, if the first digit discarded is 5 or greater than 5. Thus, if the result of a division gives 32·316, we adopt as the value 32·32 instead of 32·31. For it is evident that the four digits 32·32 more nearly represent the result of the division than the four 32·31.

Superfluous figures very frequently occur in the multiplication and division of approximate values of quantities. These have also to be discarded from the result; for if we multiply two numbers, each of which is accurate only to one part in a thousand, the result is evidently only accurate to the same degree, and hence all figures after the fourth must be discarded.

The arithmetical manipulation may be performed by using logarithms, but it is sometimes practically shorter to work out the arithmetic than to use logarithms; and in this case the arithmetical process may be much abbreviated by discarding unnecessary figures in the course of the work.

The following examples will show how this is managed:—

Example (1).—Multiply 656·3 by 4·321 to four figures.

Ordinary form	Abbreviated form
656·3	656·3
4·321	4·321
6563	(656·3 × 4) = 2625·2
13126	(656 × 3) = 196·8
19689	(65 × 2) = 13·0
26252	(6 × 1) = 6
2835·8723	2835·6
Result 2836	Result 2836

The multiplication in the abbreviated form is conducted in the reverse order of the digits of the multiplier. Each successive digit of the multiplier begins at one figure further to the left of the multiplicand. The decimal point should be fixed when the multiplication by the first digit (the 4) is completed. To make sure of the result being accurate to the requisite number of places, the arithmetical calculation should be carried to one figure beyond the degree of accuracy ultimately required.

Example (2).—Divide 65·63 by 4·391 to four figures.

Ordinary form	Abbreviated form
4·391) 65·63000 (14946	4·391) 65·630 (14948
4391	4391
21720	21720
17564	17564
·41560	(439) ·4156
39519	3951
·20410	(43) ·205
17564	172
·2846	(4) ·33
Result 14·95	Result 14·95

In the abbreviated form, instead of performing the successive steps of the division by bringing down 0's, suc-

cessive figures are cut off from the divisor, beginning at the right hand; thus, the divisors are for the first two figures of the quotient 4391; for the next figure, 439; for the next, 43. It can then be seen by inspection that the next figure is 8. The division is thus accomplished.

It will be seen that one o is added to the dividend; the arithmetic is thus carried, as before, to one figure beyond the accuracy ultimately required. This may be avoided if we always multiply the divisor mentally for one figure beyond that which we actually use, in order to determine what number to 'carry'; the number carried appears in the work as an addition to the first digit in the multiplication.

The method of abbreviation, which we have here sketched, is especially convenient for the application of small corrections (see below, p. 42). We have then, generally, to multiply a number by a factor differing but little from unity; let us take, for instance, the following:—

Example (3).—Multiply 563·6 by 1·002 to four places of decimals.

Adopting the abbreviated method we get—

$$\begin{array}{rr} & 563{\cdot}6 \\ & \underline{1{\cdot}002} \\ & 563{\cdot}6 \\ & \underline{1{\cdot}1} \\ & \underline{564{\cdot}7} \\ \text{Result} & 564{\cdot}7 \end{array}$$

or

Example (4).—Multiply 563·6 by ·9998.

In this case ·9998 = 1 − ·0002.

$$\begin{array}{rr} & 563{\cdot}6 \\ & \underline{1-{\cdot}0002} \\ & 563{\cdot}6 \\ & \underline{-1{\cdot}1} \\ & \underline{562{\cdot}5} \\ \text{Result} & 562{\cdot}5 \end{array}$$

It will be shewn later (p. 44) that dividing by ·9998 is the same, as far as the fourth place of decimals is concerned, as multiplying by 1·002, and *vice versâ*; this suggests the possibility of considerable abbreviation of arithmetical calculation in this and similar cases.

Facilitation of Arithmetical Calculation by means of Tables.—Interpolation.

The arithmetical operations of multiplication, division, the determination of any power of a number, and the extraction of roots, may be performed, to the required degree of approximation, by the use of tables of logarithms. The method of using these for the purposes mentioned is so well known that it is not necessary to enter into details here. A table of logarithms to four places of decimals is given in Lupton's book, and is sufficient for most of the calculations that we require. If greater accuracy is necessary, Chambers's tables may be used. Instead of tables of logarithms, a 'slide-rule' is sometimes employed. An explanation of the plan upon which the rule is graduated and the method of using it for making arithmetical calculations is given at the end of this chapter, pp. 51-58.

Besides tables of logarithms, tables of squares, cubes, square roots, cube roots, and reciprocals may be used. Short tables will be found in Lupton's book (pp. 1–4); for more accurate work Barlow's tables should be used. Besides these the student will require tables of the trigonometrical functions, which will also be found among Lupton's tables.

An arithmetical calculation can frequently be simplified on account of some special peculiarity. Thus, dividing by 5 is equivalent to multiplying by 2, and moving the decimal point one place to the left. Again, $\pi^2 = 9{\cdot}87 = 10 - {\cdot}13$, and many other instances might be given; but the student can only make use of such advantages by a familiar acquaintance with cases in which they prove of service.

In some cases the variations of physical quantities are also tabulated, and the necessity of performing the arithmetic is thereby saved. Thus, No. 31 of Lupton's tables gives the logarithms of $(1 + \cdot 00367\, t)$ for successive degrees of temperature, and saves calculation when the volume or pressure of a mass of gas at a given temperature is required. A table of the variation of the specific resistance of copper with variation of temperature, is given on p. 47 of the same work.

It should be noticed that all tables proceed by certain definite intervals of the varying element; for instance, for successive degrees of temperature, or successive units in the last digit in the case of logarithms; and it may happen that the observed value of the element lies between the values given in the table. In such cases the required value can generally be obtained by a process known as 'interpolation.' If the successive intervals, for which the table is formed, are small enough, the tabulated quantity may be assumed to vary *uniformly* between two successive steps of the varying element, and the increase in the tabulated quantity may be calculated as being proportional to the increase of the varying element. We have not space here to go more into detail on this question, and must content ourselves with saying that the process is strictly analogous to the use of 'proportional parts' in logarithms. We may refer to §§ 12, 19, 77 for examples of the application of a somewhat analogous method of physical interpolation.

Algebraical Approximation. Approximate Formulæ. Introduction of small Corrections.

If we only require to use a formula to give a result accurate within certain limits, it is, in many cases, possible to save a large amount of arithmetical labour by altering the form of the formula to be employed. This is most frequently the case when any small correction to the value of one of the observed elements has to be introduced, as in the case,

for instance, of an observed barometric height which has to be corrected for temperature. We substitute for the strictly accurate formula an approximate one, which renders the calculation easier, but in the end gives the same result to the required degree of accuracy.

We have already said that an accuracy of one part in a thousand is, as a rule, ample for our purpose; and we may, therefore, for the sake of definiteness, consider the simplification of algebraical formulæ with the specification of one part in a thousand, or 0·1 per cent., as the limit of accuracy desired. Whatever we have to say may be easily adapted for a higher degree of accuracy, if such be found to be necessary.

It is shewn in works on algebra that

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \text{terms involving higher powers of } x \quad . \quad . \quad . \quad (1).$$

This is known as the 'binomial theorem,' and is true for all values of n, positive or negative, integral or fractional. Some special cases will probably be familiar to every student, as:—

$$(1+x)^2 = 1 + 2x + x^2.$$
$$(1+x)^3 = 1 + 3x + 3x^2 + x^3.$$
$$(1+x)^{-1} = \frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots.$$

If we change the sign of x we get the general formula in the form

$$(1-x)^n = 1 - nx + \frac{n(n-1)}{2}x^2 - \ldots.$$

We may include both in one form, thus:—

$$(1 \pm x)^n = 1 \pm nx + \frac{n(n-1)}{2}x^2 \pm \ldots.$$

here the sign $\pm$ means that either the $+$ or the $-$ is e taken throughout the formula.

Now, if x be a small fraction, say, $1/1000$ or $0{\cdot}001$, x^2 is evidently a much smaller fraction, namely, $1/1000{,}000$, or $0{\cdot}000001$, and x^3 is still smaller. Thus, unless n is very large indeed, the term

$$\frac{n(n-1)}{2}x^2$$

will be too small to be taken account of, and the terms which follow will be of still less importance. We shall probably not meet with formulæ in which n is greater than 3. Let us then determine the value of x so that

$$\frac{n(n-1)}{2}x^2$$

may be equal to $\cdot 001$, that is to say, may just make itself felt in the calculations that we are now discussing.

Putting $n = 3$ we get

$$\begin{aligned} 3x^2 &= \cdot 001 \\ x &= \sqrt{\cdot 00033} \\ &= \cdot 02 \text{ roughly.} \end{aligned}$$

So that we shall be well within the truth if we say that (when $n = 3$), if x be not greater than $0{\cdot}01$, the third term of equation (1) is less than $\cdot 001$, and the fourth term less than $\cdot 00001$. Neither of these, nor anyone beyond them, will, therefore, affect the result, as far as an accuracy of one part in a thousand is concerned; and we may, therefore, say that, if x is not greater than $0{\cdot}01$,

$$(1+x)^3 = 1 + 3x.$$

To use this approximate formula when $x = 0{\cdot}01$ would be inadmissible, as it produces a considerable effect upon the next decimal place; and, if in the same formula, we make other approximations of a similar nature, the accumulation of approximations may impair the accuracy of the result.

In any special case, therefore, it is well to consider

whether x is small enough to allow of the use of the approximate formula by roughly calculating the value of the third term; it is nearly always so if it is less than ·005. This includes the important case in which x is the coefficient of expansion of a gas for which $x = \cdot 00367$.

If n be smaller than 3, what we have said is true within still closer limits; and as n is usually smaller than 3, we may say generally that, for our purposes,

$$(1+x)^n = 1+nx,$$

and

$$(1-x)^n = 1-nx,$$

provided x be less than 0·005.

Some special cases of the application of this method of approximation are here given, as they are of frequent occurrence:—

$$(1 \pm x)^2 = 1 \pm 2x$$
$$(1 \pm x)^3 = 1 \pm 3x$$
$$\sqrt{1 \pm x} = (1 \pm x)^{\frac{1}{2}} = 1 \pm \frac{x}{2}$$
$$\frac{1}{\sqrt{1 \pm x}} = (1 \pm x)^{-\frac{1}{2}} = 1 \mp \frac{x}{2}$$
$$\frac{1}{1 \pm x} = (1 \pm x)^{-1} = 1 \mp x$$
$$\frac{1}{(1 \pm x)^2} = (1 \pm x)^{-2} = 1 \mp 2x.$$

The formulæ for $+x$ and $-x$ are here included in one expression; the upper or lower sign must be taken throughout the formula.

We thus see that whenever a factor of the form $(1 \pm x)^n$ occurs in a formula where x is a small fraction, we may replace it by the simpler but approximate factor $1 \pm nx$; and we have already shown how the multiplication by such a factor may be very simply performed (p. 39). Cases of the application of this method occur in §§ 13, 24 etc.

Another instance of the change of formula for the pur-

poses of arithmetical simplicity is made use of in § 13. In that case we obtain a result as the geometric mean of two nearly equal quantities. It is an easy matter to prove algebraically, although we have not space to give the proof here, that the geometric mean of two quantities which differ only by one part in a thousand differs from the arithmetic mean of the two quantities by less than the millionth of either. It is a much easier arithmetical operation to find the arithmetic mean than the geometric, so that we substitute in the formula $(x+x')/2$ for $\sqrt{x\,x'}$.

The calculation of the effect upon the trigonometrical ratios of an angle, due to a small fractional increase in the angle, may be included in this chapter. We know that

$$\sin(\theta+d) = \sin\theta\cos d + \cos\theta\sin d.$$

Now, reference to a table of sines and cosines will shew that $\cos d$ differs from unity by less than one part in a thousand if d be less than $2°\ 33'$, and, *if expressed in circular measure*, the same value of d differs from $\sin d$ by one part in three thousand; so we may say that, provided d is less than $2\frac{1}{2}°$, $\cos d$ is equal to unity, and $\sin d$ is equal to d expressed in circular measure.

The formula is, therefore, for our purposes, equivalent to

$$\sin(\theta+d) = \sin\theta + d\cos\theta.$$

We may reason about the other trigonometrical ratios in a similar manner, and we thus get the following approximate formulæ :—

$$\sin(\theta\pm d) = \sin\theta \pm d\cos\theta.$$
$$\cos(\theta\pm d) = \cos\theta \mp d\sin\theta.$$
$$\tan(\theta\pm d) = \tan\theta \pm d\sec^2\theta.$$

The upper or lower sign is to be taken throughout the formula.

If d be expressed in degrees, then, since the circular

measure of $1°$ is $\pi/180$, that of $d°$ is $d\pi/180$, and the formulæ become

$$\sin(\theta \pm d) = \sin\theta \pm \frac{d \times \pi}{180}\cos\theta,$$

&c.

It has been already stated that approximate formulæ are frequently available when it is required to introduce corrections for variations of temperature, and other elements which may be taken from tables of constants. There is besides another use for them which should not be overlooked, namely, to calculate the effect upon the result, of an error of given magnitude in one of the observed elements. This is practically the same as calculating the effect of a hypothetical correction to one of the observed elements. In cases where the formula of reduction is simply the product or quotient of a number of factors each of which is observed directly, a fractional error of any magnitude in one of the factors produces in the result an error of the same fractional magnitude, but in other cases the effect is not so simply calculated. If we take one example it will serve to illustrate our meaning, and the general method of employing the approximate formulæ we have given in this chapter.

In § 75 electric currents are measured by the tangent galvanometer. Suppose that in reading the galvanometer we cannot be sure of the position of the needle to a greater accuracy than a quarter of a degree. Let us, therefore, consider the following question :—'*What is the effect upon the value of a current, as deduced from observations with the tangent galvanometer, of an error of a quarter of a degree in the reading?*'

The formula of reduction is

$$c = k \tan\theta.$$

Suppose an error δ has been made in the reading of θ, so that the observed value is

$$\begin{aligned} c' &= k\tan(\theta + \delta) \\ &= k(\tan\theta + \delta\sec^2\theta). \quad . \quad . \quad . \quad (\text{p. } 45) \\ c' - c &= k\,\delta\sec^2\theta. \end{aligned}$$

The fractional error q in the result is

$$\frac{c'-c}{c}=\frac{k\,\delta\sec^2\theta}{k\tan\theta}=\frac{\delta}{\sin\theta\cos\theta}$$
$$=\frac{2\,\delta}{\sin 2\,\theta}.$$

The error δ must be expressed in circular measure ; if it be equivalent to a quarter of a degree, we have

$$\delta=\frac{\pi}{4\times 180}=\cdot 00436,$$
$$\therefore\ q=\frac{\cdot 00872}{\sin 2\,\theta}.$$

The actual magnitude of this fraction depends upon the value of θ, that is upon the deflection. It is evidently very great when θ is very small, and least when $\theta=45°$, when it is 0·9 per cent. From which we see not only that when θ is known the effect of the error can be calculated, but also that the effect of an error of reading, of given magnitude, is least when the deflection is 45°. It is clear from this that a tangent galvanometer reading is most accurate when the deflection produced by the current is 45°. This furnishes an instance, therefore, of the manner in which the approximate formulæ we have given in this chapter can be used to determine what is the best experimental arrangement of the magnitudes of the quantities employed, for securing the greatest accuracy in an experiment with given apparatus. The same plan may be adopted to calculate the best arrangement of the apparatus for any of the experiments described below.

In concluding this part of the subject, we wish to draw special attention to one or two cases, already hinted at, in which either the method of making the experiments, or the formula for reduction, makes it necessary to pay special attention to the accuracy of some of the elements observed. In illustration of the former case we may mention the weighing of a small mass contained in a large vessel. To

fix ideas on the subject, consider the determination of the mass of a given volume of gas contained in a glass globe, by weighing the globe full and empty. During the interval between the two weighings the temperature and pressure of the air, and in consequence the apparent weight of the glass vessel, may have altered. This change, unless allowed for, will appear, when the subtraction has been performed, as an error of the same actual magnitude in the mass of the gas, and may be a very large fraction of the observed mass of the gas, so that we must here take account of the variation in the correction for weighing in air, although such a precaution might be quite unnecessary if we simply wished to determine the actual mass of the glass vessel and its contents to the degree of accuracy that we have hitherto assumed. A case of the same kind occurs in the determination of the quantity of moisture in the air by means of drying tubes (§ 42).

Cases of the second kind referred to above often arise from the fact that the formulæ contain differences of nearly equal quantities; we may refer to the formulæ employed in the correction of the first observations with Atwood's machine (§ 21), the determination of the latent heat of steam (§ 39), and the determination of the focal length of a concave lens (§ 54) as instances. In illustration of this point we may give the following question, in which the hypothetical errors introduced are not really very exaggerated.

'An observer, in making experiments to determine the focal length of a concave lens, measures the focal length of the auxiliary lens as 10·5 cm., when it is really 10 cm., and the focal length of the combination as 14·5 cm., when it is really 15 cm.; find the error in the result introduced by the inaccuracies in the measurements.'

We have the formula

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

whence

$$f_2 = -\frac{F f_1}{F - f_1},$$

putting in the true values of F and f_1.

$$f_2 = -\frac{15 \times 10}{15 - 10} = -\frac{150}{5} = -30,$$

and putting the observed values

$$f_2 = -\frac{14\cdot5 \times 10\cdot5}{14\cdot5 - 10\cdot5} = -\frac{152\cdot25}{4} = -38\cdot06.$$

The fractional error thus introduced is

$$\frac{8\cdot06}{30},$$

or more than 25 per cent., whereas the error in either observation was not greater than 5 per cent.

It will be seen that the large increase in the percentage error is due to the fact that the difference in the errors in F and f_1 has to be estimated as a fraction of $F - f_1$; this should lead us to select such a value of f_1 as will make $F - f_1$ as great as possible, in order that errors of given actual magnitude in the observations may produce in the result a fractional error as small as possible.

We have not space for more detail on this subject. The student will, we hope, be able to understand from the instances given that a large amount of valuable information as to the suitability of particular methods, and the selection of proper apparatus for making certain measurements, can be obtained from a consideration of the formulæ of reduction in the manner we have here briefly indicated.

Graphical Methods.

The results of a large number of experiments can be best expressed graphically. Examples of this method will be found in the course of the book. (See specially §§ 26, 40 41.)

The method is chiefly useful in cases in which we wish to trace the dependence of one quantity on another. Paper suitable for the purpose, ruled in small squares, can be easily obtained.[1]

In applying the method, the values of the independent variable are set down as abscissæ parallel to one set of lines, the corresponding values of the dependent variable being measured as ordinates at right angles to this. In cases in which the phenomenon under investigation is continuous in its character, a smooth curve can usually be drawn, either freehand or by the aid of a flexible ruler, so as to pass approximately through these points, and the law sought can be obtained by an investigation of the form of the curve.

Thus, suppose we are endeavouring to prove that the pressure of a given mass of gas at constant volume varies as the absolute temperature, we lay off as abscissæ the observed values of the temperature, say in degrees centigrade from freezing point as zero, and as ordinates the corresponding pressures.

On drawing the curve which best represents the experiments we find it to be a straight line; moreover, this line cuts the line of no pressure from which the ordinates are measured at a point on the negative side of the origin about 273° C. below freezing point. This point is the absolute zero, and the pressure is clearly proportional to the temperature reckoned from it.

The accuracy of a result obtained by a graphical method will, to some extent, depend on the scale adopted. Let us suppose that in the above experiment we can read the temperature to 0·1° C., and the pressure to ·5 mm. Then it is clear we must adopt such a scale for the temperature, if we wish to be accurate, as will allow 0·1° C.

[1] Messrs. Waterlow supply paper ruled in inch squares. Each inch is subdivided to tenths by fine lines, the half-inch lines being thicker than the others. For some remarks on different 'squared' papers see p. 11 of the Report on Spectrum Analysis, *B.A. Report*, 1881.

to be clearly visible. We might take 1 inch to represent 1°.

If at the same time we represent 1 cm. of pressure by 1 inch on the diagram, we can plot down the pressure to ·5 mm., and these scales will give us satisfactory results. The figure so drawn will be very large, larger than is required for the accuracy attempted in most of the experiments described.

When the diagram is to be used to represent the variations of one quantity corresponding to those of another over a small range, a wide scale can be used without making a very large diagram by using the abscissæ or ordinates, or both, to represent the respective changes and not the whole quantities. Thus, suppose we wish to represent the changes of volume of one gramme of water consequent on changes of temperature between 0° C. and 10° C.; we may regard the horizontal line through the origin as indicating volumes equal to that of one gramme of water at 4° C., and one inch of vertical height may represent a change of volume of ·00001 c.c. The line of *no volume* would, if drawn, be 100,000 inches below the horizontal through the origin. But it need not be drawn; and if one inch of horizontal distance represent 1° C., the whole diagram will be comprised in a space 10 inches square.

In drawing a diagram the horizontal and vertical scales chosen should always be very clearly set out in the diagram itself.

The Slide Rule.

The slide rule is a mechanical contrivance for performing rapidly various arithmetical operations. Its action depends in the main on the two principles that the logarithm of the product of two numbers is the sum of the logarithms of its factors, and that the logarithm of the *n*th power of a number is *n* times the logarithm of the number.

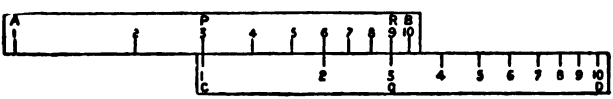
In its very simplest form a slide rule would consist of two identical scales, one of which can slide along the other. The scales are divided in such a way that the distance along either scale measured from one end—say, the left-hand—is proportional to the logarithm of the corresponding scale number. Thus the distance from the left-hand end to a reading a, say, is proportional to the logarithm of a; that to a second reading b is proportional to the logarithm of b.

One of the two scales is known as the rule; the other as the slider.

Now let P be the mark on the rule corresponding to a division a, A being the index at the left-hand end of the rule, then AP measures the logarithm of a, so that the number at A is 1. Place C, the index of the slider, which is marked 1, in contact with P, and let Q be the mark on the slider which corresponds to a division b, so that CQ measures log b. Let R be the mark on the rule opposite Q, let c be the corresponding reading; then AR = log c. Now

$$\begin{aligned} \log ab &= \log a + \log b \\ &= \text{AP} + \text{CQ} = \text{AP} + \text{PR} \\ &= \text{AR} = \log c; \\ &\therefore\ c = ab. \end{aligned}$$

In the figure as drawn, if the distance AB be taken as



unity, then AP is log 3, Q is at division 3 on the slider, and R, the corresponding division on the scale, is 9, which is equal to 3 times 3.

The above result, then, leads to the following method for obtaining the product of two or more quantities by the slide rule:—Thus, if a and b are the quantities, set the index of

the slider to division a on the rule, and read the division of the rule which corresponds to division b of the slider. This gives the product $a\,b$. The inverse of this gives us the method of division. Thus, to divide c by b, set division b of the slider opposite c of the rule, and read the division a, say, of the rule opposite the index of the slider; then, clearly, a is equal to c/b.

If, as is usually the case, the scale and the slider are of the same length, it will often happen that when the index of the slider is set to a division a, the division of the rule which corresponds to b on the slide is off the scale. The following considerations will shew us how to proceed in this case.

Let us suppose the rule is divided into ten parts, marked 1, 2, to 10, each of these being subdivided into tenths or twentieths. These subdivisions may be still further divided by eye to fifths, so that we read with fair accuracy to ·01. The divisions gradually get smaller as we go up the scale; in many rules the lower numbers are subdivided to hundredths. Thus the distance measured from the index, or division 1, of the scale to a division such as 783 gives us the logarithm of 7·83. Now $\log 783 = 2 + \log 7{\cdot}83$. Thus, to find the logarithm of 783 we have to add 2, that is, twice the length of the scale, to the distance actually given on the scale. We must suppose the scale to be produced backward to the left to twice its own length, and read from this index. Suppose, now, we want to multiply this by 85. The actual distance on the slider up to division 85 is log 8·5. To get log 85 we must add log 10 to this, and log 10 measures the length of the slider. Thus the mark on the slider which we should, according to the rule, put into coincidence with 783 would be at a distance equal to the length of the slider to the left of the index. The complete rule then would consist of a series of repetitions of the scales of both rule and slider, the first scale giving logarithms of numbers from 1 to 10, the next of

numbers from 10 to 100, and so on, and all the scales being exactly alike.

Now let us suppose the index of the slider (marked 1) to be in coincidence with a division, say 7·80, of the scale; then 10 on the slider will coincide with 78·0, 100 of the slider with 780, and so on. Also, since $7·8 \times 8·5$ is equal to 66·3, we shall find that 8·5, 85, 850, &c., of the slider coincide with 66·3, 663, and 6630 respectively.

Thus in multiplying two numbers together it is immaterial, except so far as the decimal points are concerned, which series of divisions on the rule or slider we use. We may set either division 1 or division 10 or division 100 of the slider to coincide with one of the given numbers, and look for the number on the rule which coincides with the second number read on the slider. This, with the decimal point inserted in the proper place, will be the product required.

If when the index (division 1) of the slider is made to coincide with a given division a of the rule, the division b of the slider is off the rule, we must put 10 of the slider to coincide with a, and read the coincidence with b, which will then be on the scale. This number, with the decimal point properly placed, will be the product $a\ b$.

To use a slide rule to obtain a square or square root we require two logarithmic scales, one of these being double the length of the other, and the shorter scale being repeated. In the Gravet form of rule made in celluloid, as supplied by Messrs. Davis & Son, of Derby, the two scales are placed parallel to each other, and the slider moves between them. The slider also carries two scales, the counterparts of those on the rule.

The lower scale, which is 25 cm. long, gives a scale of logarithms from 1 to 10. The left-hand half of the upper scale, 12·5 cm. long, gives a scale of logarithms from 1 to 10 of half the dimensions adopted for the lower scale. The ·ght-hand half is an exact copy of this, and gives, there-

fore, when measuring from the index of the first scale, the logarithms of numbers from 10 to 100.

A certain length measured on the lower scale gives the logarithm of a number a, say. The same length measured along the upper scale is $2 \log a$, for the unit of measurement of the upper scale is half that of the lower, also

$$2 \log a = \log a^2.$$

Thus, to find the square of a number, look out the number on the lower scale, and take the reading on the upper scale which coincides with that found on the lower.

In order to determine the coincidence, a metal slide, called the Cursor, is employed. This is equivalent to a straight-edge at right angles to the length of the scale which can slide along the scale, and thus facilitates the reading of the coincidences.

The rule can be used to find the area of a circle of given radius in the following way:—The area of a circle of radius r is πr^2. The value of $\log \pi$ ($\log 3{\cdot}142$) is marked on the slide. Set this to the index of the upper scale. Set the cursor to the value of r on the lower scale, and note the reading on the upper scale. This corresponds to $\log r^2$. Take the reading on the upper scale of the slide which coincides with this, and we obtain the value of πr^2.

The cursor may be also used to obtain a continuous product without noting the intermediate steps in the following way:—To multiply a, b, c together, read a on the rule; set the zero of the slide to this; set the cursor to b on the slide. Move the slide until its index coincides with the cursor, and read c on the slide. The corresponding division on the rule gives the value of the product.

The reverse side of the slider in the rule described contains three scales. One of these is a scale of sines. the

second a scale of tangents. These are so divided that when either of them is brought into coincidence with the corresponding scale on the rule, the divisions of the rule give respectively the sine or tangent of the angle read on the slider scale. The upper scale of the rule is used for sines, the lower for tangents. The third scale is one of equal parts, and from it the logarithm of a number can be determined. For set this scale so that its zero coincides with the index of the lower scale of the rule, and read any number, a, say, on this scale. Then, since the distances of the divisions from the index of the scale are proportional to the logarithms of the corresponding numbers, and the whole length of the scale contains 10 divisions, we have the ratio

$$\log a : \log 10 = \text{distance of } a \text{ from end} : \text{whole length of scale.}$$

Set the cursor to division a, and take the corresponding reading on the scale of equal parts; let it be x divisions. Suppose that the whole length contains d divisions; then, since $\log 10 = 1$,

$$\log a = x/d.$$

In the rule already referred to $d = 500$, so that

$$\log a = 2\,x/1000.$$

This rule also contains a device whereby the logarithms, sines, and tangents may be read without reversing the slider. On the under side of the right-hand end of the scale there is a small opening, on each side of which an index mark is seen.

When the index of the scale of equal parts, or of sines or tangents coincides with these index marks, it will be found that the scales on the upper side of the rule and slider are coincident.

Now draw out the slider, and note the reading a on the

lower scale with which its index coincides. Note also the reading x on the scale of equal parts.

This last reading gives us the distance the slider has moved—that is, the distance between the index of the lower scale and the mark a; but this distance is proportional to log a, and we have, as before,

$$\log a / \log 10 = x/d,$$

d being the number of divisions on the scale of equal parts which correspond with the full length of the logarithmic scale.

An exactly similar method applies to finding sines or tangents.

The accuracy obtainable with a slide rule depends partly on the exactness with which it is divided, partly on the possible accuracy of setting. Under favourable circumstances an accuracy of 1 part in 500 is claimed for the rule we have been describing, but this varies in different parts of the scale. Thus suppose we wish to use the rule to multiply 9·22 by, say, 8·53. There are no divisions between 9·2 and 9·25, and the actual distance between these divisions is about ·75 mm. To set the slider to this so that the error in the result may be 1 part in 500, we have to estimate to about one-fifth of the distance between the marks, or say ·15 mm. To do this requires considerable care and practice. Then, again, we have no mark on the slider between 8·5 and 8·55. We have to judge by eye the position of 8·53, and also the division on the scale which coincides with this.

The cursor is of help in this, and it is easy to see that the division required lies between 78·5 and 79. Dividing the distance between these divisions by eye with the aid of a magnifying glass, we get as the result 78·6 . . . , and the last figure will be certainly right to 1, which is about 1 in 800 in the result. As another example, suppose we wish to find the circumference of a circle 1·752 inches in diameter. To

read the last figure correctly on the scale we have to subdivide to tenths a distance of about ·5 mm.; but an error of 2 in this figure, with a corresponding error in the value of π (3·1416), will only affect the result to 1 part in 500. There are no divisions between 3·14 and 3·15, but the distance between these two can be subdivided into fifths, and we can set the cursor to 3·142, correct to ·002.

The product lies between 5·50 and 5·55, and this distance, which is well over 1 mm. in length, can be subdivided to fifths with certainty. We obtain as the result 5·51, the true value being 5·504.

Or, again, find the angle whose sine is ·8.

The divisions in the neighbourhood of 8 on the upper scale, which is used here, are about ·75 mm., and we can set the scale with fair accuracy. The angle is seen to be between 53° and 54°. To get it more nearly we have to divide a distance of about a millimetre into parts. We can do this to fifths or sixths, giving an accuracy of, say, 10 minutes, or 1 in 300. For angles above 60° the degree divisions on the scale of sines are very small, while between 70° and 80° each division is 2°, and the divisions corresponding to 80° and 90° are only about 1 mm. apart. The value of sin a changes by about 1 per cent for 1° when a is about 60°, and the setting can be done to about one-fifth or one-sixth of a degree in this position. Thus it will be seen that with care the accuracy of nearly 1 in 500 is attainable over a wide range.

CHAPTER IV.

MEASUREMENT OF THE MORE SIMPLE QUANTITIES.

LENGTH MEASUREMENTS.

THE general principle which is made use of in measuring lengths is that of direct comparison (see p. 2); in other words, of laying a standard, divided into fractional parts, against the length to be measured, and reading off from the standard the number of such fractional parts as lie between the extremities of the length in question. Some of the more important methods of referring lengths to a standard, and of increasing the accuracy of readings, may be exemplified by an explanation of the mode of using the following instruments.

1. The Calipers.

This instrument consists of a straight rectangular bar of brass, D E (fig. 1), on which is engraved a finely-divided scale.

From this bar two steel jaws project. These jaws are at right angles to the bar; the one, D F, is fixed, the other, C G, can slide along the bar, moving accurately parallel to itself. The faces of these jaws, which are opposite to each other, are planed flat and parallel, and can be brought into contact. On the sliding piece C will be observed two short scales called verniers, and when the two jaws are in contact, one end of each vernier, marked by an arrowhead in the figure, coincides with the end of the scale on the bar.[1] If then, in any other case, we determine the position of this end of the vernier with reference to the scale, we find the distance between these two flat faces, and hence the length of any object which fits exactly between the jaws.

It will be observed that the two verniers are marked 'outsides' and 'insides' respectively.[2] The distance between the

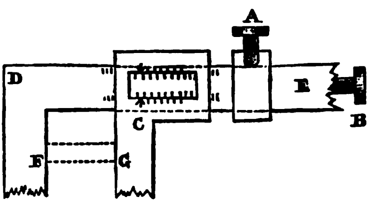
[1] If with the instrument employed this is found not to be the case, a correction must be made to the observed length, as described in § 3. A similar remark applies to § 2.

[2] See frontispiece, fig. 3.

jaws will be given by the outsides vernier. The other pair of faces of these two jaws, opposite to the two plane parallel ones, are not plane, but cylindrical, the axes of the cylinders being also perpendicular to the length of the brass bar, so that the cross section through any point of the two jaws, when pushed up close together, will be of the shape of two U's placed opposite to each other, the total width of the two being exactly one inch. When they are in contact, it will be found that the arrowhead of the vernier attached to the scale marked insides reads exactly one inch, and if the jaws of the calipers be fitted inside an object to be measured—e.g., the internal dimensions of a box—the reading of the vernier marked insides gives the distance required.

Suppose it is required to measure the length of a cylinder with flat ends. The cylinder is placed with its axis parallel to the length of the calipers. The screw A (fig. 1) is then turned so that the piece attached to it can slide freely along the scale, and the jaws of the calipers are adjusted so as nearly to fit the cylinder (which is shown by dotted lines in the diagram). The screw A is then made to bite, so that the attached piece is 'clamped' to the scale. Another screw, B, on the under side of the scale, will, if now turned, cause a slow motion of the jaw C G, and by means of this the fit is made as accurate as possible. This is considered to be attained when the cylinder is *just held firm.* This screw B is called the 'tangent screw,' and the adjustment is known as the 'fine adjustment.'

FIG. 1.



It now remains to read upon the scale the length of the cylinder. On the piece C will be seen two short scales—the 'outsides' and 'insides' already spoken of. These short scales are called 'verniers.' Their use is to increase the

accuracy of the reading, and may be explained as follows: suppose that they did not exist, but that the only mark on the piece C was the arrowhead, this arrowhead would in all probability lie between two divisions on the large scale. The length of the cylinder would then be less than that corresponding to one division, but greater than that corresponding to the other. For example, let the scale be actually divided into inches, these again into tenths of an inch, and the tenths into five parts each; the small divisions will then be $\frac{1}{50}$ inch or ·02 inch in length. Suppose that the arrowhead lies between 3 and 4 inches, between the third and fourth tenth beyond the 3, and between the first and second of the five small divisions, then the length of the cylinder is greater than $3+\frac{3}{10}+\frac{1}{50}$, i.e. $>3{\cdot}32$ inches, but less than $3+\frac{3}{10}+\frac{2}{50}$, i.e. $<3{\cdot}34$ inches. The vernier enables us to judge very accurately what fraction of one small division the distance between the arrowhead and the next lower division on the scale is. Observe that there are twenty divisions on the vernier,[1] and that on careful examination one of these divisions coincides more nearly than any other with a division on the large scale. Count which division of the vernier this is—say the thirteenth. Then, as we shall show, the distance between the arrowhead and the next lower division is $\frac{13}{20}$ of a small division, that is $\frac{13}{1000}={\cdot}013$ inch, and the length of the cylinder is therefore

$$3+\frac{3}{10}+\frac{1}{50}+\frac{13}{1000}=3{\cdot}32+{\cdot}013=3{\cdot}333 \text{ inch.}$$

We have now only to see why the number representing the division of the vernier coincident with the division of the scale gives in thousandths of an inch the distance between the arrowhead and the next lower division.

Turn the screw-head B till the arrowhead is as nearly coincident with a division on the large scale as you can make it. Now observe that the twentieth division on the vernier is coincident with another division on the large scale, and that the distance between this division and the first is nineteen small divisions. Observe also that no other

[1] Various forms of vernier are figured in the frontispiece.

divisions on the two scales are coincident. Both are evenly divided; hence it follows that twenty divisions of the vernier are equal to nineteen of the scale—that is, one division on the vernier is $\frac{19}{20}$ths of a scale division, or that one division on the vernier is less than one on the scale by $\frac{1}{20}$th of a scale division, and this is $\frac{1}{1000}$th of an inch.[1]

Now in measuring the cylinder we found that the thirteenth division of the vernier coincided with a scale division. Suppose the unknown distance between the arrowhead and next lower division is x. The arrowhead is marked 0 on the vernier. The division marked 1 will be nearer the next lower scale-division by $\frac{1}{1000}$th of an inch, for a vernier division is less than a scale division by this amount. Hence the distance in inches between these two divisions, the one on the vernier and the other on the scale, will be

$$x - \tfrac{1}{1000}.$$

The distance between the thirteenth division of the vernier and the next lower scale division will similarly be

$$x - \tfrac{13}{1000}.$$

But these divisions are coincident, and the distance between them is therefore zero; that is $x = \frac{13}{1000}$. Hence the rule which we have already used.

The measurement of the cylinder should be repeated four times, and the arithmetic mean taken as the final value. The closeness of agreement of the results is of course a test of the accuracy of the measurements.

The calipers may also be used to find the diameter of the cylinder. Although we cannot here measure surfaces which are strictly speaking flat and parallel, still the portions of the surface which are touched by the jaws of the calipers are very nearly so, being small and at opposite ends of a diameter.

Put the calipers on two low supports, such as a pair of glass rods of the same diameter, and place the cylinder on end upon the table. Then slide it between the jaws of the

[1] Generally, if n divisions of the vernier are equal to $\overline{n-1}$ of the scale, then the vernier reads to $1/n$th of a division of the scale.

calipers, adjusting the instrument as before by means of the tangent screw, until the cylinder is just clamped. Repeat this twice, reading the vernier on each occasion, and taking care each time to make the measurement across the *same diameter* of the cylinder. Next take a similar set of readings across a diameter at right angles to the former. Take the arithmetic mean of the different readings, as the result.

Having now found the diameter, you can calculate the area of the cross section of the cylinder. For this area is $\frac{\pi d^2}{4}$, d being the diameter. The volume of the cylinder can also be found by multiplying the area just calculated by the length of the cylinder.[1]

Experiments.

Determine the dimensions (1) of the given cylinder, (2) of the given sphere. Enter results thus :—

1. Readings of length of cylinder, of diameter.

	3·333 in.	Diam. 1	1·301 in.
	3·332 „		1·303 „
	3·334 „	Diam. 2	1·303 „
	3·334 „		1·302 „
Mean	3·3332 „	Mean	1·3022 „

Area = 1·3318 sq. in.
Volume = 4·4392 cu. in.

2. Readings of diameter of sphere.

Diam.	1	5·234 in.
„	2	5·233 „
„	3	5·232 „
„	4	5·233 „
Mean		5·233 „

2. The Beam-Compass.

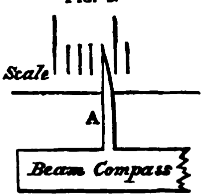
The beam-compass, like the calipers, is an instrument for measuring lengths, and is very similar to them in construction, consisting essentially of a long graduated beam

[1] A cylinder whose volume has been thus determined can be used to find the true density of water in grammes per c.c. The additional observations required are the weight of the cylinder in vacuo and in water.

with one steel compass-point fixed at one end of it, and another attached to a sliding piece provided with a fiducial mark and vernier. These compass-points take the place of the jaws of the calipers. It differs from them however in this, that while the calipers are adapted for end-measures such as the distance between the two flat ends of a cylinder, the beam-compass is intended to find the distance between two marks on a flat surface. For example, in certain experiments a paper scale pasted on a board has been taken to represent truly the centimetres, millimetres, &c. marked upon it. We now want to know what error, if any, there is in the divisions. For this purpose the beam-compass is placed with its scale parallel to the paper scale, and with the two compass points lying in a convenient manner upon the divisions. It will be found that the beam-compass must be raised by blocks of wood a little above the level of the paper scale, and slightly tilted over till the points rest either just in contact with, or just above, the paper divisions.

One of the two points is fixed to the beam of the compass; we will call this A. The other, B, is attached to a sliding piece, which can be clamped by a small screw on a second sliding piece. First unclamp this screw, and slide the point B along, till the distance A B is roughly equal to the distance to be measured. Then clamp B, and place the point A (fig. 2) exactly on one of the marks. This is best effected by gentle taps at the end of the beam with a small mallet.

FIG. 2.



It is the *inside edge* of the compass-point which has to be brought into coincidence with the mark. Now observe that, although B is clamped it is capable of a slow motion by means of a second screw called a 'tangent screw,' whose axis is parallel to the beam. Move this screw, with so light a touch as not to disturb the position of the beam-compass, until the point B is on the other mark, i.e. the inside edge of B coincides with

the division in question. Suppose that the point A is on the right-hand edge of the paper scale division, then B should also be on the right-hand edge of the corresponding division. To ensure accuracy in the coincidence of the edges you must use a magnifying-glass.

You have now only to read the distance on the beam-scale. To do this observe what are the divisions between which the arrowhead of the vernier[1] falls. Then the reading required is the reading of the lower of these divisions + the reading of the vernier. The divisions are each 1 millimetre. Hence, if the arrowhead falls between the 125th and 126th, the reading is 125 mm. + the reading of the vernier.

Observe which division of the vernier is in the same straight line with a division of the scale. Suppose the 7th to be so situated. Then the reading of the vernier is $\frac{7}{10}$ mm. and the distance between the points is 125·7 mm.

Repeat the observation twice, and suppose that 125·6 and 125·7 are the readings obtained, the mean of the three will be 125·66, which may be taken as the true distance between the marks in question.

Suppose that on the paper scale this is indicated by 126 mm., then to make the scale true we must reduce the reading by ·34 mm. This is the scale correction for this division.

Experiment.—Check by means of the beam-compass the accuracy of the divisions of the given centimetre scale.

Enter results thus :—

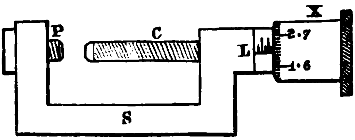
Division of scale at which A is placed	Division of scale at which B is placed	Vernier readings (mean of 3 obs.)
0	1 cm.	1·005 cm.
"	2 "	2·010 "
"	3 "	3·010 "
"	4 "	4·015 "
"	5 "	5·015 "
	etc.	

[1] See frontispiece, fig. 1.

3. The Screw-Gauge.

This instrument (fig. 3) consists of a piece of solid metal S, with two arms extending perpendicularly from its two ends. To the one arm a steel plug, P, with a carefully planed face, is fixed, and through the other arm, opposite to the plug, a screw C passes, having a plane face parallel and opposite to that of the plug. The pitch of the screw is half a millimetre, and consequently if we can count the number of turns and fractions of a turn of the screw from its position when the two plane faces (viz. that of the plug and that of the screw) are in contact, we can determine the distance in millimetres between these two parallel surfaces when the screw is in any position.

FIG. 3.



In order to do this the more conveniently, there is attached to the end of the screw farther from the plug a cap X, which slides over the cylindrical bar through which the screw passes; this cap has a bevelled edge, the circumference of which is divided into fifty equal parts. The circle on the cylindrical bar, which is immediately under the bevelled edge, when the two opposing plane surfaces are in contact, is marked L, and a line drawn parallel to the length of the cylinder is coincident (if the apparatus is in perfect adjustment) with one of the graduations on the bevelled edge which we will call the zero mark of that edge. Along this line a scale is graduated to half-millimetres, and hence one division of the scale corresponds to one complete turn of the cap and screw. Hence the distance between the parallel planes can be measured to half a millimetre by reading on this scale.

We require still to determine the fraction of a turn. We know that a complete revolution corresponds to half a millimetre; the rotating edge is divided into fifty parts, and

therefore a rotation through a single part corresponds to a separation of the parallel planes by $\frac{1}{100}$ mm. Suppose, then, that the scale or line along which the graduations on the cylinder are marked, cuts the graduations on the edge of the cap at 12·2 divisions from the zero mark; then since, when a revolution is complete, the zero mark is coincident with the line along which the graduations are carried on the cylinder, the distance between the parallel planes exceeds the number of complete revolutions read on that scale by $\frac{12 \cdot 2}{50}$ths of a turn, i.e. by ·122 mm.

If then we number every tenth division on the bevelled edge successively 1, 2, 3, 4, 5, these numbers will indicate tenths of a millimetre; 5 of them will be a complete turn, and we must go into the next turn for 6, 7, 8, 9 tenths of a millimetre. It will be noticed that on the scale graduated on the fixed cylinder the smaller scratches correspond to the odd half-millimetres and the longer ones to the complete millimetres. And on the revolving edge there are two series of numbers, 1, 2, 3, 4, 5 inside, and 6, 7, 8, 9, 10 outside. A little consideration will shew that the number to be taken is the inside or the outside one according as the last visible division on the fixed scale is a complete millimetre division or an odd half-millimetre division.

We can therefore read by this instrument the distance between the parallel planes to $\frac{1}{100}$th of a millimetre, or by estimating the tenth of a division on the rotating edge to the $\frac{1}{1000}$th of a millimetre.

We may use the instrument to measure the length of a short cylinder thus. Turn the screw-cap, holding it quite lightly, so that, as soon as the two parallel planes touch, the fingers shall slip on the milled head, and accordingly shall not strain the screw by screwing too hard.[1] Take a reading when the two planes are in contact; this gives the zero read-

[1] Special provision is made for this in an improved form of this apparatus. The milled head is arranged so that it slips past a ratchet wheel whenever the pressure on the screw-face exceeds a certain limit.

ing, which must be *added* to any observation reading if the zero of the scale has been *passed*, *subtracted* if it has *not* been *reached.* Then separate the planes and introduce the cylinder with its ends parallel to those of the gauge, and screw up again, holding the screwhead as nearly as possible with the same grip as before, so that the fingers shall slip when the pressure is as before. Then read off on the scales. Add or subtract the zero correction as the case may be; a reading of the length of the cylinder is thus obtained. Read the zero again, and then the length of the cylinder at a different part of the area of the ends, and so on for ten readings, always correcting for the zero reading.

Take the mean of the readings for the length of the cylinder, and then determine the mean diameter in the same way.

The diameter of a wire may also conveniently be found by this instrument.

The success of the method depends on the touch of the screwhead, to make sure that the two planes are pressed together for the zero reading with the same pressure as when the cylinder is between them.

Be careful not to strain the screw by screwing too hard.

Experiment.—Measure the length and diameter of the given small cylinder.

Enter result thus :—

Correction for zero	+ ·0003	cm.
Length (mean of ten)	·9957	„
True length	·9960	„

4. The Spherometer.

The instrument consists of a platform with three feet, whose extremities form an equilateral triangle, and in the middle of the triangle is a fourth foot, which can be raised or lowered by means of a micrometer screw passing perpendicularly through the centre of the platform. The readings

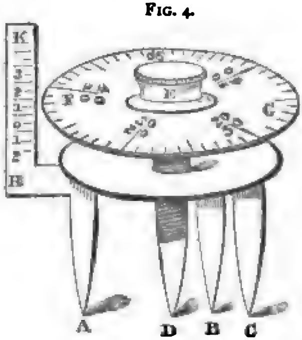
of the spherometer give the perpendicular distance between the extremity of this fourth foot and the plane of the other three.

It is used to measure the radius of curvature of a spherical surface, or to test if a given surface is truly spherical.

The instrument is first placed on a perfectly plane surface—a piece of worked glass—and the middle foot screwed down until it touches the surface. As soon as this is the case, the instrument begins to turn round on the middle foot as a centre. The pressure of the hand on the screw should be very light, in order that the exact position of contact may be observed. The spherometer is then carefully removed from the glass, and the reading of the micrometer screw is taken.

The figure (fig. 4) will help us to understand how this is done. A B C are the ends of the three fixed feet; D is the movable foot, which can be raised by turning the milled head at E. This carries round with it the graduated disc F G, and as the screw is turned the disc travels up the scale H K. The graduations of this scale are such that one complete revolution of the screw carries the disc from one graduation to the next. Thus in the figure the point F on the disc stands opposite to a division of the scale, and one complete turn would bring this point opposite the next division. In the instrument in the figure the divisions of the scale are half-millimetres, and the millimetres are marked 0, 1, 2. Thus only every second division is numbered.

FIG. 4.



But the rim of the disc F G is divided into fifty parts,

and each of these subdivided into ten. Let us suppose that division 12 of the disc is opposite to the scale at F, and that the milled head is turned until division 36 comes opposite. Then the head has been turned through 24 (i.e. 36 − 12) larger divisions; but one whole turn or fifty divisions carry the point D through $\frac{1}{2}$ mm. Thus a rotation through twenty-four divisions will carry it through $\frac{24}{50}$ of $\frac{1}{2}$ mm. or ·24 mm.

Hence the larger divisions on the disc F G correspond to tenths of a millimetre, and these are subdivided to hundredths by the small divisions.

Thus we might have had opposite to the scale in the first instance 12·6 large divisions, and in the second 36·9. Then the point D would have moved through ·243 mm.

It will be noticed that in the figure division O is in the centre of the scale H K, which is numbered 1, 2, 3, &c., from that point in both directions up and down. The divisions numbered on the disc F G are the even ones[1]—2, 4, 6, &c.—and there are two numbers to each division. One of these numbers will give the parts of a turn of the screw when it is turned so as to lower the point D, the other when it is turned so as to raise D. Thus in the figure 12 and 38 are both opposite the scale, and in the second position, 36 and 14. We have supposed the head to be turned in such a way that the point D has been lowered through ·24 mm. If the rotation had been in the opposite direction, D would have been raised through 0·26 mm.

Let us for the present suppose that all our readings are above the zero of the scale.

To take a reading we note the division of the scale next above which the disc stands, and then the division of the disc which comes opposite to the scale, taking care that we take the series of divisions of the disc which corresponds to a motion of the point D in the upward direction—the

[1] These numbers are not shewn in the figure.

inner ring of numbers in the figure. Thus the figured reading is 1·380.

If the instrument were in perfect order, the reading when it rested on a plane surface would be 0·0. This is not generally the case, so we must observe the reading on the plane. This observation should be made four times, and the mean taken. Let the result be ·460. Now take the instrument off the plane and draw the middle foot back some way. We will suppose we are going to measure the radius of a sphere from the convex side.

Place the instrument on the sphere and turn the screw E until D touches the sphere. The position of contact will be given as before, by noticing when the instrument begins to turn round D as a centre.

Read the scale and screw-head as before; let the scale reading be :—

2·5 ; and the disc ·235.

Then the reading is 2·735 mm.

Take as before four readings.

We require the distance through which the point D has been moved. This is clearly the difference between the two results, or 2·735 − ·460 ; if we call this distance a we have

$$a = 2{\cdot}275 \text{ mm.}$$

It may of course happen that the reading of the instrument when on the plane is below the zero ; in this case to find the distance a we must add the two readings.

We must now find the distance in millimetres between the feet AB or AC. We can do this directly by means of a finely divided scale; or if greater accuracy is required, lay the instrument on a flat sheet of card or paper, and press it so as to mark three dots on the paper, then measure the distance between these dots by the aid of the beam-compass (§ 2).

Let us call this length l. Then we can shew[1] that, if r be the radius required,

$$r = \frac{l^2}{6a} + \frac{a}{2}.$$

The observation of l should be repeated about four times.

If we wish merely to test if a given surface is spherical, we must measure a for different positions of the apparatus on the surface, and compare the results; if the surface be spherical, the value of a will be the same for all positions.

Experiments.

(1) Test the sphericity of the given lens by observing the value of a for four different positions.

(2) Determine the radius of the given sphere for two positions, and compare the results with that given by the calipers.

Enter results thus:—

Readings on plane	Readings on sphere
0·460	2·735
0·463	2·733
0·458	2·734
0·459	2·739
Mean 0·460	Mean 2·735

$a = 2·275$ mm.

Obs. for l 43·56 43·52 43·57 43·59. Mean 43·56.

$r = 140·146$ mm.

By calipers $r = 5·517$ in. $= 140·12$ mm.

[1] Since the triangle formed by the three feet is equilateral, the radius of the circumscribing circle is $\frac{l}{2 \sin 60°}$, i.e. $\frac{l}{\sqrt{3}}$. But a being the portion of the diameter of the sphere, radius r, cut off by the plane of the triangle, we have (Euc. iii. 35)

$$a\,(2r - a) = \frac{l^2}{3}, \text{ whence } r = \frac{l^2}{6a} + \frac{a}{2}.$$

If the distance between the centre foot and any one of the three outside feet be measured, the result is the radius of the circumscribing circle itself.

5. Measurement of a Base-Line.

The object of this experiment, which is a working model of the measurement of a geodetic base-line, is to determine with accuracy the distance between the scratches on two plugs so far apart that the methods of accurate measurement described above are inapplicable.

The general plan of the method is to lay ivory scales end to end, fixing them by placing heavy weights on them, and to read by means of a travelling reading microscope the distance between the extreme graduations of the two ivory scales, or between the mark on the plug and the extreme graduation of the ivory scale placed near it. We have then to determine the real length of the ivory scales, and by adding we get the total length between the plugs.

The experiment may therefore be divided into three parts.

(1). *To determine the Distance between the End Graduations of the Ivory Scales placed end to end.*

This is done by means of the travelling microscope. Place the scales with their edges along a straight line drawn between the two marks perpendicular to the scratches, and fix them so that the extreme graduations are within $\frac{1}{8}$th inch. Next place the microscope (which is mounted on a slide similar to the slide-rest of a lathe, and moved by a micrometer screw the thread of which we will suppose is $\frac{1}{50}$th of an inch) so that the line along which it travels on its stand is parallel to the base line, and focus it so that one of its cross-wires is parallel and coincident with *one edge* of the image of the end graduation of the one ivory scale. (It is of no consequence which edge is chosen, provided it be always the same in each case.)

Read the position of the microscope by its scale and micrometer screw, remembering that the fixed scale along which the divided screw-head moves is graduated to 50ths of an inch, and the circumference of the screw-head into

200 parts; each part corresponds, therefore, to $\frac{1}{10000}$ inch. So that if the reading on the scale be 7, and on the screw-head 152, we get for the position—

$$\begin{aligned} 7 \text{ divisions of the scale} &= \tfrac{7}{50} \text{ in.} = 0{\cdot}14 \text{ in.} \\ 152 \text{ divisions of the screw-head} &= 0{\cdot}0152 \text{ in.} \\ \text{Reading} &= 0{\cdot}1552 \text{ in.} \end{aligned}$$

Or if the scale reading be 5 and the screw-head reading 15, the reading similarly is 0·1015 in.

Next turn the micrometer screw-head until the last division on the other ivory scale comes into the field of view, and the *corresponding* edge of its image is coincident with the cross-wire as before. Read again; the difference of the two readings gives the required distance between the two graduations.

In the same way the distance between the scratch on the plug and the end division of the scale may be determined.

Place one ivory scale so that one extremity is near to or coincident with the scratch on the plug; read the distance between them; then place the other scale along the line and end-on with the first, and measure the distance between the end divisions of the two scales. Then transfer the first scale to the other end of the second; measure the distance between them again; and so on.

(2). *To Estimate the Fraction of a Scale over.*

This may be done by reading through the microscope the division and fraction of a division of the scale corresponding to the scratch on the second plug. This gives the length of a portion of the scale as a fraction of the true length which is found in (3).

(3). *To Determine the true Length of the Ivory Scales.*

This operation requires two reading microscopes. Focus these two, one on each extreme division of the scales to be measured, taking care that the same edge of the scratch is used as before. Then remove the scale, introduce a standard whose graduation can be assumed to be accurate,

or whose true length is known, and read by means of the micrometer the exact length, through which the microscopes have to be moved in order that their cross-wires may coincide with two graduations on the standard the distance between which is known accurately.[1]

The lengths of all the separate parts of the line between the marks, which together make up the whole distance to be measured have thus been expressed in terms of the standard or of the graduations of the micrometer screw. These latter may be assumed to be accurate, for they are only used to measure distances which are themselves small fractions of the whole length measured (see p. 41). All the data necessary to express the whole length in terms of the standard have thus been obtained.

Experiment.—Measure by means of the two given scales and the microscope the distance between the two given points.

Enter the results thus :—

Distance from the mark on first plug to the end graduation of Scale A		0·1552 in.
Distance between end graduations of Scales A and B	(1)	0·1015 „
„ „ „	(2)	0·0683 „
„ „ „	(3)	0·0572 „
„ „ „	(4)	0·1263 „
„ „ „	(5)	0·1184 „
Total of intervals		·6269 in.
Reading of Scale B at the mark on the second plug		10·631 „
True length of Scale A		12·012 „
„ „ B		11·993 „

Total distance between the marks

$$= 3 \times 12{\cdot}012 + 2 \times 11{\cdot}993 + 10{\cdot}631 + 0{\cdot}6269$$
$$= 71{\cdot}280 \text{ in.}$$

Observations of similar character will enable us to compare together two scales, such as a metre and a yard. For this purpose two travelling microscopes are required. The slides of the two are mounted on a board so as to be

[1] For less accurate measurements the length of the scales may also be determined by the use of the beam-compass, § 2.

parallel and in the same straight line, the distance between the two being about a yard. Each slide is furnished with a scale of millimetres, and verniers reading to one-tenth of a millimetre are attached to the microscopes. Cross-wires are fixed in the eye-piece of the microscopes.

Place the yard-measure on the board parallel to the slides, and focus each microscope on marks on the measure.

Set the cross-wire so as to bisect the broad image of a division of the measure, the cross-wire being parallel to the division. Do not observe the actual end of the measure—it is difficult to focus this satisfactorily—but choose some division near the end, say, one inch from the end in each case. To determine which division is chosen, move a piece of paper on the scale until its edge appears just to coincide with the cross-wires, and then note the division by looking at the scale directly. The distance between the cross-wires of the microscopes is now known in terms of the divisions of the measure. Let us suppose that this distance is 34 inches.

Read the scale and vernier attached to each microscope; let the readings be a and b, a being that of the left-hand microscope, and suppose the scales read from left to right. Let l be the distance between the cross-wires of the two microscopes where the scale reading of each is zero.

$$\text{Then} \qquad 34 = l + b - a,$$
$$\therefore l = 34 \text{ in.} + a - b.$$

Now remove the yard-measure and replace it by the metre scale. Set the cross-wires, as before, on two suitable divisions. This should be done without altering the focus of the microscopes; if the scale when placed in position is not distinct, it can be raised and supported by wedges of wood of proper thickness. Determine as before the divisions on which the cross-wires are set. Suppose them to be 860 millimetres apart, and let the readings of the microscope scales be a' and b'; then, as before,

$$l = 860 \text{ mm.} + a' - b',$$
$$\therefore 34 \text{ inches} = 860 \text{ mm.} + a' - a - (b' - b).$$

By this experiment we determine the number of millimetres in an inch, assuming the scale of the microscopes and the metre scale to be accurate. Test the accuracy of the slide-scale by comparing it with the metre scale, and then express (1) the yard, (2) the foot, (3) the inch in millimetres, comparing your results with the recognised values.

Enter the results thus :—

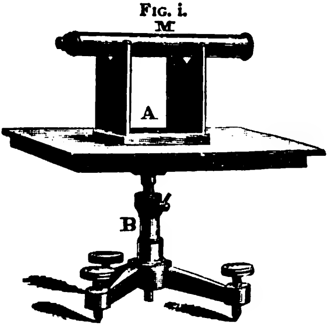
Distance between selected marks on the yard scale .	34	inches
First reading of left-hand microscope . . .	8·2	mm.
„ „ right-hand microscope . . .	9·9	„
Distance between marks on metre scale . . .	860	„
Second reading of left-hand microscope . . .	12·5	„
„ „ right-hand microscope . .	10·4	„

$$34 \text{ inches} = 860 + (12{\cdot}5 - 8{\cdot}2) - (10{\cdot}4 - 9{\cdot}9)$$
$$= 860 + 3{\cdot}8 = 863{\cdot}8 \text{ mm.}$$

A. The Kathetometer Microscope.

This is an instrument devised by Prof. Quincke for measuring with great accuracy small vertical heights. A metal stand (A, fig. i) carries a microscope M, resting horizontally in two Y-shaped supports ; the under side of the metal stand is cemented to a piece of flat glass. The microscope has a fine micrometer scale in its eye-piece. Resting on three levelling-screws is a small table B. The upper surface of this table is flat glass, on which the stand A rests.

FIG. i.



The stand can be easily moved about into any position, there being very little friction between the two glass sur-

faces. By dusting lycopodium over the table the adjustment is facilitated.

In using the instrument the glass table is first levelled by the aid of the screws and a spirit-level; the micrometer scale is then set vertical, and the value of a division determined; to do this a finely divided scale is required. This scale is set vertical; for this purpose it is convenient to have it attached to a small levelling-table, with a circular level, in such a way that when the level is set the scale is vertical. The microscope is focussed on the scale, and the readings of the micrometer divisions corresponding to the consecutive divisions of the scale are taken; from these the value of one micrometer division is found. The instrument may now be used to determine small differences in vertical height; if the two marks, the height-difference of which is required, are so placed that they can be brought into the field of view of the microscope simultaneously, the difference of their heights can be read off directly on the scale. When this is not the case, by moving the microscope, bring one mark into focus, and read off its position on the scale; then, without altering the position of the microscope in the Ys, slide the stand A over the horizontal glass plate until the second mark is in focus, and read its position on the scale. The height of the axis of the microscope above the glass plate and the inclination of the axis to the horizon remain unaltered by this motion, and thus the difference between the two readings gives the difference of height between the two marks.

6. The Kathetometer.

This instrument consists of a vertical beam carrying a scale. Along the scale there slides a brass piece, supporting a telescope, the axis of which can be adjusted so as to be horizontal. The brass slide is fitted with a vernier

which reads fractions of the divisions of the scale, thus determining the position of the telescope.

The kathetometer is used to measure the difference in height between two points.

To accomplish this, a level fitted so as to be at right angles to the scale is permanently attached to the instrument, and the scale is placed vertical by means of levelling screws on which the instrument rests.

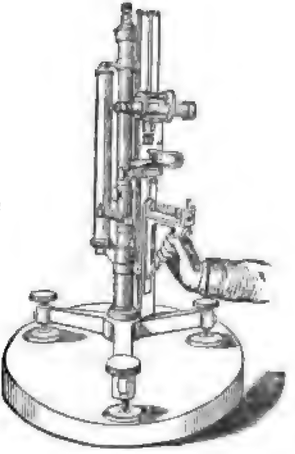
Let us suppose the instrument to be in adjustment, and let P, Q be the two points, the vertical distance between which is required.

The telescope of the instrument has, as usual, cross-wires in the eye-piece. Focus the telescope on the mark P, and adjust it until the image of P coincides with the horizontal cross-wire. Then read the scale and vernier.

Let the reading be 72·125 cm.

Raise the telescope until Q comes into the field, and adjust again till the image of Q coincides with the cross-wire; let the reading be 33·275 cm.

FIG. 5.



The difference in level between P and Q is

72·125 − 33·275, or 38·850 cm.

The adjustments are :—(1) To level the instrument so that the scale is vertical in all positions. (2) To adjust the telescope so that its axis is horizontal. (3) To bring the cross-wire in the focal plane of the telescope into coincidence with the image of the mark which is being observed.

(1) The scale must be vertical, because we use the instrument to measure the vertical height between two points.

The scale and level attached to it (fig. 5) can be turned

round an axis which is vertical when properly adjusted, carrying the telescope with them, and can be clamped in any position by means of a screw.

(*a*) *To test the Accuracy of the Setting of the Scale-level and to set the Axis of Rotation vertical.*

If the scale-level is properly set it is perpendicular to the axis of rotation; to ascertain whether or not this is so, turn the scale until its level is parallel to the line joining two of the foot-screws and clamp it; adjust these screws until the bubble of the level is in the middle. Unclamp, and turn the scale round through 180°. If the bubble is still in the middle of the level, it follows that this is at right angles to the axis of rotation; if the bubble has moved, then the level and the axis of rotation are not at right angles. We may make them so by adjusting the screws which fix the level to the instrument until the rotation through 180° produces no change, or, without adjusting the level, we may proceed to set the axis of rotation vertical if, instead of adjusting the levelling screws of the instrument until the bubble stands in the centre of the tube, we adjust them until the bubble does not move relatively to the tube when the instrument is turned through 180°.

This having been secured by the action of two of the screws, turn the scale until the level is at right angles to its former position and clamp. Adjust now in the same manner as before, using only the third screw.

It follows then that the bubble will remain unaltered in position for all positions of the instrument, and that the axis about which it turns is vertical.

If the scale of the instrument were parallel to the axis, it, too, would be vertical, and the instrument would be in adjustment.

(*b*) *To set the Scale vertical.*

To do this there is provided a metallic bracket-piece. One arm of this carries a level, while the other is a flat surface at right angles to the axis of the level, so that when

the level is horizontal this surface is truly vertical. The adjustment can be tested in the following manner. The level can rotate about its axis, and is weighted so that the same part of the tube remains uppermost as the bracket is rotated about the axis of the level. Place then the flat face of the bracket with the level uppermost against a nearly vertical plane surface; notice the position of the bubble. Then reverse it so that the level is lowest, and read the position of the bubble again. If it has not changed the level is truly set, if any displacement has taken place it is not so.

The scale of the instrument can be adjusted relatively to the axis of rotation and fixed by screws.

Press the flat surface of the bracket-piece against the face of the scale. If the scale be vertical, the bubble of the level on the bracket-piece will occupy the middle of its tube. Should it not do so, the scale must be adjusted until the bubble comes to the central position. We are thus sure that the scale is vertical.

For ordinary use, with a good instrument, this last adjustment may generally be taken as made.

Now turn the telescope and, if necessary, raise or lower it until the object to be observed is nearly in the middle of the field of view.

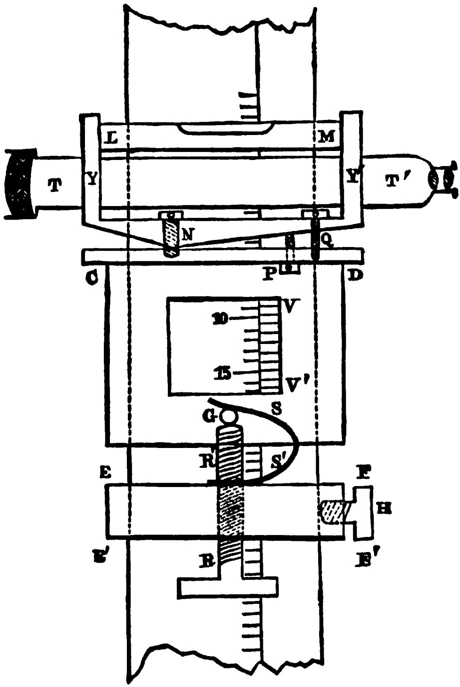
(2) It is necessary that the axis of the telescope should be always inclined to the scale at the same angle, for if, when viewing a second point Q, the angle between the axis and the scale has changed from what it was in viewing P, it is clear that the distance through which the telescope has been displaced will not be the vertical distance between P and Q.

If, however, the two positions of the axis be parallel, the difference of the scale readings will give us the distance we require.

Now the scale itself is vertical. The safest method, therefore, of securing that the axis of the telescope shall be always inclined at the same angle to the scale is to adjust

the telescope so that its axis shall be horizontal. The method of doing this will be different for different instruments. We shall describe that for the one at the Cavendish Laboratory in full detail; the plan to be adopted for other instruments will be some modification of this.

FIG. 6.



In this instrument (fig. 6) a level L M is attached to the telescope T T'. The telescope rests in a frame Y Y'. The lower side of this frame is bevelled slightly at N; the two surfaces Y N, Y' N being flat, but inclined to each other at an angle not far from 180°.

This under side rests at N on a flat surface C D, which is part of the sliding-piece C D, to which the vernier V V′ is fixed.

A screw passes through the piece Y Y′ at N, being fixed into C D. The hole in the piece Y Y′ is large and somewhat conical, so that the telescope and its support can be turned about N, sometimes to bring N Y into contact with C N, sometimes to bring N Y′ into contact with N D.

Fitted into C D and passing freely through a hole in N Y′ is a screw Q; P is another screw fitted into C D, which bears against N Y′. Hidden by P and therefore not shown in the figure is a third screw just like P, also fitted into C D, and bearing against N Y′. The screws N, P, and Q can all be turned by means of a tommy passed through the holes in their heads. When P and Q are both screwed home, the level and telescope are rigidly attached to the sliding-piece C D.

Release somewhat the screw Q. If now we raise the two screws P, we raise the eye-piece end of the telescope, and the level-bubble moves towards that end. If we lower the screws P, we lower the eye-piece end, and the bubble moves in the opposite direction.

Thus the telescope can be levelled by adjusting the screws P. Suppose the bubble is in the centre of the level. Screw down the screw Q. This will hold the telescope fixed in the horizontal position.

If we screw Q too firmly down, we shall force the piece N Y′ into closer contact with the screws P, and lower the eye-piece end. It will be better then to adjust the screw P so that the bubble is rather too near that end of the tube. Then screw down Q until it just comes to the middle of the tube, and the telescope is level.

(3) To bring the image of the object viewed to coincide with the cross-wires.

The piece C D slides freely up and down the scale. E F F′E′ is another piece of brass which also slides up and down.

H is a screw by means of which E F′ can be clamped fast to the scale. A screw R R′ passes vertically upwards through E F′ and rests against the under side of a steel pin G fixed in C D. Fixed to E F′ and pressing downwards on the pin G so as to keep it in contact with the screw R R′ is a steel spring S S′. By turning the screw R R′, after clamping H, a small motion up or down can be given to the sliding piece C D and telescope.

Now loosen the screw H and raise or lower the two pieces C D, E F′ together by hand, until the object viewed is brought nearly into the middle of the field of view. Then clamp E F′ by the screw H.

Notice carefully if this operation has altered the level of the telescope; if it has, the levelling must be done again.

By means of the screw R R′ raise or lower the telescope as may be needed until the image is brought into coincidence with the cross-wire. Note again if the bubble of the level is in its right position, and if so read the scale and vernier.

It may happen that turning the screw R R′ is sufficient to change the level of the telescope. In order that the slide C D may move easily along the scale, a certain amount of play must be left, and the friction between R′ and the pin is sometimes sufficient to cause this play to upset the level adjustment. The instrument is on this account a troublesome one to use.

The only course we can adopt is to level and then adjust R R′ till the telescope is in the right position, levelling again if the last operation has rendered it necessary.

This alteration of level will produce a small change in the position of the line of collimation of the telescope relatively to the vernier, and thus introduce an error, unless the axis round which the telescope turns is perpendicular both to the line of collimation and to the scale. If, however, the axis is only slightly below the line of collimation and the change of level small, the error will be very small indeed and may safely be neglected.

It is clear that the error produced by an error in levelling

will be proportional to the distance between the instrument and the object whose height is being measured. We should therefore bring the instrument as close to the object as is possible.

Experiment.—Adjust the kathetometer, and compare by means of it a length of 20 cm. of the given rule with the scale of the instrument.

Hang the rule up at a suitable distance from the kathetometer, and measure the distance between divisions 5 cm. and 25 cm.

The reading of the kathetometer scale in each position must be taken *three* times at least, the telescope being displaced by means of the screw R R′ between successive observations.

Enter results as below :—

	Kath. reading, upper mark	Kath. reading, lower mark
	25·315	45·325
	25·305	45·335
	25·320	45·330
Mean	25·3133	45·330
Difference	20·0167	

Mean error of scale between divisions 5 and 25, ·0167 cm.

MEASUREMENT OF AREAS.

7. Simpler Methods of measuring Areas of Plane Figures.

There are four general methods of measuring a plane area:—

(*a*) If the geometrical figure of the boundary be known, the area can be calculated from its linear dimensions—e.g. if the boundary be a circle radius r.

$$\text{Area} = \pi r^2 \text{ where } \pi = 3·142.$$

A table of areas which can be found by this method is given in Lupton's Tables, p. 7.

The areas of composite figures consisting of triangles and circles, or parts of circles, may be determined by addition of the calculated areas of all the separate parts.

In case two lengths have to be measured whose product determines an area, they must both be expressed in the same unit, and their product gives the area expressed in terms of the square of that unit.

(*b*) If the curve bounding the area can be transferred to paper divided into known small sections, e.g. square millimetres, the area can be approximately determined by counting up the number of such small areas included in the bounding curve. This somewhat tedious operation is facilitated by the usual grouping of the millimetre lines in tens, every tenth line being thicker. In case the curve cuts a square millimetre in two, the amount must be estimated; but it will be generally sufficient if portions greater than a half be reckoned a whole square millimetre and less than a half zero.

(*c*) By transferring the curve of the boundary to a sheet of paper or metal of *uniform* thickness and cutting it out, and cutting out a square of the same metal of known length of side, say 2 inches, and weighing these two pieces of metal. The ratio of their weights is the ratio of the areas of the two pieces of metal. The one area is known and the other may therefore be determined.

(*d*) By the planimeter. A pointer is made to travel round the boundary, and the area is read off directly on the graduated rim of a wheel.

For the theory of this instrument see Williamson's Integral Calculus (§ 149). Practical instructions are issued by the makers.

Experiment.—Draw a circle of 2 in. radius. Calculate or determine its area in all four ways, and compare the results.

Enter results thus :—

Method *a*	*b*	*c*	*d*
12·566 sq. in.	12·555 sq. in.	12·582 sq. in.	12·573 sq. in.

Orthogonal Projection.

Suppose that through all points of the boundary of an area, S, lines are drawn perpendicular to a given plane, the feet of these lines will trace out a curve in the plane ; this curve is said to be the orthogonal projection of the boundary of the given area, and the area bounded by the curve is the orthogonal projection of S.

It is easy to see that in orthogonal projection parallel straight lines are projected into parallel straight lines, and the ratio of their lengths is unaltered ; and also that the orthogonal projection of a finite straight line on a plane is equal in length to the length of the projected line multiplied by the cosine of its inclination to the straight line or plane.

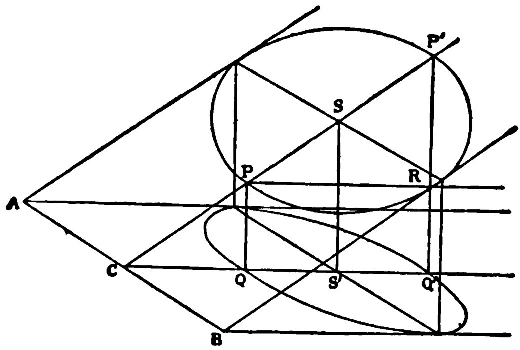
If S be an area cut out of a sheet of metal or cardboard, the form of its orthogonal projection can be obtained thus :—

Place a piece of paper on a horizontal drawing-board, and secure the area S in the required position above it. Then hold a plumb-line, made of a piece of thin silk, or cotton, and a shot, so that the plummet is just above the paper, while the line is made to touch in succession a number of points on the boundary of S, and mark the corresponding position of the plummet with a pencil-dot on the paper. If a sufficient number of points are taken, a curve can be drawn through them, and this curve will be the orthogonal projection of the boundary of S.

If the original area be plane, and if α be the angle between its plane and that of the projection, we may shew that the area of the projection is $S \cos \alpha$. For let AB (fig. ii) be the line in which the planes of the two areas intersect. Let a line PP', drawn perpendicular to AB, cut the boundary of S in P and P', and let Q, Q' be the projections of P and P'. Then $P'P$ and $Q'Q$ when produced meet AB in the same point C, and the angle PCQ is α. Let PR, drawn parallel to QQ', meet $P'Q'$ in R. Then $QQ' = PR = PP' \cos \alpha$.

Now the area S may be considered as made up of a large number of very narrow parallelograms with their lengths

FIG. 11.



parallel to P P' and their breadth parallel to A B. Each of these will be projected into a corresponding parallelogram of the same breadth, but of length Q Q' or P P' cos α. These projected parallelograms make up the projected area S'; the area of each parallelogram is decreased by projection in the ratio of cos α to unity. Thus the whole area S projects into an area S cos α.

The projection of a circle is a curve called an ellipse. Many of the most important geometrical properties of the ellipse can be very simply deduced by the method of projection from the corresponding properties of a circle (see Clifford's 'Elements of Dynamic,' chap. i.).

Experiment.—Cut out a circle of 3 inches radius. Fix it at an angle of between 30° and 60° to the horizon, and project it on to a piece of squared paper.

Find the angle α between the area and the horizon, and shew that the area of the projection = area of circle × cos α.

Measure also the maximum and minimum semi-diameters, a and b, of the ellipse, and shew that the area $= \pi a b$.

8. Determination of the Area of the Cross-Section of a Cylindrical Tube.—Calibration of a Tube.

The area of the cross-section of a narrow tube is best determined indirectly from a measurement of the volume of mercury contained in a known length of the tube. The principle of the method is given in Section 9. The tube should first be ground smooth at each end by rubbing on a stone with emery-powder and water, and then very carefully cleaned, first with nitric acid, then with distilled water, then with caustic potash, and finally rinsed with distilled water, and *very carefully dried* by passing air through it, which has been freed from dust by passing through a plug of cotton-wool and dried by chloride of calcium tubes.[1] If any trace of moisture remain in the tube, it is very difficult to get all the mercury to run out of it after it has been filled.

The tube is then to be filled with *pure*[2] mercury ; this is best done by immersing it in a trough of mercury of the necessary length. [A deep groove about half an inch broad cut in a wooden beam makes a very serviceable trough for the purpose.] When the tube is quite full, close the ends with the forefinger of each hand, and after the small globules of mercury adhering to the tube have been brushed off, allow the mercury to run into a small beaker, or other convenient vessel, and weigh it. Let the weight of the mercury be w. Measure the length of the tube by the calipers or beam-compass, and let its length be l. Look out in the table (33) the density of mercury for the temperature (which may be taken to be that of the mercury in the trough), and

[1] For this and a great variety of similar purposes an aspirating pump attached to the water-supply of the laboratory is very convenient. The different liquids may be drawn up the tube by means of an air-syringe.

[2] A supply of pure mercury may be maintained very conveniently by distillation under very low pressure in an apparatus designed by Weinhold (see Carl's *Rep.* vol. 15, and *Phil. Mag.* Jan. 1884).

let this be ρ. Then the volume v of the mercury is given by the equation

$$v=\frac{w}{\rho},$$

and this volume is equal to the product of the area A of the cross-section and the length of the tube. Hence

$$A=\frac{v}{l}=\frac{w}{\rho l}$$

If the length be measured in centimetres and the weight in grammes, the density being expressed in terms of grammes per c.c., the area will be given in sq. cm.

The length of the mercury column is not exactly the length of the tube, in consequence of the fingers closing the tube pressing slightly into it, but the error due to this cause is very small indeed.

This gives the mean area of the cross-section, and we may often wish to determine whether or not the area of the section is uniform throughout the length. To do this, carefully clean and dry the tube as before, and, by partly immersing in the trough, introduce a thread of mercury of any convenient length, say about 5 centimetres long. Place the tube along a millimetre scale, and fix it horizontally so that the tube can be seen in a telescope placed about six or eight feet off.

By slightly inclining the tube and scale, adjust the thread so that one end of it is as close as possible to the end of the tube, and read its length in the telescope. Displace the thread through 5 cm. and read its length again ; and so on, until the thread has travelled the whole length of the tube, taking care that no globules of mercury are left behind. Let l_1, l_2, l_3 . . . . be the successive lengths of the thread. Then run out the mercury into a beaker, and weigh as before. Let the weight be w, and the density of the mercury be ρ.

Then the mean sectional areas of the different portions of the tube are

$$\frac{w}{\rho\, l_1},\ \frac{w}{\rho\, l_2},\ \frac{w}{\rho\, l_3},\ \ldots\ \text{etc.}$$

The mean of all these values of the area should give the mean value of the area as determined above. The accuracy of the measurements may thus be tested.

On a piece of millimetre sectional paper of the same length as the tube mark along one line the different points which correspond to the middle points of the thread in its different positions, and along the perpendicular lines through these points mark off lengths representing the corresponding areas of the section, using a scale large enough to shew clearly the variations of area at different parts of the length. Join these points by straight lines. Then, the ordinates of the curve to which these straight lines approximate give the cross-section of the tube at any point of its length.

Experiment.—Calibrate, and determine the mean area of the given tube.

Enter the result thus :—

[The results of the calibration are completely expressed by the diagram.]

Length of tube	25·31	cm.
Weight of beaker . . .	10·361	gm.
Weight of beaker and mercury .	11·786	gm.
Weight of mercury . . .	1·425	gm.
Temperature of mercury .	·14°	C.
Density of mercury (table 33)	13·56	

$$\text{Mean area of section} = \frac{1·425}{25·31 \times 13·56} \text{ sq. cm.}$$

$$= 0·415 \text{ sq. mm.}$$

Mean of the five determinations for calibration 0·409 sq. mm.

MEASUREMENT OF VOLUMES.

The volumes of some bodies of known shape may be determined by direct calculation from their linear dimensions; one instance of this has been given in the experiment with the calipers.

A Table giving the relations between the volume and linear dimensions in those cases which are likely to occur most frequently will be found in Lupton's Tables, p. 7.

9. Determination of Volumes by Weighing.

Volumes are, however, generally determined from a knowledge of the mass of the body and the density of the material of which it is composed. Defining 'density' as the mass of the unit of volume of a substance, the relation between the mass, volume and density of a body is expressed by the equation $M = V\rho$, where M is its mass, V its volume, and ρ its density. The mass is determined by means of the balance (see p. 123), and the density, which is different at different temperatures, by one or other of the methods described below (see pp. 139–143). The densities of certain substances of definitely known composition, such as distilled water and mercury, have been very accurately determined, and are given in the tables (Nos. 32, 33), and need not therefore be determined afresh on every special occasion. Thus, if we wish, for instance, to measure the volume of the interior of a vessel, it is sufficient to determine the amount and the temperature of the water or mercury which exactly fills it. This amount may be determined by weighing the vessel full and empty, or if the vessel be so large that this is not practicable, fill it with water, and run the water off in successive portions into a previously counterpoised flask, holding about a litre, and weigh the flask thus filled. Care must be taken to dry the flask between the successive fillings; this may be rapidly and easily done by using a *hot* clean cloth. The capacity of vessels of very considerable

size may be determined in this way with very great accuracy.

All the specific gravity experiments detailed below involve the measurement of a volume by this method.

Experiment.—Determine the volume of the given vessel Enter results thus :—

	Weight of water	
Filling 1 . .	1001·2 gms.	
2 . .	998·7 ,,	
3 . .	1002·3 ,,	
4 . .	999·2 ,,	
5 . .	798·1 ,,	
Total weight .	4799·5 gms.	Temperature of water
Volume . .	4803·5 c.c.	in vessel, 15°.

10. Testing the Accuracy of the Graduation of a Burette.

Suppose the burette to contain 100 c.c. ; we will suppose also that it is required to test the capacity of each fifth of the whole.

The most accurate method of reading the burette is by means of a *float*, which consists of a short tube of glass loaded at one end so as just to float vertically in the liquid in the burette ; round the middle of the float a line is drawn, and the change of the level of the liquid is determined by reading the position of this line on the graduations of the burette. The method of testing is then as follows :—

Fill the burette with water, and read the position of the line on the float. Carefully dry and weigh a beaker, and then run into it from the burette about $\frac{1}{5}$th of the whole contents ; read the position of the float again, and weigh the amount of water run out into the beaker. Let the number of scale divisions of the burette be 20·2 and the weight *in grammes* 20·119. Read the temperature of the water ; then, knowing the density of water at that temperature (from table 32), and that 1 gramme of water at 4° C. occupies 1 c.c.,

we can determine the actual volume of the water corresponding to the 20·2 c.c. as indicated by the burette, and hence determine the error of the burette. Proceeding in this way for each $\frac{1}{5}$th of the whole volume, form a table of corrections.

Experiment.—Form a table of corrections for the given burette. Enter results thus :—

Burette readings	Error
0 – 5 c.c.	–·007 c.c.
5 – 10 „	–·020 „
10 – 15 „	–·011 „
15 – 20 „	·000 „
20 – 25 „	–·036 „

MEASUREMENT OF ANGLES.

The angle between two straight lines drawn on a sheet of paper may be roughly measured by means of a protractor, a circle or semi-circle with its rim divided into degrees. Its centre is marked, and can therefore be placed so as to coincide with the point of intersection of the two straight lines; the angle between them can then be read off on the graduations along the rim of the protractor. An analogous method of measuring angles is employed in the case of a compass-needle such as that required for § 69. Angles traced on a diagram may be determined by measuring lines from which one or other of the trigonometrical ratios can be calculated (see Chap. V.*).

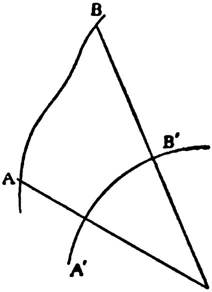
The more accurate methods of measuring angles depend on optical principles, and their consideration is accordingly deferred until the use of the optical instruments is explained (see §§ 62, 71).

MEASUREMENT OF SOLID ANGLES.

The angle which a plane curve joining any two points subtends at a third point O in the plane of the curve, as given by its 'circular measure,' may be found thus :—

Let A B be the curve. Join O A, O B, and with O as centre and any radius describe a circle A′ B′, cutting O A, O B in A′ and B′.

FIG. iii.

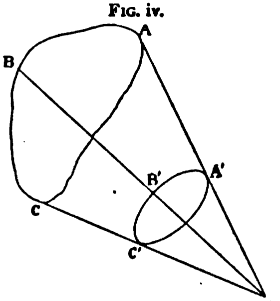


The ratio of the arc A′ B′ to the radius O A′ is the same for all values of the radius O A′, and is the measure of the angle A O B in 'radians'; if the radius O A′ be unity, then the arc A′ B′ measures the angle. The circular measure of an angle is the number of units of length in the arc of a circle of unit radius subtended by the angle.

A corresponding method is employed to measure the 'solid angle' subtended at a point by a surface.

Let O be the point, A B C the surface (fig. iv). With O as centre and any radius describe a sphere, and consider a line passing through O which moves so as to trace out the boundary of the area A B C. It will thus describe a cone cutting the sphere in a closed curve A′ B′ C′, and we can shew that the ratio of this area to the square of the radius O A′ is the same for all values of the radius.

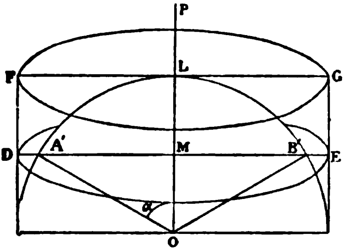
FIG. iv.



This ratio is adopted as defining the measure of the solid angle at O. If we take a sphere of unit radius, the ratio becomes the measure of the area A′ B′ C′, and we thus find that the solid angle subtended by an area at a point is measured by the number of units of area intercepted from a sphere of unit radius by a cone with the given point as vertex and the given area as base. If the area as seen from the given point appears circular in form, the cone is a right circular cone and the boundary A′ B′ C′ on the sphere is a circle. Let O L P (fig. v)

be the axis of this cone, and let O A′, the radius to any point on the circle, be inclined at an angle α to O P. Describe a cylinder with its axis parallel to O P touching the sphere. The circle A B C lies in a plane perpendicular to O P. Let this circle cut the cylinder in the circle D E, and let a plane touching the unit sphere in L cut the cylinder in F G. Then by an application of the method of projection it may be shewn that the area of the belt of the cylinder between D E and F G is equal to the corresponding area L A′ B′ C′ on the sphere, and this last measures the required solid angle at O. Let M be the centre of the circle A′ B′ C′.

FIG. v.



$$\begin{aligned} \text{The solid angle} &= \text{area of belt FDEG} \\ &= 2\pi\, DM \,.\, LM; \\ LM &= LO - OM = LO - OA' \cos\alpha \\ &= 1 - \cos\alpha; \end{aligned}$$

for $LO = OA' = 1$, the sphere being of unit radius.

Also $DM = 1$.

$\therefore$ Solid angle $= 2\pi(1 - \cos\alpha)$.

This expression, of course, only holds when the solid angle in question is that of a right circular cone.

It is clear from the above that a 'solid angle' is not an angle at all, but is only so named from analogy, being related to a sphere of unit radius in a manner similar to the relation between the circular measure of an angle and the circle of unit radius.

MEASUREMENTS OF TIME.

The time-measurements most frequently required in practice are determinations of the period of vibration of a needle. To obtain an accurate result some practice in the use of the 'eye and ear method' is required. The experi-

ment which follows (§ 11) will serve to illustrate the method and also to call attention to the fact that for accurate work any clock or watch requires careful 'rating,' *i.e.* comparison of its rate of going with some timekeeper, by which the times can be referred to the ultimate standard—the mean solar day. The final reference requires astronomical observations.

Different methods of time measurement will be found in §§ 21 and 28. The 'method of coincidences' is briefly discussed in § 20.

11. Rating a Watch by means of a Seconds-Clock.

The problem consists in determining, within a fraction of a second, the time indicated by the watch at the two instants denoted by two beats of the clock with a known interval between them. It will be noticed that the seconds-finger of the clock remains stationary during the greater part of each second, and then rather suddenly moves on to the next point of its dial. Our object is to determine to a fraction of a second the time at which it just completes one of its journeys.

To do this we must employ both the eye and ear, as it is impossible to read both the clock and watch at the same instant of time. As the watch beats more rapidly than the clock, the plan to be adopted is to watch the latter, and listening to the beating of the former, count along with it until it can be read. Thus, listening to the ticking of the watch and looking only at the clock, note the exact instant at which the clock seconds-finger makes a particular beat, say at the completion of one minute, and count along with the watch-ticks from that instant, beginning 0, 1, 2, 3, 4, . . and so on, until you have time to look down and identify the position of the second-hand of the watch, say at the instant when you are counting 21. Then we know that this time is 21 ticks of the watch after the event (the clock-beat) whose

time we wished to register; hence, if the watch ticks 4 times a second, that event occurred at $\frac{2}{4}$ seconds before we took the time on the watch.

We can thus compare to within $\frac{1}{4}$ sec. the time as indicated by the clock and the watch, and if this process be repeated after the lapse of half an hour, the time indicated by the watch can be again compared, and the amount gained or lost during the half-hour determined. It will require a little practice to be able to count along with the watch.

During the interval we may find the number of ticks per second of the watch. To do this we must count the number of ticks during a minute as indicated on the clock. There being 4 or 5 ticks per second, this will be a difficult operation if we simply count along the whole way; it is therefore better to count along in groups of either two or four, which can generally be recognised, and mark down a stroke on a sheet of paper for every group completed; then at the end of the minute count up the number of strokes; we can thus by multiplying, by 2 or 4 as the case may be, obtain the number of watch-ticks in the minute, and hence arrive at the number per second.

Experiment.—Determine the number of beats per second made by the watch, and the rate at which it is losing or gaining.

Enter results thus:—

No. of watch-ticks per minute, 100 groups of 3 each.
No. of ticks per second, 5.

	hr.	m.	s.
Clock-reading	11	38	3
Estimated watch-reading, 11 hr. 34 m. and 10 ticks =	11	34	2
Difference .		4	1
Clock-reading	12	8	3
Estimated watch-reading, 12 hr. 4 m. and 6 ticks =	12	4	1·2
Difference		4	1·8

Losing rate of watch, 1·6 sec per hour.

CHAPTER V.

MEASUREMENT OF MASS AND DETERMINATION OF SPECIFIC GRAVITIES.

12. The Balance.

General Considerations.

THE balance, as is well known, consists of a metal beam, supported so as to be free to turn in a vertical plane about an axis perpendicular to its length and vertically above its centre of gravity. At the extremities of this beam, pans are suspended in such a manner that they turn freely about axes, passing through the extremities of the beam, and parallel to its axis of rotation. The axes of rotation are formed by agate knife-edges bearing on agate plates. The beam is provided with three agate edges; the middle one, edge downwards, supporting the beam when it is placed upon the plates which are fixed to the pillar of the balance, and those at the extremities, edge upwards; on these are supported the agate plates to which the pans are attached.

The effect of hanging the pans from these edges is that wherever in the scale pan the weights be placed, the vertical force which keeps them in equilibrium must pass through the knife-edge above, and so the effect upon the balance is independent of the position of the weights and the same as if the whole weight of the scale pan and included masses were collected at some point in the knife-edge from which the pan is suspended.

In order to define the position of the beam of the balance, a long metal pointer is fixed to it, its length being perpendicular to the line joining the extreme knife-edges. A small scale is fixed to the pillar of the balance, and the motion of the beam is observed by noting the motion of the pointer along this scale. When the balance is in good adjustment, the scale should be in such a position that the pointer is

opposite the middle division when the scale-beam is horizontal. The only method at our disposal for altering the relative position of the scale and pointer is by means of the levelling screws attached to the case. Levels should be placed in the case by the instrument-maker, which should shew level when the scale is in its proper position.

In the investigation below we shall suppose the zero position of the balance to be that which is defined by the pointer being opposite the middle point of its scale, whether the scale is in its proper position, and the pointer properly placed or not.

The other conditions which must be satisfied if the balance is in perfect adjustment are :—

(1) The arms must be of equal length.

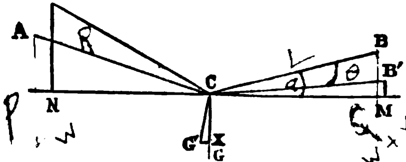
(2) The scale pans must be of equal weight.

(3) The centre of gravity of the beam must be vertically under the axis of rotation when the beam is in its zero position. This can always be ensured by removing the scale pans altogether, and by turning the small flag of metal attached to the top of the beam until the latter comes to rest with the pointer opposite the middle of its scale. Then it is obvious from the equilibrium that the centre of gravity is vertically under the axis of support.

On the Sensitiveness of a Balance.

Let us suppose that this third condition is satisfied, and that the points A, C, B (fig. 7) represent the points in which the three knife-edges cut a vertical plane at right angles to their edges, and let C A, C B make angles α, α' with a horizontal line through C. [If the balance is in perfect adjustment $\alpha = \alpha'$.]

FIG. 7.



We may call the lengths C A, C B the lengths of the arms

of the balance, and represent them by R, L respectively. Let the masses of the scale pans, the weights of which act vertically downward through A and B respectively, be P and Q. Let G, the centre of gravity of the beam, be at a distance h, vertically under C, and let the mass of the beam be K. If the balance be in adjustment, R is equal to L, and P to Q. Now let us suppose that a mass w is placed in the scale pan P, and a mass $w+x$ in Q, and that in consequence the beam takes up a new position of equilibrium, arrived at by turning about C through an angle θ, and denoted by B′ C A′, and let the new position of the centre of gravity of the beam be G′.

Then if we draw the vertical lines B′ M, A′ N to meet the horizontal through C in M and N, a horizontal line through G′ to meet C G in X, and consider the equilibrium of the beam, we have by taking moments about the point C

$$(\mathrm{Q}+w+x)\ \mathrm{CM} = (\mathrm{P}+w)\ \mathrm{CN} + \mathrm{K}\,.\,\mathrm{G'X}.$$

Now

$$\mathrm{CM} = \mathrm{CB'}\cos(\alpha'-\theta) = \mathrm{L}(\cos\alpha'\cos\theta + \sin\alpha'\sin\theta).$$
$$\mathrm{CN} = \mathrm{CA'}\cos(\alpha+\theta) = \mathrm{R}(\cos\alpha\cos\theta - \sin\alpha\sin\theta).$$
$$\mathrm{G'X} = \mathrm{CG'}\sin\theta = h\sin\theta.$$

Hence we get

$$\mathrm{L}(\mathrm{Q}+w+x)(\cos\alpha'\cos\theta+\sin\alpha'\sin\theta)$$
$$=\mathrm{R}(\mathrm{P}+w)(\cos\alpha\cos\theta-\sin\alpha\sin\theta)+\mathrm{K}h\sin\theta.$$

Since θ is very small, we may write $\tan\theta=\theta$,

$$\therefore\ \theta=\tan\theta=\frac{\mathrm{L}.(\mathrm{Q}+w+x)\cos\alpha'-\mathrm{R}(\mathrm{P}+w)\cos\alpha}{\mathrm{K}h-\mathrm{L}(\mathrm{Q}+w+x)\sin\alpha'-\mathrm{R}(\mathrm{P}+w)\sin\alpha}. \quad (1)$$

This gives us the position in which the balance will rest when the lengths of the arms and masses of the scale pans are known, but not necessarily equal or equally inclined to the horizon; and when a difference x exists between the masses in the scale pans.

It is evident that θ may be expressed in pointer scale divisions when the angle subtended at the axis of rotation by one of these divisions is known.

DEFINITION.—The number of scale divisions between the position of equilibrium of the pointer when the masses are equal and its position of equilibrium when there is a given small difference between the masses is called the sensitiveness of the balance for that small difference. Thus, if the pointer stand at 100 when the masses are equal and at 67 when there is a difference of ·001 gramme between the masses, the sensitiveness is 33 per milligramme.

We have just obtained a formula by which the sensitiveness can be expressed in terms of the lengths of the arms, &c.

Let us now suppose that the balance is in adjustment, i.e.

$$\mathrm{L}=\mathrm{R},\ \mathrm{Q}=\mathrm{P},\quad a=a'$$

$$\therefore \theta=\frac{\mathrm{L}x\cos a}{\mathrm{K}h-\mathrm{L}(2\mathrm{P}+2w+x)\sin a}\quad . \quad . \quad . \quad (2)$$

Hence the angle turned through for a given excess weight x increases proportionally with x, and increases with the length of the arm.

Let us consider the denominator of the fraction a little more closely. We see that it is positive or negative according as

$$\mathrm{K}h > \text{or} < \mathrm{L}\,(2\mathrm{P}+2w+x)\sin a.$$

Now it can easily be shewn that the equation

$$\mathrm{K}h=\mathrm{L}(2\mathrm{P}+2w+x)\sin a$$

leads to the condition that if x be zero, C is the centre of gravity of the beam and the weights of the scale pans &c. supposed collected at the extremities of the arms. In this case with equal weights w in the scale pans, the balance would be in equilibrium in any position.

If $\mathrm{K}\,h$ be less than $\mathrm{L}(2\mathrm{P}+2w+x)\sin a$, tan θ is negative, which shews that there is a position of equilibrium with the centre of gravity of the whole, above the axis; but it is reached by moving the beam in the opposite direction to that

in which the excess weight tends to move it: it is therefore a position of unstable equilibrium. We need only then discuss the case in which $\text{K}\,h$ is $> \text{L}(2\text{P}+2w+x)\sin a$, i.e. when the centre of gravity of the whole is below the axis of rotation.

With the extreme knife-edges above the middle one, a is positive and the denominator is evidently diminished, and thus the sensitiveness increased, as the load w increases; but if the balance be so arranged that $a=0$, which will be the case when the three knife-edges are in the same plane, we have

$$\tan\theta=\frac{\text{L}\,x}{\text{K}\,h},$$

or the sensitiveness is independent of the load; if the extreme knife-edges be below the mean, so that a is negative, then the denominator increases with the load w, and consequently the sensitiveness diminishes. Now the load tends to bend the beam a little; hence in practice, the knife-edges are so placed that when half the maximum load is in the scale pans, the beam is bent so that all the knife-edges lie in a plane, and the angle a will be positive for loads less than this and negative for greater loads. Hence, in properly made balances, the sensitiveness is very nearly independent of the load, but it increases slightly up to the mean load, and diminishes slightly from the mean to the maximum load.

The Adjustment of a Balance.

I. Suppose the balance is not known to be in adjustment.

Any defect may be due to one of the following causes:—

(1) The relative position of the beam and pointer and its scale may be wrong. This may arise in three ways: (a) the pointer may be wrongly fixed, (β) the balance may not be level, (γ) the pointer when in equilibrium with the pans unloaded may not point to its zero position. We

always weigh by observing the position of the pointer when at rest with the scale pans empty, and then bring its position of equilibrium with the pans loaded back to the same point. It is clear that this comes to the same thing as using a pointer not properly adjusted. In all these cases a will not be equal to a' in equation (1).

(2) The arms may not be of equal length, *i.e.* L not equal to R.

(3) The scale pans may not be of equal weight.

We may dispose of the third fault of adjustment first. If the scale pans be of equal weight, there can be no change in the position of equilibrium when they are interchanged; hence the method of testing and correcting suggests itself at once (see p. 101).

The first two faults are intimately connected with each other, and may be considered together. Let the pointer be at its mean position when there is a weight w in P and $w'+x$ in Q, w and w' being weights which are nominally the same, but in which there may be errors of small but unknown amount,

Then $\theta=0$ $\therefore$ $\tan\theta=0$ $\therefore$ from (1) (assuming P=Q)

$$\text{L}(\text{P}+w'+x)\cos a'=\text{R}(\text{P}+w)\cos a \quad . \quad . \quad . \quad (3)$$

Interchange the weights and suppose now that w in Q balances $w'+y$, in P, then

$$\text{L}(\text{P}+w')\cos a'=\text{R}(\text{P}+w'+y)\cos a \quad . \quad . \quad . \quad (4)$$

And if the pointer stands at zero when the pans are unloaded, we have

$$\text{L}\,.\,\text{P}\cos a'=\text{R}\,.\,\text{P}\cos a \quad . \quad . \quad . \quad . \quad (5)$$

Hence equations (3) and (4) become

$$\text{L}(w'+x)\cos a'=\text{R}\,w\cos a.$$
$$\text{L}\,w'\cos a'=\text{R}(w'+y)\cos a.$$

Multiplying

$$L^2 \cos^2 a' \, (w' + x) = R^2 \, (w' + y) \cos^2 a \quad . \quad . \quad . \quad (6)$$

$$\therefore \frac{L \cos a'}{R \cos a} = \sqrt{\frac{w' + y}{w' + x}}$$

$$= \sqrt{\frac{1 + \frac{y}{w'}}{1 + \frac{x}{w'}}}$$

$$= 1 + \frac{y - x}{2\,w'} \text{ approximately (p. 44).}$$

It will be seen on reference to the figure that L cos a' and R cos a are the projections of the lengths of the arms on a horizontal plane—i.e. the practical lengths of the arms considered with reference to the effect of the forces to turn the beam.

If the balance be properly levelled and the pointer straight $a = a'$, and we obtain the ratio of the lengths of the actual arms. We thus see that, if the pointer is at zero when the balance is unloaded, but the balance not properly levelled, the error of the weighing is the same as if the arms were unequal, provided that the weights are adjusted so as to place the pointer in its zero position. The case in which $a = -a'$ and therefore $\cos a = \cos a'$ will be an important exception to this; for this happens when the three knife-edges are in one plane, a condition which is very nearly satisfied in all delicate balances. Hence with such balances we may get the true weight, although the middle point of the scale may not be the equilibrium position of the pointer, provided we always make this equilibrium position the same with the balance loaded and unloaded. If we wish to find the excess weight of one pan from a knowledge of the position of the pointer and the sensitiveness of the balance previously determined, it will be

a more complicated matter to calculate the effect of not levelling.

We may proceed thus: Referring to equation (1), putting $P = Q$ we get

$$\tan\theta = \frac{L(P+w+x)\cos\alpha' - R(P+w)\cos\alpha}{Kh - L(P+w+x)\sin\alpha' - R(P+w)\sin\alpha}.$$

And since $\theta = 0$ when no weights are in the pans, we get

$$L\,P\cos\alpha' = R\,P\cos\alpha.$$

$$\therefore \tan\theta = \frac{L\,x\cos\alpha'}{Kh - L(w+P+x)\sin\alpha' - R(w+P)\sin\alpha}.$$

Since α and α' are always very small, we may put $\cos\alpha' = 1$ and $\sin\alpha' = \alpha'$, and so on, the angles being measured in circular measure (p. 45).

$$\therefore \tan\theta = \frac{L\,x}{Kh - L(P+w+x)\alpha' - R(P+w)\alpha}$$

$$= \frac{L\,x}{Kh}\left[1 + \frac{L(w+P+x)\alpha' + R(w+P)\alpha}{Kh}\right]$$

Neglecting x and the difference between L and P, in the bracket, since these quantities are multiplied by α or α', we have

$$\tan\theta = \frac{L\,x}{Kh}\left[1 + \frac{L(w+P)(\alpha'+\alpha)}{Kh}\right]$$

The error thus introduced is small, unless

$$\frac{L(w+P)}{Kh}$$

is a very large quantity, compared with α, and it well may be so, since h is small and $w+P$ may be many times K; but α in a well-made balance is generally so small that the effect is practically imperceptible, and if the knife-edges be in a plane, so that $\alpha = -\alpha'$, the correction vanishes.

Practical Details of Manipulation. Method of Oscillations.

All delicate balances are fitted with a long pointer fixed to the beam, the end of which moves over a scale as the beam turns.

The middle point of this scale should be vertically below the fulcrum of the beam, and if the balance be in perfect adjustment, when the scale pans are empty and the beam free, the end of the pointer will coincide with the middle division of the scale. This coincidence, however, as we have seen, is not rigorously necessary.

To weigh a body we require to determine first at what point of the scale the pointer rests when the pans are empty. We then have to put the body to be weighed in one pan and weights in the other, until the pointer will again come to rest opposite to the same division of the scale. The weight of the body is found by adding up the weights in the scale pan.

We shall suppose that the weights used are grammes, decigrammes, &c.

The weights in the boxes usually supplied are some of them brass and the others either platinum or aluminium.

The brass weights run from 1 gramme to 50, 100 or 1000 grammes in different boxes.

We may divide the platinum and aluminium weights into three series :—

The first includes,	·5,	·2,	·1,	·1	gramme
The second	·05,	·02,	·01,	·01	„
The third	·005,	·002,	·001,	·001	„

that is, the first series are decigrammes, the second centigrammes, and the third milligrammes.

The weights should never be touched with the fingers ; they should be moved by means of the small metal pliers provided for the purpose. In the larger boxes a brass bar is provided for lifting the heavier weights.

When the balance is not being used, the beam and the scale pans do not rest on the knife-edges but on independent

supports provided for them. The balance is thrown into action by means of a key in the front of the balance case. This must always be turned slowly and carefully, so as to avoid any jarring of the knife-edges from which the beam and scale pans hang.

When it is necessary to stop the beam from swinging, wait until the pointer is passing over the middle of the scale, and then turn the key and raise the frame till it supports the beam. The key must not be turned, except when the pointer is at the middle of the scale; for if it be, the supporting frame catches one end of the beam before the other, and thus jars the knife-edges.

The weights or object to be weighed when in the scale pans must never be touched in any way while the beam is swinging; thus, when it is required to change the weights, wait until the pointer is passing across the middle point of the scale, turn the key, and fix the beam, then move the weights from the scale pan.

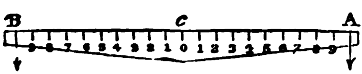
In the more delicate balances, which are generally enclosed in glass cases, it will be seen that the length of each arm of the beam is divided into ten parts.

Above the beam, and slightly to one side of it, there is a brass rod which can be moved from outside the balance case. This rod carries a small piece of bent wire, which can, by moving the rod, be placed astride the beam. This piece of wire is called a 'rider.' The weight of the rider is usually one centigramme.

Let A C B, fig. 8, be the beam, C being the fulcrum; the divisions on the arm are reckoned from C.

Suppose now we place the centigramme rider at division 1, that is one-tenth of the length of the arm away from the fulcrum, it will clearly require one-tenth of its own weight to be placed in the scale pan suspended from B, to balance it. The effect on the balance

FIG. 8.



beam of the centigramme rider placed at division 1, is the same as that of a weight of $\frac{1}{10}$ centigramme or 1 milligramme in the pan at A. By placing the rider at division 1, we practically increase the weight in the pan at A by 1 milligramme. Similarly, if we place the rider at some other division, say 7, we practically increase the weight in A by 7 milligrammes.

The rider should not be moved without first fixing the balance beam.

Thus without opening the balance-case we can make our final adjustments to the weights in the scale pan by moving the rider from outside.

The object of the case is to protect the balance from draughts and air currents. Some may even be set up inside the case by opening it and inserting the warm hand to change the weights ; it is therefore important in delicate work to be able to alter the weight without opening the case.

We proceed now to explain how to determine at what point of the graduated scale the pointer rests when the pans are empty. If the adjustments were quite correct, this would be the middle point of the scale. In general we shall find that the resting-point is somewhere near the middle.

We shall suppose for the present that the stand on which the balance rests is level. This should be tested by the spirit-level before beginning a series of weighings, and if an error be found, it should be corrected by moving the screw-feet on which the balance-case rests.

We shall find that the balance when once set swinging will continue in motion for a long period. The pointer will oscillate across the scale, and we should have to wait for a very long time for it to come to rest.

We require some method of determining the resting-point from observations of the oscillations.

Let the figure represent the scale, and suppose, reckoning from the left, we call the divisions 0, 10, 20, 30. . . .

A little practice enables us to estimate tenths of these divisions.

Watch the pointer as it moves; it will come for a moment to rest at P_1 suppose, and then move back again. Note the division of the scale, 63, at which this happens.[1] The pointer swings on past the resting-point, and comes to instantaneous rest again in some position beyond it, as P_2, at 125 suppose.

FIG. 9.

Now if the swings on either side of the resting-point were equal, this would be just half-way between these two divisions, that is at 94; but the swings gradually decrease, each being less than the preceding. Observe then a third turning point on the same side as the first, P_3 suppose, and let its scale reading be 69.

Take the mean 66, between 69 and 63. We may assume that this would have been the turning-point on that side at the moment at which it was 125 on the other, had the pointer been swinging in the opposite direction. Take the mean of the 125 and 66, and we have 95·5 as the value of the resting-point.

Thus, to determine the resting point:—

Observe three consecutive turning points, two to the left and one to the right, or *vice versâ*. Take the mean of the two to the left and the mean of this and the one to the right; this gives the resting-point required.

The observations should be put down as below.

	Turning-points		Resting-point
	Left	Right	
Mean 66	63 69	125	95·5

We may, if we wish, observe another turning-point to the right, 120 suppose; then we have another such series.

[1] A small mirror is usually fixed above the scale, the plânes of the two being parallel. When making an observation the observer's eye is placed so that the pointer exactly covers its own image formed in the mirror; any error due to parallax is thus avoided.

Proceeding thus we get a set of determinations of the resting-point, the mean of which will give us the true position with great accuracy.

Having thus found the resting point with the pans empty, turn the key or lever, and fix the beam; then put the object to be weighed in one scale pan. Suppose it to be the left-hand, for clearness in the description. Then put on some weight, 50 grammes say, and *just begin* to turn the key to throw the balance into action. Suppose the pointer moves sharply to the left, 50 gms. is too much. Turn the key back, remove the 50 and put on 20; just begin to turn the key; the pointer moves to the right, 20 is too little. Turn the key back, and add 10; the pointer still moves to the right; add 10 more, it moves to the left; 40 is too much. Turn the key back, remove the 10 and add 5. Proceed in this way, putting on the weights in the order in which they come, removing each weight again if the pointer move sharply to the left, that is, if it be obviously too much, or putting on an additional weight if the pointer move to the right.

There is no necessity to turn the key to its full extent to decide if a weight be too much or too little until we get very nearly the right weight; the first motion of the pointer is sufficient to give the required indication.

It saves time in the long run to put on the weights in the order in which they come in the box.

Caution.—The beam must always be fixed before a weight is changed.

Suppose now we find that with 37·68 grammes the pointer moves to the right, shewing the weight too little, and that with 37·69 the motion is to the left, shewing that it is too much. Close the balance-case, leaving on the lighter weight, 37·68 grammes. Turn the key, and notice if the pointer will swing off the scale or not. Suppose it is quite clear that it will, or that the resting-point will be quite at one end near the division 200. Fix the beam, and put on the rider say

at division 2. This is equivalent to adding ·002 gm. to the weights in the scale pan, so that the weight there may now be reckoned as 37·682 gms. Release the beam, and let it oscillate, and suppose that this time the pointer remains on the scale.

Read three turning-points as before.

	Turning-points		Resting-point
	Left	Right	
Mean 170	172 168	98	134

Thus we find that with no weights in the scale pans, the resting-point is 95·5—we may call this 96 with sufficient accuracy—while, with the object to be weighed in the left pan, and 37·682 grammes in the right, the resting-point is 134.

Hence 37·682 gms. is too small, and we require to find what is the exact weight we must add to bring the resting point from 134 to 96, that is, through 38 divisions of the scale.

To effect this, move the rider through a few divisions on the beam, say through 5; that is, place it at division 7. The effective weight in the scale pan is now 37·687 gms.; observe as before.

	Turning-points		Resting-point
	Left	Right	
Mean 46	44 48	102	74

The addition of ·005 gramme has moved the resting-point from 134 to 74; that is, through 60 divisions.

We have then to determine by simple proportion what weight we must add to the 37·682 in order to move the resting-point through the 38 divisions; that is, from 134 to 96. The weight required is $\frac{38}{60} \times$ ·005 or ·00316 gm. If then we add ·00316 gm. to the 37·682, the resting-point will be 96, the same as when the scale pans were empty.

Thus the weight of the body is 37·68516 gms.

We have not been working with sufficient accuracy to make the last figure at all certain; we will therefore discard it, and take the weight as 37·6852 grammes (p. 37).

One or two other points require notice.

In each case we have supposed the pointer to swing over from 60 to 70 divisions; this is as large a swing as should be allowed.

We have supposed the resting point, when the balance was unloaded, to lie between those for the two cases in which the load was 37·682 and 37·687; the weights should always be adjusted so that the like may be the case.

We have supposed that the weight for which we first observe the swing is too small. It is more convenient that this should be so; it is not absolutely necessary: we might have started from the heavier weight, and then moved the rider so as to reduce the weight in the right-hand pan.

We must be careful to make no mistake as to the weights actually in the scale pan. It is generally wise for beginners to add them up as they rest on the pan, putting down each separately, grouping those weights together which belong to each separate digit, thus arranging them in groups of grammes, decigrammes, centigrammes, and milligrammes, and then to check the result by means of the vacant places left in the box.

When the weighing is completed see that the weights are replaced in their proper positions in the box, and that the beam is not left swinging.

We shall in future refer to this method of weighing as the 'method of oscillations.'

The alteration produced in the position of the resting point for a given small addition to the weights in the pan is called, as we have seen, the sensitiveness of the balance for that addition (p. 102).

Thus in our case the resting-point was altered by 60 for an addition of ·005 gramme.

The sensitiveness, then, is 60/5 or 12 per milligramme.

The load in the pans in this case was nearly 38 grammes.

We should find by experiment that the sensitiveness depends slightly on the load in the pans. (See p. 102).

Experiments.

(1) Determine the position of the resting-point four times when the balance is unloaded.

(2) Weigh the given body twice.

(3) Determine the sensitiveness for loads of 10, 50, and 100 gms.

Enter results thus:—

(1) Balance unloaded.	Resting-point .	.	95·5
			95·8
			96·1
			95·4
	Mean .	.	95·7
(2) Weight of the body.	1st weighing .	.	37·6852
	2nd „		37·6855
	Mean .	.	37·68535

(3) Sensitiveness.

Weight in right-hand pan		Resting point	Sensitiveness per milligramme
10 grammes	. . .	134	9·6
10·005 „	. . .	86	
50 „	. . .	128	11·6
50·005 „	. . .	70	
100 „	. . .	129	10·6
100·005 „	. . .	76	

13. Testing the Adjustments of a Balance.

The method of weighing which we have described in the preceding section requires the balance to be in perfect adjustment. But the only precaution for that purpose to which attention was called in the description was the levelling of the balance case. We previously mentioned, however (p. 100), that the centre of gravity of the beam could be made to be vertically under its axis of rotation by adjusting the metal flag attached to the beam, and we have, moreover, shewn (pp. 104, 106) that the effect upon the weighings of the pointer not being properly placed, or of our not using the middle point of its scale as the zero, is

inappreciable. We need consider, therefore, only the adjustment to equality of the weights of the scale pans and of the lengths of the arms. The former may, if necessary, be made equal by filing one of them until the necessary equality is attained, while the latter can be adjusted by means of the screws which attach the end knife-edges to the beam.

We have, however, said nothing as yet about adjusting the sensitiveness of the balance. A delicate balance is generally provided with a small sphere fixed to the beam vertically above the middle knife-edge, whose height can be altered by means of the vertical screw passing through its centre, by which it is supported. By raising or lowering this sphere, called the inertia bob, we can diminish or increase the value of h in equation (1) (p. 101), and thus increase or diminish the sensitiveness of the balance. At the same time the moment of inertia (see p. 190) of the beam about the axis of rotation is correspondingly increased, and with it the time of swing of the pointer. Now a long period of swing involves spending a long time over the weighings, and this is a disadvantage; it is therefore not advisable to make the sensitiveness so great that the time of swing is inconveniently long.

The usual period of swing is about 15 seconds. Lord Rayleigh has, however, recently suggested (Brit. Assoc. 1883) that the same accuracy of weighing with considerable saving of time may be secured by loading the pointer of the balance so that the time of swing is about 5 seconds, and using a magnifying glass to read the turning points of the pointer, and thus making up for the diminished sensitiveness by increased accuracy of reading.

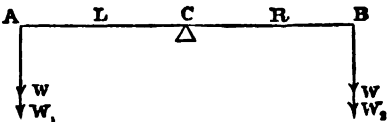
None of these adjustments should be carried out by any but practised observers with the balance, and not by them except after consultation with those who are responsible for the safe custody of the instrument. It is, however, very important for every observer to be able to tell whether or not the balance is in adjustment, and we therefore proceed

to give practical directions for testing in such a manner as to measure the errors produced and enable us to allow for them.

(1) *To determine the Ratio of the Arms of a Balance, and to find the true Weight of a Body by means of a Balance with unequal Arms.*

Let A C B be the beam, and let R and L be the lengths of the arms C B and C A.

FIG. 10.



Weigh a body, whose true weight is w, in the right-hand scale pan, and let the apparent weight be w_1.

Then weigh it in the left-hand pan, and let the apparent weight be w_2.

The weighing must be done as described in the previous section.

Then we have

$$w \times R = w_1 \times L \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$w_2 \times R = w \times L \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Provided that $P \times R = Q \times L$, where P and Q are the weights of the scale pans—i.e. provided the balance pointer stands at zero with the pans unloaded. In practice this condition must first be satisfied by adding a counterpoise to one of the pans.

Multiplying (1) by (2)

$$w_2 \times R^2 = w_1 \times L^2,$$

or

$$\frac{R^2}{L^2} = \frac{w_1}{w_2} \quad \frac{R}{L} = \sqrt{\frac{w_1}{w_2}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Dividing (1) by (2)

$$\frac{w}{w_1} = \frac{w_2}{w}$$

$$w^2 = w_1 \times w_2 \qquad w = \sqrt{w_1 \times w_2} \quad . \quad . \quad . \quad (4)$$

When w_1 and w_2 are nearly the same, we may put

for $\sqrt{W_1 W_2}$, $\frac{1}{2}(W_1 + W_2)$, since the error depends on $\{\sqrt{W_1} - \sqrt{W_2}\}^2$, and this quantity is very small. (See p. 45.)

Thus, if W_1, W_2 be the apparent weights of W in the two pans right and left respectively, the ratio of the arms is the square root of the ratio of W_1 to W_2. The true value of W is the square root of the product $W_1 \times W_2$.

Thus, if when weighed in the right pan, the apparent weight of a body is 37·686 grammes, and when weighed in the left, it is 37·592,

$$\frac{R}{L} = \sqrt{\frac{37 \cdot 686}{37 \cdot 592}} = 1 \cdot 00125.$$

$$W = \sqrt{37 \cdot 686 \times 37 \cdot 592} = 37 \cdot 635 \text{ grammes.}$$

The true weight of a body may also be determined in a badly adjusted balance by the following method, known as the method of taring. Place the body in one scale pan and counterpoise it, reading the position of equilibrium of the pointer with as great accuracy as possible ; then, leaving the same counterpoise, replace the body by standard weights, until the position of equilibrium of the pointer is the same as before. The mass which thus replaces the body is evidently that of the body, no matter what state the balance may be in. (This is called Borda's method.)

(2) *To Compare the Weights of the Scale Pans.*

Let a be the length of the arms supposed equal, S the weight of one pan, and $S + \omega$ that of the other.

Weigh a body whose weight is Q first in the pan whose weight is S ; let the apparent weight be W.

Then interchange the scale pans and weigh Q again ; let the weight be W′.

Then
$$(S + Q)\,a = (W + S + \omega)a$$
$$a(S + \omega + Q) = (W' + S)a.$$

Divide each by a, and subtract ; then $\omega = W' - W - \omega$, or $\omega = \frac{1}{2}(W' - W)$.

Thus, weigh the body in one pan; let its weight be W. Interchange the scale pans and weigh the body again in the other scale pan, but on the same side of the fulcrum; let the weight be W′, then the difference in the weight of the scale pans is $\frac{1}{2}(W'-W)$.

This will be true very approximately, even if the arms be not equal; for let one be R and the other L. Then we have

$$(S+Q)R=(W+S+\omega)L$$
$$(S+\omega+Q)R=(W'+S)L$$
$$\therefore \qquad \omega=(W'-W-\omega)\frac{L}{R}.$$

Now $\frac{L}{R}$ is nearly unity; we may put it equal to $1+\rho$, where ρ is very small.

$$\omega=(W'-W-\omega)(1+\rho)$$
$$=W'-W-\omega+\rho(W'-W-\omega).$$

But we suppose that ω, and therefore $W'-W$, is very small. Thus $\rho(W'-W-\omega)$, being the product of two small quantities, may be neglected, and we get

$$\omega=W'-W-\omega, \text{ or}$$
$$\omega=\tfrac{1}{2}(W'-W).$$

Experiments.

(1) Determine the ratio of the arms of the given balance.

(2) Determine the difference between the weights of the scale pans.

Enter as below:—

(1) Weight in right-hand pan	= 37·686	gms.
" left-hand pan	= 37·592	"
$\frac{R}{L}$	= 1·00125	"
W	= 37·650	"
(2) Weight in left-hand pan	= 37·592	"
" pans interchanged	= 37·583	"

∴ Left-hand pan − right-hand pan = ·0045 gm.

14. Correction of Weighings for the Buoyancy of the Air.

The object of weighing a body is to determine its mass, and the physical law upon which the measurement depends is that the weights of bodies are proportional to their masses, if they are sufficiently near together.

Now we have all along assumed that when an adjusted balance-beam was in equilibrium, the force of gravity upon the weights was equal to the force of gravity upon the body weighed, i.e. that their weights were equal, and this would have been so if we had only to deal with the force of gravity upon these bodies. But the bodies in question were surrounded by air, and there was accordingly a force upon each acting vertically upwards, due to the buoyancy of the air; and it is the resultant force upon the weights which is equal to the resultant force upon the body weighed. But the forces being vertical in each case, their resultant is equal to their difference; and the force due to the displacement of air by the body is equal to the weight of the air displaced, i.e. it bears the same ratio to the weight of the body as the specific gravity of air does to the specific gravity of the body; while the same holds for the weights.

Thus, if w be the weight of the body, σ its specific gravity, and λ the specific gravity of air at the pressure and temperature of the balance-case, the volume of air displaced is w/σ and its weight $w\lambda/\sigma$ (p. 121). Hence the resultant force on the body is $w\left(1-\frac{\lambda}{\sigma}\right)$; similarly, if ω be the weights, and ρ their density, the force on the weights is $\omega\left(1-\frac{\lambda}{\rho}\right)$.

These two are equal, thus

$$w=\frac{\omega\left(1-\frac{\lambda}{\rho}\right)}{\left(1-\frac{\lambda}{\sigma}\right)}=\omega\left(1-\frac{\lambda}{\rho}+\frac{\lambda}{\sigma}\right) \text{ approximately,}$$

since in general $\frac{\lambda}{\sigma}$ is very small.

The magnitude of the correction for weighing in air depends therefore upon the specific gravities of the weights, the body weighed, and the density of the air at the time of weighing, denoted by ρ, σ, and λ respectively. The values of ρ and σ may be taken from the tables of specific gravities (tables, 17, 80) if the materials of which the bodies are composed are known. If they are not known, we must determine approximately the specific gravity. We may as a rule neglect the effect of the buoyancy of the air upon the platinum and aluminium weights, and write for ρ, 8·4, the specific gravity of brass, the larger weights being made of brass. The value of λ depends upon the pressure and temperature of the air, and upon the amount of moisture which it contains, but as the whole correction is small, we may take the specific gravity of air at 15° C. and 760 mm., when half-saturated with moisture, as a sufficiently accurate value of λ. This would give $\lambda = \cdot 0012$.

Cases may, however, arise in which the variation of the density of the air cannot be neglected. We will give one instance. Suppose that we are determining the weight of a small quantity of mercury, say 3 grammes, in a glass vessel of considerable magnitude, weighing, say, 100 grammes. Suppose that we weigh the empty vessel when the air is at 10° C. and 760 mm., and that we weigh it with the mercury in at 15° C. and 720 mm. deducing the weight of the mercury by subtracting the former weight from the latter. We may neglect the effect of the air upon the weight of the mercury itself, but we can easily see that the correction for weighing the glass in air has changed in the interval between the weighings from 22 mgm. to 20·5 mgm. The difference between these, 1·5 mgm., will appear as an error in the calculated weight of the mercury, if we neglect the variation in density of the air, and this error is too considerable a fraction of the weight of the mercury to be thus neglected.

Experiment.

Determine the weight in vacuo of the given piece of platinum.

Enter results thus :—

Weight in air at 15°C. and 760 mm. with brass weights 37·634 gm. Specific gravity of platinum 21·5. Weight in vacuo, 37·632.

DENSITIES AND SPECIFIC GRAVITIES.[1]

DEFINITION 1.—The density of a substance at any temperature is the mass of a unit of volume of the substance at that temperature ; thus the density of water at 4° C. is one gramme per cubic centimetre.

DEFINITION 2.—The specific gravity of a substance at any temperature is the ratio of its density at that temperature to the density of some standard substance, generally the maximum density of water (i.e. the density of water at 4° C.).

DEFINITION 3.—The specific gravity of a body is the ratio of the mass of the body to the mass of an equal volume of some standard substance, generally water at 4° C.

It evidently follows from these definitions that, if ρ be the density of a substance, σ its specific gravity, and ω the maximum density of water, $\rho=\sigma\omega$, and if M be the mass of a body consisting of the substance, whose volume is V, then $M=V\rho=V\sigma\omega$, and the mass of a volume of water equal to

[1] It is unfortunate that in many physical text-books the terms 'density' and 'specific gravity' are used synonymously, the former being generally employed for gases and liquids, the latter for solids. It is quite evident that there are two very distinct ideas to be represented, namely (1) the mass of the unit of volume, a quantity whose numerical value depends of course on the units chosen for measuring masses and volumes ; and (2) the *ratio* of the mass of any volume to the mass of an equal volume of water at 4° C. ; this quantity being a ratio, is altogether independent of units. There being now also two names, 'density' and 'specific gravity', it seems reasonable to assign the one name to the one idea and the other name to the other idea, as suggested by Maxwell, 'Theory of Heat' (ed. 1872, p. 82). When there is no danger of confusion arising from using the term density when specific gravity is meant, there may be no harm in doing so, but beginners should be careful to use the two words strictly in the senses here defined.

the volume of the body $= v\,\omega$. The maximum density of water is 1 gramme per cubic centimetre. If we use the gramme as the unit of mass, and the cubic centimetre as the unit of volume, the numerical value of ω is unity and the equations we have written become $\rho = \sigma$ and $M = v\,\sigma$. Thus, the numerical value of the density of a substance on the C.G.S. system of units is the same as the number which expresses the specific gravity of the substance, this latter being of course a ratio, and therefore independent of units. And for the C.G.S. system of units, moreover, the numerical value of the mass of a body is equal to the number which expresses its volume multiplied by its specific gravity.

These relations are only true for the C.G.S. system, and any other systems in which the unit of mass is the mass of the unit of volume of water at 4° C.; but whatever be the system, the density of water at 4° C. is accurately known, although its numerical value may not be unity. Hence, in order to calculate the volume of a body whose mass is known, or *vice versâ*, we require only to know its specific gravity, and hence the practical importance of determinations of specific gravity. It is generally an easy matter to determine experimentally the ratio of the mass of a body to the mass of an equal volume of water at the same temperature, but it would not be easy or convenient always to keep the water at its temperature of maximum density, throughout the experiment. The densities of bodies are therefore not usually experimentally compared directly with the maximum density of water in determining specific gravities, and the necessity for doing so is obviated by our knowing with great accuracy the density of water at different temperatures, (this is given in table 32); so that we are enabled, when we know the mass of a volume of water at any temperature, to calculate from the table the mass of the same volume at 4° C., and thus obtain the specific gravity required. We proceed to describe some of the practical methods in general use.

15. The Hydrostatic Balance.

The specific gravity of a substance is determined by the hydrostatic balance by weighing the substance in air, and also in water.

One scale pan is removed from the balance, and replaced by a pan suspended by shorter strings from the beam. This pan has a hook underneath, and from the hook the substance to be weighed is suspended by a piece of very fine wire.

(1) *To determine the Specific Gravity of a Solid heavier than Water.*

We must first make sure that the beam is horizontal when the balance is loaded only with the wire which is to carry the substance.

Turn the key or lever gently to release the beam ; the pointer will probably move sharply across the scale, showing that one pan is heavier than the other.

Fix the beam again, and put shot or pieces of tinfoil into the lighter scale until it becomes nearly equal in weight to the other, then let it swing, and observe a resting-point as in § 12. The weights put in should be so adjusted that this resting-point may be near the centre of the scale.

Do not counterpoise with weights which you may subsequently require in order to weigh the object.

Hang the object whose specific gravity you require—a piece of copper suppose—by the fine wire from the hook above mentioned, and weigh it twice or three times by the method of oscillations (§ 12). Let its weight be 11·378 grammes.

Fill a vessel with distilled water, and bring it under the end of the beam so that the copper may dip completely into the water.

Be careful that no air-bubbles adhere to the copper; if there be any, remove them by means of a small brush or feather, or a fibre of glass. It is well to use water that has

been freed from dissolved air either by boiling or by means of an air-pump. Any very small bubbles not easily removable by mechanical means will then be dissolved by the water.

Be careful also that the wire which supports the copper cuts the surface of the water only once; there is always a certain amount of sticking, due to surface tension, between the wire and the surface of the water, and this is increased if a loose end of the wire be left which rises through the surface. To completely avoid the effect of surface tension the diameter of wire should not be greater than ·004 inch.

Weigh the copper in the water; it will probably be found that the pointer will not oscillate, but will come to rest almost immediately. Observe the resting-point, and by turning the key set the beam swinging again, and take another observation. Do this four times, and take the mean.

Add some small weight, say ·01 gramme, to the weight, and observe another resting-point, and from these observations calculate, as in § 12, the weight of the copper in water; it will be about 10·101 grammes. Observe at the same time the temperature of the water with a thermometer. Suppose it is 15°.

Then it follows that the weight of the water displaced is 11·378—10·101 grammes, or 1·277 gramme.

Now the specific gravity of a substance is equal to

$$\frac{\text{weight of substance}}{\text{weight of equal vol. water at } 4^\circ\text{C.}}.$$

In all cases, if we know the weight of a volume of water at t°, we can find its weight at 4° C., by dividing the weight at t° by the specific gravity of water at t°.

$$\text{Thus, weight at } 4^\circ = \frac{\text{weight at } t^\circ}{\text{specific gravity at } t^\circ}.$$

The specific gravity of water at t° may be taken from (32).

In this case, the weight of the equal volume of water at 15° C. is 1·277 gramme, and the specific gravity of water at 15° is ·99917.

∴ The weight of the equal volume of water at 4° C.

$$=\frac{1\cdot277}{\cdot99917}=1\cdot278.$$

Thus, the specific gravity of copper

$$=\frac{11\cdot378}{1\cdot278}=8\cdot903.$$

It is well to pour the water into the beaker or vessel that is to hold it, before beginning the experiment, and leave it near the balance, so that it may acquire the temperature of the room.

If greater accuracy be required, we must free the water used from air. This can be done by putting it under the receiver of an air-pump and exhausting, or by boiling the water for some time and then allowing it to cool.

We have neglected the effect of the wire which is immersed in the water; we can, if we need, correct for this.

We have also neglected the correction to the observed weight, which arises from the fact that the weights used displace some air, so that the observed weight in air is really the true weight minus the weight of air displaced.

(2) *To determine the Specific Gravity of a Solid lighter than Water.*

If we wish to find the specific gravity of a solid lighter than water, we must first weigh the light solid in air, then tie it on to a heavier solid, called a sinker, whose weight and specific gravity we know. The combination should be such that the whole will sink in water.

Let W and σ be the weight in air, and the specific gravity of the light solid—a piece of wax, for instance—W', σ' corresponding quantities for the sinker, $\overline{W}$, $\overline{\sigma}$ for the combina-

tion ; w', $\bar{w}$ the weights in water of the sinker and the combination respectively.

Then, using C.G.S. units, w/σ represents the volume of the wax, w'/σ' that of the sinker, $\bar{\mathrm{w}}/\bar{\sigma}$ that of the combination.

Since the volume of the wax is equal to that of the combination minus that of the sinker, we get

$$\frac{\mathrm{w}}{\sigma}=\frac{\bar{\mathrm{w}}}{\bar{\sigma}}-\frac{\mathrm{w}'}{\sigma'}.$$

But, with the proper temperature corrections,

$$\frac{\bar{\mathrm{w}}}{\bar{\sigma}}=\bar{\mathrm{w}}-\bar{w}$$

and

$$\frac{\mathrm{w}'}{\sigma'}=\mathrm{w}'-w'$$

$$\therefore\quad \frac{\mathrm{w}}{\sigma}=\bar{\mathrm{w}}-\bar{w}-(\mathrm{w}'-w'),$$

or remembering that $\bar{\mathrm{w}}=\mathrm{w}+\mathrm{w}'$

$$\sigma=\frac{\mathrm{w}}{\mathrm{w}-\bar{w}+w'}.$$

w, w', $\bar{w}$ can each be observed, and thus the specific gravity of the wax determined.

If it is convenient to tie the sinker so that it is immersed while the solid itself is out of the water, the following method is still simpler.

Weigh the solid in air and let its weight be w.

Attach the sinker below the solid, and weigh the combination with the former only immersed. Let the weight be w_1.

Raise the vessel containing the water so that the solid is immersed as well as the sinker, and let the weight be w_2.

Then, if the temperature of the water be $t°$, the specific gravity required

$$=\frac{w}{w_1 - w_2} \times \text{specific gravity of water at } t°$$

(3) *To determine the Specific Gravity of a Liquid.*

Weigh a solid in air; let its weight be w. Weigh it in water; let the weight be w_1. Weigh it in the liquid; let its weight be w_2. The liquid must not act chemically on the solid. $w-w_1$ is the weight of water displaced by the solid, and $w-w_2$ is the weight of an equal volume of the liquid. Thus, the specific gravity of the liquid at 0°, if it expand by heat equally with water, and if the temperature of the two observations be the same, is the ratio of these weights.

To find the specific gravity of the liquid at the temperature of the observation, $\tau°$ say, we must multiply this ratio by the specific gravity of water at the temperature at which the solid was weighed in water; let this be $t°$. Hence the specific gravity of the liquid at $\tau°$

$$=\frac{w-w_2}{w-w_1} \times \text{specific gravity of water at } t°.$$

Experiments.

(1) Determine the specific gravity of copper.
(2) Determine the specific gravity of wax.

Enter results as below, indicating how often each quantity has been observed.

(1) Specific gravity of copper.

Weight in air . . .	11·378 gm. (mean of 3)
Weight in water . . .	10·101 gm. (mean of 3)
Weight of water displaced .	1·277 gm.
Temperature of water . .	15° C.
Specific gravity . . .	8·903

(2) Specific gravity of wax. Using the piece of copper (1) as sinker.

Weight of wax in air (w)	. . .	26·653 gm.
Weight of sinker (w′)	. . .	11·378 "
Weight of combination ($\overline{W}$)	. .	38·031 "
Weight of sinker in water (w')	. .	10·101 "
Weight of combination in water ($\overline{w}$)	.	9·163 "
Temperature of water	. . .	15° C.
Specific gravity of wax	. . .	0·965

16. The Specific Gravity Bottle.

(1) *To determine the Specific Gravity of small Fragments of a Solid by means of the Specific Gravity Bottle.*

We shall suppose that we require to know (1) the weight of the solid, (2) the weight of the empty bottle, (3) the weight of water which completely fills the bottle, and (4) the weight of the contents when the solid has been put inside and the bottle filled up with water. Strictly speaking, if the weight of the solid fragments can be independently determined, the difference of (4) and (3) is all that is necessary, and the weight of the empty bottle is not required; but in order to include under one heading all the practical details referring to the specific gravity bottle we have added an explanation of the method of obtaining or allowing for the weight of the bottle. The student can easily make for himself the suitable abbreviation if this is not required.

We shall also suppose the temperature to be the same throughout the experiment.

If it consists of only a few fragments of considerable size we may find the weight of the solid by the method of oscillations; let it be 5·672 grammes.

Dry the bottle thoroughly before commencing the experiment.

The necessity of drying the interior of vessels occurs so frequently in laboratory practice, that it will be well to men-

tion here the different methods which are suitable under different circumstances in order that we may be able to refer to them afterwards. We may take for granted that all the water that can be removed by shaking or by soaking up with slips of filter paper, has been so got rid of.

An ordinary bottle or flask can for most purposes be sufficiently dried by drawing air through it by means of a tube passing to the bottom of the bottle and connected with an aspirator or the aspirating pump referred to in the note (p. 89), and at the same time gently warming the bottle by means of a spirit lamp. If there be any considerable quantity of water to be got rid of, the process can be considerably shortened by first rinsing out the bottle with alcohol. If more careful drying is necessary, as, for instance, for hygrometric experiments, the mouth of the vessel should be closed by a cork perforated for two tubes, the one opening at one end and the other at the other end of the vessel, and a current of perfectly dry air kept passing through the vessel for some hours. The air may be dried by causing it to pass first through U-tubes filled with fused chloride of calcium, which will remove the greater part of the moisture, and finally through a tube containing phosphoric anhydride or fragments of ignited pumice moistened with the strongest sulphuric acid.

If there be no opening in the vessel sufficiently large to allow of two tubes passing, the following plan may be adopted :—Connect the tube which forms the prolongation of the plug of a three-way tap [1] with an air-pump. The water air-pump before referred to is very convenient for the purpose if there be a sufficient head of water on the water-

[1] A three-way tap is a simple, but in many ways very useful, contrivance. In addition to the two openings of an ordinary tap, it has a third, formed by a tubular elongation of the plug, and communicating with that part of the conical face of the plug which is on the same cross-section as the usual holes, but at one end of a diameter perpendicular to the line joining them. Such taps may now be obtained from many of the glass-blowers.

supply to give efficient exhaustion. Connect the other openings of the tap with the vessel to be dried and the drying tubes respectively. Then, by turning the tap, connection can be made alternately between the pump and the vessel and between the vessel and the drying tubes, so that the vessel can be alternately exhausted and filled with dried air. This process must be repeated very many times if the vessel is to be completely dried.

Having by one of these methods thoroughly dried the bottle, place it on one of the scale pans of the balance, and counterpoise on the other either with the brass weight provided for the purpose, or by means of shot or pieces of lead. Observe the resting-point of the pointer by the method of oscillations, taking two or three observations.

Meanwhile a beaker of distilled water, which has been freed from air either by boiling or by being enclosed in the exhausted receiver of an air-pump, should have been placed near the balance, with a thermometer in it, in order that the water used may have had time to acquire the temperature of the room and that the temperature may be observed.

Fill the bottle with the water, taking care that no air-bubbles are left in. To do this the bottle is filled up to the brim, and the stopper well wetted with water. The end of the stopper is then brought into contact with the surface of the water, taking care that no air is enclosed between, and the stopper pushed home.

All traces of moisture must be carefully removed from the outside of the bottle by wiping it with a dry cloth.

Observe the temperature of the water before inserting the stopper; let it be 15° C. The bottle should be handled as little as possible, to avoid altering its temperature.

Replace the bottle on the scale pan, and weigh; let the weight observed be 24·975 grammes.

This weighing, like every other, should be done twice or three times, and the mean taken.

This is the weight of the water in the bottle only, for we

have supposed that the bottle has been previously counterpoised.

Open the bottle and introduce the small fragments of the solid which have been weighed, taking care to put all in.

Again fill the bottle, making sure by careful shaking that no air-bubbles are held down by the pieces of the solid; if any are observed, they must be removed by shaking or by stirring with a clean glass rod; or, if great accuracy is required, by placing the bottle under the receiver of an air-pump and then exhausting.

Replace the stopper, carefully wiping off all moisture, and weigh again, twice or three times; let the weight be 27·764 grammes.

This is clearly the weight of the substance + the weight of the bottleful of water − the weight of water displaced by the substance.

Thus the weight of water displaced is equal to the weight of the substance + the weight of the bottleful of water − 27·764 grammes

$$= 30{\cdot}647 - 27{\cdot}764 = 2{\cdot}883 \text{ grammes.}$$

Now we require the weight of water which would be displaced were the temperature 4° C.; for the specific gravity of a substance is equal to

$$\frac{\text{weight of substance}}{\text{weight of equal vol. water at } 4^\circ};$$

but the weight of any volume of water at 4°

$$= \frac{\text{weight of equal vol. at } t^\circ}{\text{specific gravity water at } t^\circ}$$

Thus the specific gravity of the substance

$$= \frac{\text{weight of substance}}{\text{weight of equal vol. water at } t^\circ} \times \text{spec. grav. water at } t^\circ.$$

Taking from the table (32) the specific gravity of water at 15°, we find the specific gravity of the substance to be

$$\frac{5{\cdot}672}{2{\cdot}883} \times {\cdot}99917 = 1{\cdot}966.$$

If greater accuracy be required, we must free the water used from air by boiling or the use of the air-pump. We should also require to correct the weighings for the air displaced.

(2) *To find the Specific Gravity of a Powder.*

The process of finding the specific gravity of a powder is nearly identical with the foregoing. The only modification necessary is to weigh the powder in the bottle. The order of operations would then be—

(1) Counterpoise the dry bottle.

(2) Introduce a convenient amount of the powder, say enough to fill one-third of the bottle, and weigh.

(3) Fill up with water, taking care that none of the powder is floated away, and that there are no air-bubbles, and weigh again. If it be impossible to make all the powder sink, that which floats should be collected on a watch-glass, dried, and weighed, and its weight allowed for

(4) Empty the bottle, and then fill up with water and weigh again

The method of calculation is the same as before.

(3) *To determine the Specific Gravity of a Liquid by the Specific Gravity Bottle.*

Fill the bottle with water, as described above, and weigh the water contained, then fill with the liquid required, and weigh again. Each weight should of course be taken twice.

The ratio of the two weights is the specific gravity of the liquid at 4° C. if it expand by heat equally with water.

If we require the specific gravity of the liquid at the temperature of the experiment, we must note the temperature of the water, and reduce its weight to the weight of an

equal volume at 4° C.; that is, we must multiply the above ratio by the specific gravity of water at the temperature of the observation.

Thus, the specific gravity of a liquid

$$= \frac{\text{weight of liquid}}{\text{weight of equal vol. water at } t^\circ} \times \text{spec. grav. water at } t^\circ.$$

Experiments.

(1) Determine the specific gravity of the given solid.

(2) Determine the specific gravity of the given liquid.

Enter as below, indicating the observations made of each quantity:—

(1) Specific gravity of solid.

Weight of solid	5·672 gm.
Weight of water in bottle . . .	24·975 gm.
Weight of water with solid . .	27·764 gm.

Temperature, 15° C.

Specific gravity, 1·966.

(2) Specific gravity of liquid.

Weight of water in bottle . .	24·975 gm.
Weight of liquid	23·586 gm.
Temperature . . .	15° C.
Specific gravity of liquid . . .	·9430.

17. Nicholson's Hydrometer.

This instrument is used (1) to determine the specific gravity of small solids which can be immersed in water; (2) to determine the specific gravity of a liquid.

(1) *To find the Specific Gravity of a Solid.*

Taking care that no air-bubbles adhere to it, place the hydrometer in a tall vessel of distilled water recently boiled, and put weights on the upper cup until it just sinks to the mark on the stem.

To avoid the inconvenience caused by the weights falling into the water, a circular plate of glass is provided as a cover

for the vessel in which the hydrometer floats. This has been cut into two across a diameter, and a hole drilled through the centre, through which the stem of the instrument rises.

It will generally be found that with given weights on the cup the hydrometer will rest in any position between certain limits; that there is no one definite position of flotation, but many. The limits will be closer together and the experiment more accurate if the surface of the instrument, especially that of the stem, be thoroughly clean and free from grease. It is well therefore carefully to rub the stem and upper part of the bulb with some cotton-wool soaked in methylated spirit.

Suppose now it is floating with the mark on the stem just below the surface. Take off some weights until the mark just rises past the surface; let the weights then on be 8·34 grammes. Put on weights until the mark just sinks below the surface, and then let the weight be 8·35 grammes. Do this several times, and take the mean as the weight required to sink the mark to the surface.

Let the mean be 8·345 grammes.

Remove the weights and put the solid in the upper cup. Then add weights until the mark again just comes to the surface, estimating the weight required as before. Let this be 2·539 grammes. The weight of the solid in air is the difference between these, or 5·806 grammes.

Now put the solid in the lower cup[1] and weights in the upper one until the mark sinks to the surface. Estimate as before. Let the mean of the weights be 5·462 grammes. The difference between this and the weight 8·345, put on originally to sink the hydrometer, gives the weight in water.

Thus, the weight in water = 2·883 grammes.

And the weight of water displaced = weight in air − weight in water = 2·923 grammes.

[1] If the solid be lighter than water it must be fastened down to the cup either by a wire or by being enclosed in a cage fixed to the instrument.

The specific gravity, therefore, referred to water at the temperature of experiment

$$=\frac{5\cdot806}{2\cdot923}=1\cdot987.$$

To determine the true specific gravity—water at 4° C. being taken as the standard—we must multiply this number by the specific gravity of the water at the time of the experiment.

This may be taken from the table (32), if we know the temperature. Thus, we must observe the temperature of the water at the time of the experiment. Let it be 15°. Then the specific gravity required

$$=1\cdot987\times\cdot99917=1\cdot985 \text{ approximately.}$$

(2) *To determine the Specific Gravity of a Liquid.*

Let the weight of the instrument itself be 11·265 grammes. This must be determined by weighing it in a balance.

Place it in the water, and put weights on the upper pan until it just floats up to the mark on the stem. Let the weight be 8·345 grammes. This of course must be estimated as in experiment (1).

The sum of these two weights is the weight of a volume of water equal to that of the instrument up to the mark on the stem. Thus, the weight of this volume of water is 19·610 grammes.

Now place the instrument in the liquid and add weights till the mark is just in the surface. Let the weight be 9·875 grammes.

Then the weight of the volume of liquid displaced is

$$11\cdot265+9\cdot875 \text{ or } 21\cdot140 \text{ grammes.}$$

The specific gravity of the liquid referred to water at the temperature of the experiment is therefore

$$\frac{21\cdot140}{19\cdot610}=1\cdot078.$$

Let the temperature of the water be 15° C.; that of the liquid 11·5° C. Then the specific gravity of liquid at 11·5° C. is

$$1{\cdot}078 \times {\cdot}99917 = 1{\cdot}077.$$

Experiments.

(1) Determine the specific gravity of sulphur by Nicholson's Hydrometer.

(2) Make a 20 per cent. solution[1] of common salt in water, and determine its specific gravity by Nicholson's Hydrometer

Enter results thus :—

(*a*) Specific gravity of sulphur.

Mean weight required to sink the hydrometer to the mark	8·345 gms.
Mean weight required to sink the hydrometer with sulphur on upper pan	2·539 "
Mean weight required to sink the hydrometer with sulphur on lower pan	5·462 "

Temperature of the water, 15° C.

Sp. gr. of sulphur = 1·985.

(*b*) Specific gravity of salt solution.

Weight of salt used	539·0	gms.
Weight of water used	2156·0	"
Weight of hydrometer	11·265	"
Weight required to sink the instrument to the mark in water at 15°	8·345	"
Weight required to sink instrument in solution at 11°·5 C.	9·875	"
Specific gravity of solution	1·077	"

18. Jolly's Balance.

The apparatus consists of a long spiral spring carrying a pan into which weights or the object to be weighed can be put.

[1] A 20 per cent. solution is one which contains 20 parts by weight of salt in 100 parts of the solution. It may therefore be made by adding the salt to water in the proportion of 20 grammes of salt to 80 grammes of water.

From this there hangs, by a fine thread, a second pan which is always kept immersed in water.

Behind the spring is a millimetre scale engraved on a strip of looking-glass, and just above the pan is a white bead, which can be seen directly reflected in the glass.

By placing the eye so that the top of the bead just appears to coincide with its own image, the division of the scale which is opposite to the top of the bead can be read with great accuracy.

(1) *To weigh a small Body and find its Specific Gravity.*

Place the object to be weighed in the upper pan, taking care that the lower pan is well below the surface of the water, and that the vessel in which the water is, is sufficiently large to allow the pan to hang clear of the sides.

Note the division of the scale which coincides with the top of the bead. Suppose it is 329.

Remove the object from the pan and replace it by weights until the bead occupies the same position as before. Let the weights be 7·963 grammes.

It may be impossible with given weights to cause the bead to come to exactly the same position.

Thus, we may find that 7·963 gms. causes it to stand at 330, while 7·964 gms. brings it to 327·5. The true weight lies between these two; and the addition of ·001 gramme lowers the bead through 2·5 mm. We require the bead to be lowered from 330 to 329—that is, through 1 mm. We must therefore add to our weight

$$\frac{1}{2\cdot5} \text{ of } \cdot001 \text{ gramme, or } 0\cdot0004 \text{ gramme.}$$

The true weight then would be 7·9634 grammes.

The water should be adjusted so that its surface is above the point of junction of the three wires which carry the lower pan.

Next place the small object in the lower pan, and put weights into the upper till the bead again comes to the

same point on the scale. Let the weights be 3·9782 grammes.

This is clearly the weight of the water displaced by the object, and its specific gravity referred to water at the temperature of the observation is therefore

$$\frac{7·9634}{3·9782} \text{ or } 2·002.$$

To obtain the true specific gravity, we must multiply this by the specific gravity of the water at the temperature of the observation. Let this be 15°.

The specific gravity of water at 15° is ·99917, so that the specific gravity of the solid is

$$2·002 \times ·99917, \text{ or } 2·000.$$

(2) *To determine the Specific Gravity of a Liquid.*

Take a small solid which will not be acted on by the liquid, and place it in the upper pan. Note the point to which the bead is depressed, the lower pan being in water.

Now place the solid in the lower pan and put weights into the upper until the bead comes opposite the same mark. Let the weight be 3·596 grammes. This is the weight of the water displaced by the solid.

Remove the water and replace it by the liquid. Put the solid into the upper pan, and note the division opposite to which the bead stands. Let it be 263.

Put the solid into the lower pan, and put weights into the upper until the bead comes opposite to 263. Let the weight be 4·732 grammes. This is the weight of the liquid displaced by the solid.

Thus, the specific gravity of the liquid

$$= \frac{4·732}{3·596} = 1·316.$$

This must be corrected for temperature as usual.

Experiments.

(1) Determine by means of Jolly's Balance the specific gravity of the given small crystal.

(2) Determine by means of Jolly's Balance the specific gravity of the given liquid.

Enter the results thus :—

(1) Specific gravity of crystal.

Scale reading with the crystal in the upper pan	329 mm.
Weight required to bring the bead to same position	7·9634 gms.
Weight required with crystal in lower pan .	3·9782 „

Temperature of water 15° C.

Sp. gr. of crystal 2·000.

(2) Specific gravity of liquid.

Scale reading with solid in upper pan, lower pan in water	329 mm.
Weight required to bring the bead to the same reading with the solid in water . . .	3·596 gms.
Scale reading with the solid in the upper pan, lower pan in the liquid	263 mm.
Weight required to bring the bead to the same reading with the solid in the liquid . .	4·732 gms.

Temperature of the water 15° C.

Specific gravity of liquid = 1·315.

19. The Common Hydrometer.

The specific gravity of a liquid may be most easily determined to within 0·1 per cent. by the use of the common hydrometer.

This instrument consists of a glass bulb with a cylindrical stem, loaded so that it floats in any liquid whose specific gravity lies within certain limits, with the stem vertical and partly immersed. The depth to which it requires to be immersed in order to float is defined by the condition that the weight of the liquid displaced is equal to the weight of the hydrometer. For any liquid, therefore,

within the limits, there is a definite point on the stem to which the instrument will sink, depending on the specific gravity; and the stem can be graduated in such a manner that the graduation reading gives the specific gravity at once. This is generally done by a scale attached to the inside of the stem, and hence all that has to be done to determine the specific gravity of a liquid is to float in it a suitable hydrometer, and take the scale reading at the surface. The temperature correction is to be allowed for as usual.

An instrument sensitive to such slight variations of density as 0·1 per cent. would require to have too long a stem if used for the whole range of density commonly occurring. Hydrometers are, therefore, usually obtained in sets of three or four, each suitable for one portion only of the range. The case in which they are kept contains a long cylindrical vessel, which is convenient for floating them in and also a thermometer.

The hydrometers, vessel, and thermometer should be carefully washed and dried before replacing them in the case.

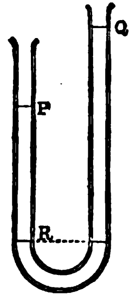
The graduation of the scale is a comparatively difficult matter, as equal increments in the length of the stem immersed do not correspond to equal differences of density. The scales are graduated by the instrument-makers, and we require to be able to test the accuracy of the graduation.

We can do this by taking the hydrometer readings in liquids whose specific gravities are known. Distilled water would naturally be a suitable one for the purpose. The hydrometer when floating in distilled water at 15° C. should read 0·999. The specific gravity of any other suitable liquid could be determined by one of the methods already described. The following experiment, however, serves as a very instructive method of comparing the density of any liquid with that of water, and it is, therefore, suggested as a means of testing the accuracy of the hydrometer scale.

To compare the Densities of two Liquids by the Aid of the Kathetometer.

If we have a U tube (fig. 11) and fill one leg with one liquid standing up to the level P, and the other with a second up to the level Q, and if R be the common surface of the liquids in the two legs P R, Q R, their densities are inversely proportional to the vertical distances between P and R, Q and R.[1] These can be accurately measured by the kathetometer, and the densities thus compared. If the kathetometer be not available, the heights may be measured by scales placed behind the tubes, which are read by a telescope placed at a distance and roughly levelled for each observation.

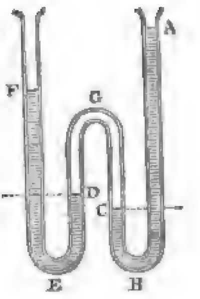
FIG. 11.



This arrangement supposes that the two liquids do not mix. The following apparatus is therefore more generally available :—

A B C, D E F are two U tubes, the legs B C, D E being the shorter. These legs are connected together by a piece of india-rubber tubing C G D.

FIG. 12.



One liquid is poured into the tube A B, and then the other into the tube F E. This, as it runs down the tube, compresses the air below it, thus increasing the pressure on the surface of the first liquid, and forcing it up the leg B A. The quantity poured into F E must not be sufficient to rise over the end D of the tube.

Now pour more of the first liquid into A B. This forces up the level of the liquid in E F, and after one or two repetitions of this

[1] See below, chap. vii. p. 197.

operation the levels of the liquid in one tube will be at A and C, those in the other being at F and D.

The pressure at C and D, being that of the enclosed air, is the same.

The excess of the pressure at C above the atmospheric pressure is due to a column of liquid of height equal to the vertical distance between A and C, that at D is due to a column of the second liquid of height equal to the distance between F and D.

These distances can be observed by the kathetometer, and the densities of the two liquids are inversely proportional to them.

The surface of the liquids in the tubes will be curved, owing to capillary action. In measuring, either the bottom or the top of the meniscus, whichever be most convenient, may be observed, but it is necessary to take the same at each end of the column. The bottom will, if the liquid wet the tube, give the more accurate result.

It is well to hang up behind the tubes a sheet of white or grey paper, to afford a good background against which to see the liquids.

It is important that the temperature should remain the same during the experiment; for if it increase the pressure in the portion C G D increases, and the air there expands, thus forcing up the columns of liquid. We may avoid the difficulty this causes by the following method of taking the measurements:

Observe the height of A, then the height of C, and finally the height of A again.

Then, if the temperature has changed uniformly and the intervals between the successive measurements have been the same, the mean of the two observed heights of A will give its height at the time when the observation of the height of C was made, and the difference between these two, the mean of the observed heights of A and the height of C, will give the true height of the column.

If one liquid be water at a temperature, say, of 15° C., the ratio of the two heights gives us the specific gravity of the second liquid, for its temperature at the time of the observation, referred to water at 15° C.

If we wish to find the true specific gravity of the liquid at the temperature of the observation, 15° C., we must multiply the above ratio by the specific gravity of water at 15° C.

Suppose the second liquid is also at 15° C., and that its coefficient of expansion by heat does not differ greatly from that of water. Then the same ratio gives us the specific gravity of the liquid at 4° C. referred to water at 4° C., or the true specific gravity of the liquid at 4° C. without any correction.

Experiment.—Determine the specific gravity of the given liquid by means of the hydrometer, testing the accuracy of the results.

Enter results thus:—

Specific gravity by hydrometer 1·283.
Tube AC water; tube DF liquid.

Height of A	Mean	Height of C
23·51 23·535	23·522	86·460
		Difference 62·938

Temperature of the water, 15°C.

Height of F	Mean	Height of D
35·63 35·605	35·618	84·365
		Difference 48·747

Temperature of the liquid 13·5 C.

$$\text{Specific gravity of liquid} = \frac{62{\cdot}938}{48{\cdot}747} \times {\cdot}99917 = 1{\cdot}290$$

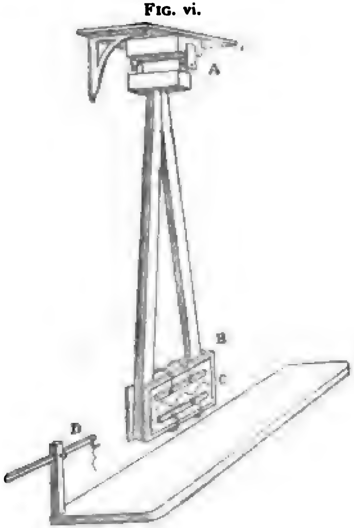
CHAPTER V.*

MEASUREMENT OF VELOCITY AND ACCELERATION.

B. To Measure the Velocity of a Pendulum.

WHEN a body is moving uniformly in a straight line its velocity is measured by measuring the distance it traverses in a measured interval. When the velocity is changing, this method is no longer applicable. The measurement of the distance traversed in a known interval gives only the *mean* velocity for that interval, but by making the interval sufficiently short the result represents adequately the actual velocity during the interval. We shall in this chapter shew how by making use of the very short intervals corresponding to the time of vibration of a tuning-fork a fair measure of the velocity of a moving body can be obtained, and shall further shew how by geometrical methods upon a true scale-diagram of the path of a moving body the velocity and acceleration of the body can be determined. The velocity of the pendulum is not uniform at any point of its path; but when near its lowest position it has a maximum value which varies very slowly. This maximum value may be found thus in the case of a heavy pendulum

FIG. vi.



(A B, fig. vi) mounted so as to move in one plane. A glass plate C is attached to it, and a piece of smoked card or metallic paper is fixed on the plate in such a way that the plane of the card is the plane in which the pendulum moves. D is an adjustable rod fitted with a small movable hook, to which the pendulum can be secured in any desired position. On pulling the hook down by a string attached to it, the pendulum is released and starts swinging. The card in its motion is just touched by a light metallic pointer; if the pointer be fixed, an arc of a circle is traced on the card by the pointer as the plate swings past. This pointer is attached to the prong of a large tuning-fork, which vibrates in a vertical plane. The tuning-fork is not shewn in the figure, but rests on the stand below the pendulum. If when the tuning-fork is set in motion the pendulum is again started, the pointer traces out on the card a sinuous curve cutting the circle in a number of points—1, 2, 3, 4, &c. Each point corresponds to the passage of the fork through its position of equilibrium. Now the characteristic property of the tuning-fork is that the interval between successive passages through the equilibrium position is constant. This constant value is not greatly altered by the friction between the pointer and the card. Let us suppose its value is known for the fork in question, and that it is $1/n$ of a second; n may conveniently be about 60, so that the fork is making 60 vibrations per second.

Then the distances between the successive points 1, 2, 3, &c., are the distances moved over by the pendulum in successive $n$ths of a second.

The distances will vary slightly, but towards the centre of the trace, where the pendulum is moving at its maximum rate, the variations will be small and the waves longer than at either end. Select some of these waves of maximum length, and measure a number of them with a fine scale and dividers, or by the aid of reading-microscopes. Let the

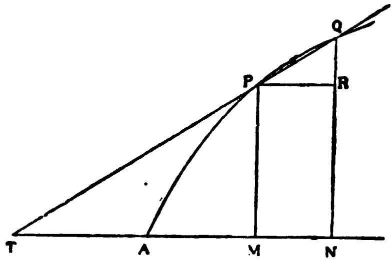
length be x centimetres; this distance was traversed in $1/n$ of a second, and thus the *mean* velocity for that period is nx centimetres per 1 second. Measure the vertical distance h between the position of a point P on the pendulum at the bottom of its swing and the point from which P started, and by a series of observations verify the law that v^2 is proportional to h.

When the velocity of the pendulum has been determined thus by the aid of a known fork, the same apparatus may be used to determine the period of a tuning-fork, for if we repeat the experiment using a fork of unknown frequency n', and if x' be the lengths of the waves as measured, then $x'n'$ = velocity of pendulum at the lowest point of its swing $= xn$, and hence $n' = nx/x'$.

Again, the trace of the fork may be used so as to measure in a similar way the velocity at other parts of the swing, and thus the rate of change in velocity can be determined. But the rate of change of velocity is the acceleration, and we can thus verify the fact that the acceleration is proportional to the displacement from the equilibrium position. To obtain the acceleration plot a curve on squared paper, taking the times as abscissæ and the velocities as ordinates (see p. 50). The velocity is proportional to the length of the waves. We may thus take the horizontal divisions of the paper to represent the period of the fork—one-sixtieth of a second suppose—and at the end of each division draw a vertical ordinate proportional to the corresponding measured wave-length. We thus obtain a curve such as APQ (fig. vii), AMN being the time line. Let PM, QN be ordinates at two instants close together. Draw PR parallel to AMN to meet QN in R; then RQ is the increase in velocity in time MN, and the ratio QR/MN measures the average acceleration or rate of increase of velocity during that interval. Now when Q is very near to P, QP becomes ultimately PT, the tangent to the curve at P, and the ratio QR/MN measures tan PTA or tan θ. Thus

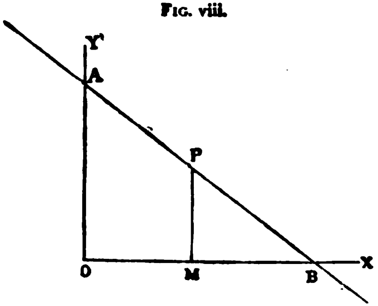
the acceleration at each point is measured by the tangent of the angle which the tangent to the velocity curve makes with the time axis

FIG. vii.



A curve may therefore be plotted in a similar way for the acceleration, and will be found similar to the velocity curve, but with the maxima in a different position. Now in the velocity curve the space described in any interval is represented by the area between the curve, the time line, and the two ordinates at the ends of the interval. Calculate the value of this area for the times 1, 2, 3 . . ., reckon-

FIG. viii.



ing from some convenient instant, and determine the values of the acceleration, or tan θ, at the same instants. Then plot a third curve with the values of the acceleration as ordinates and the distances as abscissæ. It will be found

to be a straight line, as in fig. viii. Thus P M, the acceleration after moving a distance O M, is proportional to M B, the distance from some point B. This point B represents the equilibrium position.

C. To trace the Curve described by a Falling Body and the character of its downward Acceleration.

Take a bottle provided with an aperture in one side near the bottom. A small glass nozzle is fitted into this, so that water issuing from the bottle emerges in a horizontal direction. A pipe passes through a cork in the top of the bottle and reaches about two-thirds of the way down. The bottle is partly filled with water. As the water runs out through the nozzle air enters in bubbles through the pipe; the pressure at the bottom of the pipe is equal to the atmospheric pressure, and hence the pressure of the issuing jet remains constant so long as the surface of the water is above the bottom of the pipe; thus the stream is steady. Under these circumstances the curve described by the water remains unchanged, and this curve is the same as that described by a falling body projected from the nozzle with the velocity of the issuing jet.

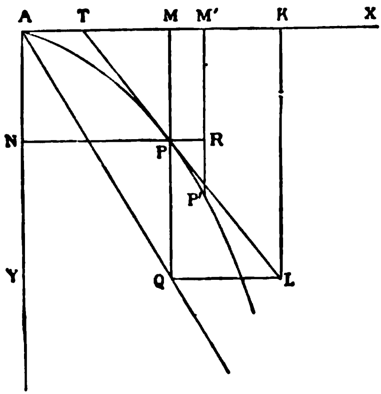
Place near the water-jet, and parallel to its plane, a sheet of glass, and at the farthest convenient distance on the other side place a powerful lamp which will throw a shadow of the jet on the glass. Hold up a sheet of paper against the glass, standing on the side of the glass remote from the lamp; the shadow will be clearly seen, and can be traced on the paper with a pencil. Fix a ruler in a horizontal position so that it casts a shadow on the paper, and thus draw a horizontal line on the paper.[1]

[1] A better curve for the purposes of measurement can be obtained by first drawing a parabola on paper and then adjusting the pressure of the water until the shadow exactly corresponds with the drawn curve.

Let A (fig. ix) be the highest point on the curve corresponding to the shadow of the nozzle from which the jet issues horizontally Through A draw horizontal and vertical lines A M X, A N Y. Let P be any point on the jet, and P M, P N vertical and horizontal. Now the horizontal velocity remains unchanged; thus, as P moves along the jet, M moves uniformly along the line A X, and we may take A M to represent the time of travelling from A to P.

P M is, of course, the vertical distance traversed in this

FIG. ix.



time. Now let P′ be a neighbouring point, and let P′ M′ be drawn vertical to meet A X in M′. Draw P R horizontally to meet P′ M′ in R. Then P′ R is the vertical distance traversed in time M M′. The average vertical velocity during this time is therefore the ratio P′ R/M M′. Let P T, the tangent at P, meet A X in T; then when M M′ is very small P′ P coincides with P T, and the limiting value of the ratio P′ R/M M′ is P M/M T, or the tangent of the angle P T M. This, then, measures the vertical velocity at P.

On P M take a point Q, such that Q M is proportional to tan P T M. Then Q M will represent the vertical velocity at P, and corresponding to each point such as P

a point Q can be found. A curve can be drawn through these points, and this curve will be the vertical velocity-curve for the falling body. The simplest method of determining the position of Q is to set off any convenient constant length T K, say *a* centimetres, from T along T X, and then through K draw K L vertical to meet T P in L; from L draw L Q horizontal to meet M P produced. Then

$$\text{QM} = \text{LK} = \text{TK} \tan \text{PTM} = a \tan \text{PTM}.$$

Thus Q M represents the vertical velocity. Now if the figure be carefully drawn, it will be found that the curve traced out by Q is a straight line passing through A. Thus the vertical velocity increases uniformly with the time, and the vertical acceleration is therefore a constant, and is represented by the tangent of Q A X.

We can represent the results symbolically thus. Let u represent the constant horizontal velocity, v the vertical velocity, t the time from A to P.

Let $\text{AM}=x$, $\text{PM}=y$, $\text{QM}=z$, $\text{PTM}=\theta$, and $\text{QAX}=\psi$.

Then

$$x=ut,\ dx=udt,\ v=\frac{dy}{dt}=u\,\frac{dy}{dx}=u\tan\theta=\frac{uz}{a}$$

Let g be the vertical acceleration.

$$g=\frac{dv}{dt}=\frac{u}{a}\,\frac{dz}{dt}=\frac{u^2}{a}\,\frac{dz}{dx}=\frac{u^2}{a}\tan\psi.$$

If we know the value of g, this result gives the initial horizontal velocity.

It follows from the above results that the curve traced out by P is a parabola. This can readily be verified from the figure, for on measuring the values of P M and P N for different positions of P, it will be found that PN^2 is always proportional to P M, and this is the fundamental property of a parabola with its vertex at A and A Y for its axis.

The curve may be shewn to possess the other characteristic properties of a parabola, and, conversely, some of the known properties of the parabola may be employed to find the focus, axis, and direction of the curve. Thus, if a series of chords be drawn parallel to the tangent at any point P, the diameter, which bisects all these chords, will be a straight line parallel to the axis. If Q V be one-half of one of the chords, the property $QV^2 = 4\,SP, PV$ may be employed to determine S P, the distance of the focus from P, in terms of lengths that can be measured in the figure. Determining in this way the value of S P for two points, P_1, P_2, the focus can be obtained as the intersection of two circles with radii P_1S, P_2S respectively. The axis is the line through the focus drawn parallel to any diameter. The directrix is the locus of intersections of tangents at right angles, the tangent at the vertex is the locus of the foot of the perpendicular from the focus on the tangents, and thus each of these lines can be drawn when the curve[1] only is figured on the paper.

[1] The curve can be shewn experimentally to be described by a pendulum-bob with a long Y-suspension, when the distance of the bob from the junction of the strings is one-quarter of the whole vertical distance of the bob from the points of support of the strings; and also to be the boundary of the shadow of a circle thrown upon a horizontal plane by a point of light on a level with the top of the rim of the circle.

CHAPTER VI.

MECHANICS OF SOLIDS.

20. The Pendulum.

(1) *To determine the Value of* g *by Observations with the Pendulum.*

IF t be the time of a complete oscillation of a *simple* pendulum whose length is l, and g the acceleration due to gravity, then it can be shewn that

$$t = 2\pi\sqrt{\frac{l}{g}}.$$

(See Maxwell, 'Matter and Motion,' Chap. VII.)

Thus,

$$g = \frac{4\pi^2 l}{t^2}.$$

We can therefore find the value of g by observing t, the time of a complete oscillation, and l the length of the pendulum.

A heavy sphere of metal suspended by a fine wire is, for our purposes, a sufficiently close representation of a simple pendulum. Corrections for the mass of the suspending wire, &c., can be introduced if greater accuracy be required.

To observe t, focus a telescope so that the wire of the pendulum coincides when at rest with the vertical cross-wire. A sheet of white paper placed behind the wire forms a suitable background. Set the pendulum swinging, and note by means of a chronometer or clock the times of some six consecutive transits, in the same direction, of the pendulum across the wire of the telescope.

To obtain these, the best plan is to listen for the ticks of the clock, and count in time with them, keeping one eye at the telescope. Then note on paper the number of the tick at which each successive transit takes place.

Thus, suppose the clock beats half-seconds, we should obtain a series of numbers as follows :—

No. of transit	(1)	(2)	(3)	(4)	(5)	(6)
Time noted, 11 hrs. 10 min.	2,	9,	17,	26,	34,	43 ticks.

Thus, successive transits in the same direction occur at the following times :—

No. of transit	(1)	(2)	(3)	(4)	(5)	(6)
Time, 11 hrs. 10 min. . . .	1,	4·5,	8·5,	13,	17,	21·5 sec.

Wait now for one or two minutes,[1] and observe again :—

Transit	(7)	(8)	(9)	(10)	(11)	(12)
Time, 11 hrs. 14 min. . . .	9,	13·5,	17,	22,	26,	30 sec.

Subtracting the time (1) from (7), (2) from (8), &c., we get the times of a certain large but unknown number of oscillations—viz., 4 min. 8 sec., 4 min. 9 sec., 4 min. 8·5 sec., 4 min. 9 sec., 4 min. 9 sec., 4 min. 8·5 sec. ; the mean of these is 4 min. 8·66 sec. So that in 248·66 sec. there is a large whole number of complete oscillations. We have now to find what that number is.

From our first series of observations we may see that five complete oscillations occupy 20·5 sec. Thus, the time of an oscillation deduced from this series is $\frac{1}{5}$ of 20·5 or 4·1 sec. ; from the second series $\frac{1}{5}$ of 21, or 4·2 sec. Thus, the time of a complete oscillation deduced from these two sets of observations is 4·15 sec.

If this were the true time of an oscillation, it would divide 248·66 sec. exactly. On doing the division, the quotient obtained is 59·92 sec. This is very nearly 60, and since there has been a whole number of oscillations in the 248·66 sec. the whole number may have been 60, and, in consequence, the time of an oscillation 248·66/60—i.e. 4·144 sec.

This method of measuring accurately the time of an oscillation turns upon measuring roughly the time of oscillation and then determining the exact number of oscillations in a considerable interval by dividing the interval by the approximate measure of the time of oscillation, and selecting the nearest integer. One important point requires notice.

[1] The rule for determining the proper interval which should be allowed is given later, p. 154.

The rough value of the time of oscillation was determined by observing the time of five oscillations with a clock shewing half-seconds. We must therefore consider the observation of the first and sixth transit as each liable to an error of half a second; that is, the time of the five oscillations is liable to an error of one second, and the calculated time of one is only to be regarded as accurate within 0·2 sec.

All we can be sure of, therefore, is that the time of an oscillation lies between 3·95 sec. and 4·35 sec. Now the nearest integer to 248·66/3·95 is 63, and the nearest integer to 248·66/4·35 is 57; hence, without more observations than have been indicated above, we are not justified in taking 60 as the proper integral number of oscillations during the interval. All we really know is that the number is one of those between 57 and 63.

In order that there may be no doubt about the proper integer to select, the possible error in the rough value of the time of oscillation, when multiplied by the integer found, must give a result less than half the time of an oscillation; thus in the instance quoted the inference drawn is a safe one, provided 4·15 sec. represents the period of one oscillation to the thirtieth of a second. If this be the case the method given above will indicate the proper integer to select as representing the number of oscillations in 248 sec., and therefore give the time of an oscillation correct to about the 250th of a second.

There are two ways of securing the necessary accuracy in the observed time of an oscillation: (1) by making a series of thirty-one transit observations instead of 6, as indicated above; and (2) by repeating the process sketched, using intervals sufficiently small for us to be certain that we can select the right integer.

Thus, suppose six transit observations are made, the second series must be made after an interval not greater than 20 sec., a third after an interval of 60 sec. from the first, a fourth after an interval of 140 sec. From the original

series a result will be obtained accurate to 0·2 sec.; with the first and second the accuracy can be carried to 0·1 sec., with the first and third to 0·05 sec.; and so proceeding in this way, we can with complete security carry the accuracy to any extent desired.

To determine l, we measure the length of the suspending wire by means of a tape, and add one half of the diameter of the bob as measured by the calipers. If the value of gravity is to be expressed in C.G.S. units (cm. per sec. per sec.), the length must be given in centimetres.

Thus the values of t and l have been found. Substituting these in the formula for g, its numerical value may be found. The value of π may be taken as 3·142.

(2) *To compare the Times of Oscillation of two Pendulums. Method of Coincidences.*

The method is only applicable in the case of two pendulums whose periods of oscillation are very nearly in some simple ratio which can be roughly identified.

The two pendulums are arranged one behind the other, and a screen is placed in front with a narrow vertical slit.

A telescope is arranged so as to view through the slit the nearer of the two wires. The second one is not visible, being covered by the first.

Let us suppose that the shorter pendulum vibrates rather more than twice as fast as the longer.

Start the two pendulums swinging; the two wires will appear to cross the slit at different moments. After a few swings they will cross in the same direction at the same moment.

We may notice that the shorter pendulum, besides executing two oscillations while the longer executes one, gradually gains on the latter, but after a time the two again cross simultaneously in the same direction. Let us suppose that this happens after 12 oscillations of the long pendulum; then there have been clearly 25 oscillations of the shorter

in the same interval. Thus, the time of oscillation of the shorter pendulum is

$$\frac{12}{25} \times 4{\cdot}144, \text{ or } 1{\cdot}9891 \text{ sec.}$$

If the longer pendulum had been gaining on the shorter, the latter would have lost one oscillation during the interval, and the ratio of the times of oscillation would have been 12 : 23.

As an example of the method of coincidences for nearly equal times of swing, we may take the accurate determination of g by the aid of Kater's pendulum. Consider the vibration of a body in the form of a long metal rod, fitted with a spherical ball which can slide along it and be secured in any desired position. A knife-edge is fitted to one end of the rod in such a way that when it rests on a pair of horizontal plates the rod hangs vertically and can oscillate about the knife-edge. The other end of the rod is prolonged to form a wire pointer. The rod is placed in front of the pendulum of a clock, and a telescope adjusted to view the two, which are arranged in such a way that when hanging vertically the rod-pendulum is exactly in front of, and hides, some definite and easily recognised mark on the clock pendulum. To obtain such a mark a small silvered bead may be permanently attached to the clock pendulum, and a lamp arranged in such a position that the light from the bead as it passes through the lowest point of its path it reflected into the telescope. Thus at each transit of the bead an observer sees a bright flash of light. If, however, at the same moment the other pendulum is also at the lowest point of its swing, this flash is cut off and does not appear in the field of view of the telescope.

In practice it may, of course, happen that the flash is not entirely eclipsed at any transit. As the observer watches he will see it grow dimmer and then again become brighter; the transit at which the brightness was least will

give the nearest approach to coincidence. Or, again, the flash may be obscured for more than one transit; by taking the mean of the times for which this happens the time of coincidence may be found.

Now let us suppose the clock pendulum to be vibrating rather the more rapidly of the two. Watch the flash through the telescope, and, after noting the time, count seconds in time with the ticks of the clock until coincidence occurs. Write this time down. Do the same for a second, third, and fourth coincidence for motion in the same direction as the first, and thus find the interval, by taking the mean interval for several observations, between two coincidences. Suppose that during this interval n swings of the clock pendulum have occurred. In each swing the clock pendulum has gained a little on the other, and when it has completed n swings the other pendulum has made one less. So that the time of $n - 1$ swings of the latter pendulum is equal to that of n swings of the clock. Thus if t be the time of swing of the clock, T that of the other pendulum,

$$(n-1)\,T = n\,t,$$

$$T = \frac{n}{n-1}\,t.$$

If, on the other hand, the other pendulum is going the more rapidly of the two, we should get $T = n\,t/(n+1)$.[1]

The time of swing of the clock pendulum is obtained from astronomical observations. Direct observation is usually sufficient to determine whether the clock pendulum or the other is going at the quicker rate. The method

[1] If we denote by T and t the times of half-vibrations of the pendulums, and consider only transits in one direction, when a coincidence occurs again the one pendulum has lost or gained one whole or two half vibrations, and thus we get $nt = (n+2)\,T$, or $(n-2)\,T$, as the case may be, and in this case $T = \frac{n}{n+2}\,t$, or $\frac{n}{n-2}\,t$, instead of as above, n being the number of *half-swings* of the clock in the interval between coincidences.

gives the time of the free pendulum, if it does not differ greatly from that of the clock, with great accuracy. Thus, suppose there is coincidence once in every 300 swings of the clock, then we have

$$\begin{aligned} T &= \frac{300}{299} t, \\ &= t \times 1{\cdot}003344 \quad . \quad . \quad . \quad . \quad (1) \end{aligned}$$

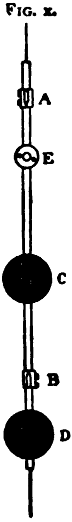
Whereas if we had found that the coincidence occurred every 301 swings, we should have obtained the value

$$T = t \times 1{\cdot}003333 \quad . \quad . \quad . \quad . \quad (2)$$

Thus the error made by a mistake of one in the number of swings between two coincidences is only ·000011 of the time of swing.

It must be remembered in the above that t is the time of a complete oscillation, i.e. the interval between two transits in the same direction.

FIG. x.



For the application of the method in Kater's pendulum the brass rod of the pendulum is fitted with knife-edges (A, B, fig. x) at each end. In general the times of swing about the two knife-edges will be different; but by adjusting the sliding weight E they can be made equal. If, when this is the case, h is the distance between the knife-edges, and T the time of swing, we can make use of the formula

$$T^2 = \frac{4\pi^2 h}{g}.$$

Now if the pendulum be so constructed that T is very nearly one second, its value can be found with great accuracy by the method of coincidences, while the value of h can easily be determined by reading-microscopes (§ 5). The above formula thus gives us an accurate value for g. In practice it is not easy to adjust the pendulum so that the

times of swing from the two knife-edges are exactly equal; if they differ slightly, a small correction to the above formula is required.

It is shewn in books on dynamics that if l_1 l_2 are the distances between the knife-edges and the centre of gravity, T_1 T_2 the times of swing about the two knife-edges, then

$$\frac{4\pi^2}{g} = \frac{T_1^2 + T_2^2}{2(l_1 + l_2)} + \frac{T_1^2 - T_2^2}{2(l_1 - l_2)}.$$

Now $l_1 + l_2$ is equal to h, and can be found accurately; the position of the centre of gravity may be roughly determined by balancing the pendulum, and thus approximate values obtained, l_1 and l_2. If T_1 is nearly equal to T_2, these approximate values are sufficient, for the last term $\frac{T_1^2 - T_2^2}{2(l_1 - l_2)}$ will be very small, unless l_1 is too nearly equal to l_2.

Experiments.

(1) Determine by observations on a simple pendulum the value of g.

(2) Compare the times of oscillation of the two pendulums.

Enter results thus:—

(1) Approximate value of t (from 31 transits)	4·15	sec.
Corrected value from an interval of 4 min. 8·66 sec.	4·144	"
Length of suspending wire	421·2	cm.
Radius of bob	4·5	"
Value of l	425·7	"

$$g = 980 \frac{cm}{(\text{sec.})^2}.$$

(2) Ratio of times from rough observations . 2·1

Interval between coincidences twelve complete oscillations of the longer (the shorter pendulum gaining on the other).

Ratio of times . . . 2·083.

(3) Determine by the method of coincidences the times of

vibration of the given pendulum when supported from the two knife-edges in turn.

Arrange the sliding weight so as to make these times more nearly equal, and hence determine the value of g.

Enter the results thus :—

Pendulum adjusted so that the time between two coincidences was approximately 24 seconds, the coincidences in one direction only being observed. The period of the clock pendulum is 2 seconds. An approximate value of n is therefore 12.

Pendulum erect. Coincidences observed at 15 m. 55 s., 19 m. 10 s. . . . . . 30 m. 9 s.

There have been 8 coincidences in the first interval, and $n = 12{\cdot}2$.

Using this value, we find there have been 35 coincidences in the second interval, and $n = 12{\cdot}2$.

Pendulum inverted. Coincidences at 35 m. 35 s., 38 m. 48 s., ... 49 m. 45 s.

From the first two a more approximate value of n is $12{\cdot}05$, while from the first and third we obtain the more accurate value $n = 12{\cdot}145$.

From these we find— $\quad T_1 = 1{\cdot}8484,$

$$T_2 = 1{\cdot}8478.$$

Also

$$l_1 + l_2 = 84{\cdot}88 \text{ cm.}$$

$$l_1 - l_2 = 55{\cdot}46 \quad ,,$$

$$\frac{\pi^2}{g} = {\cdot}010053 + {\cdot}000003,$$

$$g = 981{\cdot}48 \text{ (cm.) sec}^{-2}.$$

21. Atwood's Machine.

Two equal weights each of mass M are hung by a fine string over a pulley.

A third weight of mass R is allowed to ride on one of these two, thus causing it to descend. After it has fallen through a measured distance, R is removed by means of a ring, through which the weight carrying it can pass, while R cannot.

The time which it takes for the weights to fall through this measured distance is noted.

After R has been removed, the other weights continue moving, and the time they take to pass over another measured distance is observed.

Now, let us suppose that the height through which R falls before being removed is a centimetres and that the time it takes in falling is t seconds.

Let the space through which the weight continues to move downwards be c centimetres, and the time taken t_1 seconds.

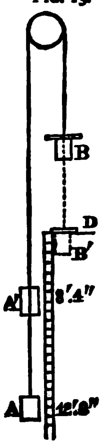
Then, if for the present we neglect the friction and momentum of the pulley which carries the weights, the mass moved is $2M+R$; and the force producing motion is the weight of the mass R; and hence, if f be the acceleration

$$(2M+R)f = Rg;$$

whence
$$f = \frac{Rg}{2M+R}.$$

Also, since R descends through a space a in t seconds, $a=\frac{1}{2}ft^2$; and if v be the velocity acquired by the weights at the time when R is removed, $v=ft$ and $v^2=2fa$.

FIG. 13.



Thus, so long as the weights and rider R remain the same, we must have a proportional to the square of t.

The distance a is easily measured by means of a measuring tape.

Thus, let D (fig. 13) be the ring by which R is removed, and let A and B be the weights in their initial position.

Lower the tape from D to the ground, and note the division with which the top of A coincides. Then release the string and allow the weight to fall, noting the interval t. Next, by pulling the string, raise the weight B until its top comes level with the ring, and note the division of the tape opposite to which A stands.

The difference between these two readings gives the distance *a*.

Thus, in the figure, A stands at 12 ft. 8 in., when B comes to B′ just passing the ring D, A has arrived at A′, and the reading is 8 ft. 4 in. Thus

$$a = 12 \text{ ft. } 8 \text{ in.} - 8 \text{ ft. } 4 \text{ in.} = 4 \text{ ft. } 4 \text{ in.} = 132{\cdot}08 \text{ cm.}$$

We must now shew how the time *t* may be conveniently measured.

This may be done by means of a metronome, a clock-work apparatus, which by adjusting a movable weight can be made to tick any required number of times—within certain limits—in a second. Adjust the weight so that the rate of ticking is as rapid as can conveniently be observed, and count the number of ticks in the time of fall. It will be an advantage if the metronome can be so adjusted that this shall be a whole number. Then determine the number of ticks per second, either by the graduations of the metronome or by taking it to a clock and counting the ticks in a known intèrval, and thus express the time of fall in seconds.

If a metronome is not obtainable, fairly accurate results may be obtained by allowing mercury to flow from a small nozzle through a hole in the bottom of a large flat dish, and catching in a weighed beaker, and then weighing the mercury which flows out while the weight is falling. The weight of mercury which flows out in a known interval of time is also observed, and by a comparison of the two weights the time required is determined.

The time *t* should be observed at least twice for the same fall *a*.

Now make the same observations with a different fall, *a′* suppose, and shew that the law that the space traversed varies as the square of the time is true.[1]

[1] If the apparatus can be arranged so that the distance *a* can be varied, more accurate results may be obtained by determining the value

Now, let the weight B, after falling through the distance a, deposit R upon the ring D, and observe the time required by the weights A, B to pass over a further distance c; let it be t_1 seconds.

The weights move over the space c with uniform velocity v; thus t_1, the time of fall, is inversely proportional to v.

Now, v is the velocity acquired by falling through the distance a; thus v is proportional to the square root of a.

Thus, t_1 should be inversely proportional to the square root of a, or t_1^2 proportional to $1/a$.

Thus, $a t_1^2$ should be constant, and equal to $c^2/2f$.

Observe the value of t_1 for various values of a, and shew that $a t_1^2$ is constant.

From the last observations we can calculate the value of g, the acceleration due to gravity.

For if f be the acceleration produced by the weight of the mass R,

$$f = \frac{Rg}{2M+R},$$

$$v^2 = 2fa, \qquad c = v t_1;$$

$$\therefore \quad c^2 = v^2 t_1^2 = 2 f a t_1^2$$

$$f = \frac{c^2}{2a t_1^2} = \frac{Rg}{2M+R},$$

$$g = \frac{2M+R}{R} \times \frac{c^2}{2a t_1^2}.$$

M and R are the number of grammes in the weights used.

We have neglected the effect of the momentum produced in the pulley and of friction.

We can allow for the former in the following manner:—

of a, for which the time t is an exact multiple of the period of the clock or metronome.

It can be shewn theoretically that its effect is practically to increase the mass moved without altering the force tending to produce motion. Thus we should include in the mass moved a quantity W, which we can calculate by theory, or better determine by experiment.

Thus, if f as before be the acceleration,

$$f = \frac{\text{R}g}{2\text{M}+\text{R}+\text{W}} = \frac{c^2}{2at_1^2}.$$

Repeat the observations, using the same value of c and a, but altering the rider to R′; t_1 will be changed to t_1', and the acceleration will be f' where

$$f' = \frac{\text{R}'g}{2\text{M}+\text{R}'+\text{W}} = \frac{c^2}{2at_1'^2};$$

$$\therefore \quad \frac{2ag}{c^2}\text{R}'t_1'^2 = 2\text{M}+\text{R}'+\text{W}.$$

But

$$\frac{2ag}{c^2}\text{R}t_1^2 = 2\text{M}+\text{R}+\text{W}.$$

Hence

$$\frac{2ag}{c^2}(\text{R}t_1^2 - \text{R}'t_1'^2) = \text{R}-\text{R}',$$

and

$$g = \frac{(\text{R}-\text{R}')c^2}{2a(\text{R}t_1^2-\text{R}'t_1'^2)}.$$

To eliminate the effect of friction we may determine experimentally the least mass which we must attach to the weight B in order just to start the apparatus. Let this be F grammes. Then, if we assume the friction effect to be constant throughout the experiment, the part of R which is effective in producing acceleration is R − F; we must therefore substitute R − F for R throughout.

It is probably not true that the frictional effect is the same throughout; the apparatus is, however, so constructed

that it is very small, and a variation from uniformity is unimportant.

The string by which the weights are hung is generally thin; be careful therefore lest it break.

Experiments.

(1) Shew from three observations that the space through which a mass falls in a given time is proportional to the square of the time.

(2) Shew with the above notation from three observations that at_1^2 is a constant.

(3) Determine the value of g, using two or three different masses as riders.

(4) Obtain from your results with two of these riders a value for g corrected for the inertia of the pulley.

(5) Correct your result further for the friction of the pulley.

Enter results as below:—

Exp. 1.

	Value of a	Value of t	Ratio $\frac{a}{t^2}$
(1)	400 cm.	7·5 sec.	7·1
(2)	300 „	6·5 „	7·1
(3)	200 „	5·4 „	6·9

Exp. 2.

	Value of a	Value of t_1	Product of at_1^2
(1)	400 cm.	4·3 sec.	739
(2)	300 „	4·9 „	720
(3)	200 „	6·1 „	744

Exp. 3.

	a = 400 cm.	c = 450 cm.
	M = 300 gm.	
(1)	R = 10 „	t_1 = 4·3 sec.
(2)	R′ = 8 „	t_1' = 4·5 „
(3)	R″ = 6 „	t_1'' = 5·2 „

Values of g respectively—

945 942 946

D. The Fly-wheel.

The kinetic energy of a particle of mass m moving with velocity v is $\frac{1}{2} m v^2$. If the particle be describing a circle of radius r with angular velocity ω, then $v = r \omega$, and the kinetic energy becomes $\frac{1}{2} m r^2 \omega^2$. The momentum of the particle is $m v$, or $m r \omega$.

The moment of this momentum about the centre, or the angular momentum of the particle, is $m r v$, or $m r^2 \omega$. If the particle form part of a rigid body rotating about a fixed axis, then ω, the angular velocity, is the same for all the particles.

Thus the whole angular momentum of the rotating body is $\omega \Sigma (m r^2)$, or $\text{K} \omega$; and the whole kinetic energy is $\frac{1}{2} \omega^2 \Sigma (m r^2)$, or $\frac{1}{2} \text{K} \omega^2$; M being the mass of the body, and K its moment of inertia about the axis.[1] Let h be the distance of its centre of gravity from the axis.

[1] *Moment of Inertia.*—The moment of inertia of a body about a given axis may be defined physically as follows:—If a body oscillate about an axis under the action of forces which, when the body is displaced from its position of equilibrium through an angle θ, produce a couple tending to bring it back again, whose moment about the axis of rotation is $\mu\theta$, then the time of a complete oscillation of the body about that axis will be given by the formula

$$t = 2\pi \sqrt{\frac{\text{K}}{\mu}}$$

where K is a 'constant' which depends upon the mass and configuration of the oscillating body, and is called the moment of inertia of the body about the axis of rotation.

It is shewn in works on Rigid Dynamics that the relation between the moment of inertia K and the mass and configuration of the body is arrived at thus:—K is equivalent to the sum of the products of every small elementary mass, into which the body may be supposed divided, into the square of its distance from the axis about which the moment of inertia is required, or in analytical language $\text{K} = \Sigma m r^2$ (Routh's 'Rigid Dynamics,' chap. iii.).

The following are the principal propositions which follow from this relation (Routh's 'Rigid Dynamics,' chap. i.):—

(1) The moment of inertia of a body about any axis is equal to the sum of the moments of inertia of its separate parts about the same axis.

(2) The moment of inertia of a body about any axis is equal to the moment of inertia of the body about a parallel axis through the centre of

Again denote by $\dot{v}$ the rate of change of velocity, i.e. the acceleration in the direction of motion, and by $\dot{\omega}$ the angular acceleration Then $\dot{v} = l\,\dot{\omega}$.

Let F be the force acting in the direction of motion.

Then, since rate of change of momentum is equal to the impressed force,

$$F = m\,\dot{v} = m\,r\,\dot{\omega},$$
$$\therefore\ \Sigma\,(F\,r) = \dot{\omega}\,\Sigma\,(m\,r^2)$$
$$= K\,\dot{\omega}.$$

Now among the forces F we must reckon those which arise from the mutual reactions of the particles of the body. But if P, Q be any two particles of the body, and if Q act on P with a certain force F in any direction, then since action and reaction are equal and opposite; P acts on Q with a force $-$ F.

Thus the mutual reactions contribute nothing to the product $\Sigma\,(r\,F)$, which therefore measures the moment about the axis of the impressed forces.

gravity together with the moment of inertia of a mass equal to the mass of the body, supposed collected at its centre of gravity, about the original axis.

(3) The moment of inertia of a sphere of mass M and radius a about a diameter is $M\frac{2}{5}a^2$.

(4) The moment of inertia of a right solid parallelepiped, mass M, whose edges are $2a$, $2b$, $2c$, about an axis through its centre perpendicular to the plane containing the edges b and c is

$$M\frac{b^2+c^2}{3}.$$

(5) The moment of inertia of a solid cylinder mass M and radius r about its axis of figure is

$$M\frac{r^2}{2},$$

about an axis through its centre perpendicular to the length of the cylinder,

$$M\left(\frac{l^2}{3}+\frac{r^2}{4}\right),$$

where $2l$ is the length of the cylinder.

It is evident from the fact that in calculating the moment of inertia the mass of each element is multiplied by the *square* of its distance from the axis, the moment of inertia will in general be different for different distributions of the same mass with reference to the axis.

Thus we have as the equation of motion of a body about an axis the following :—

Moment of impressed forces

= Moment of inertia multiplied by angular acceleration

= Rate of change of angular momentum.

This statement, then, is the expression of the second law of motion applied to a body rotating about an axis.[1]

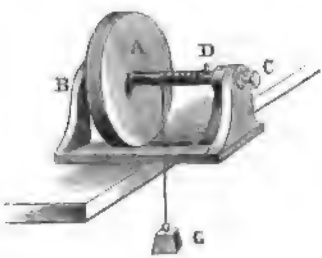
The principle of the conservation of energy also tells us that the increase in the kinetic energy of the body is equal to the work done by the impressed forces. Thus, if we suppose the body to start from rest we have the result

$$\tfrac{1}{2}\,\mathrm{K}\,\omega^2 = \text{Work done by the impressed forces.}$$

This result can readily be deduced from the former.

We may exemplify the above by considering the motion of a flywheel mounted so as to turn on a horizontal axis B C (fig. xi) without much friction.

FIG. xi.



A loop at the end of a piece of string is passed round a pin D on the axle, and a weight G attached to the other end of the string wound up by turning the wheel. When the weight is released the string is unwound off the axle, thus turning the wheel as the weight descends ; when the string is completely unwound the loop is released from the peg and the weight falls freely ; the wheel continues to rotate until stopped by friction.

[1] The above statement has been deduced after the usual method from the second law of motion as applied to linear motion. It may be noticed that the science of dynamics may be based upon it as a fundamental law analogous to the second law of motion. Substituting couple for force, angular velocity and acceleration for linear velocity and acceleration, moment of inertia for mass, we get for rotation about an axis a series of propositions exactly corresponding to those for linear motion. Both systems give, of course, the same expression for kinetic energy when the moment of inertia is expressed as $\Sigma m r^2$.

Now it is shewn by the results of experiments that when the wheel is started the friction remains nearly the same, and is independent of the velocity, so that the work done by the friction in each turn is the same at whatever rate the wheel is moving. Let this work be F; let ω be the angular velocity of the wheel and K its moment of inertia.

The first step is to find F.

Wind up the weight, then release it, and after the string has fallen off let the wheel make N complete turns before coming to rest; let ω be the angular velocity of the wheel at the moment the string falls off. The kinetic energy of the wheel at that moment was $\frac{1}{2} K \omega^2$, and by the time the wheel stops this has been used in doing work against the friction. The amount of work so done is F . N, and hence

$$F N = \frac{1}{2} K \omega^2.$$

If, then, we know K, and can observe ω, we can find the work done against friction.

To find ω, a strip of metallic paper is fastened on to the rim of the wheel by india-rubber bands or otherwise, and a tuning-fork, carrying a light metallic style and vibrating in a horizontal plane, is arranged so that the style can be readily brought into contact with the paper by pushing the stand of the tuning-fork against fixed stops on the table. The tuning-fork is set vibrating, and as the weight falls off it is moved so that the style just touches the paper. A wave-curve is thus drawn on the paper, and, as in § B, the wave-length gives the distance traversed by a point on the rim of the wheel during one vibration of the fork. Multiplying this by the number of vibrations per second, we get the velocity u, say, of the rim at that moment, and dividing this by a, the radius of the wheel, we have the required angular velocity ω.

Again, suppose we consider the motion while the weight is still on the string, after the wheel has made n revolutions

from the start, and the weight has descended a distance z cm. Let m be the mass of the weight. Let r be the radius of the axle, v the velocity of the falling weight. This is, of course, the same as that of a point on the axle.

Since a length $2\pi r$ of string is unwound at each turn, the weight descends through this distance in one turn, and since z is the fall in n turns,

$$z = 2n\pi r.$$
$$\text{Also } v = r\,\omega.$$

Now in descending a distance z the weight has lost an amount of potential energy mgz; this has been used (1) in giving kinetic energy to the wheel; (2) in giving kinetic energy to the weight; (3) in overcoming the friction. Thus

$$mgz = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\mathrm{K}\,\omega^2 + \mathrm{F}\,n,$$
$$\therefore\ 2n\pi\,rmg = \tfrac{1}{2}(mr^2 + \mathrm{K})\,\omega^2 + \mathrm{F}\,n.$$

We have already seen how to find F. If we substitute its value in this equation we can determine ω, and then verify the result by finding the same experimentally.

The quantity K includes the moment of inertia of the axle as well as that of the wheel. If M be the mass of the wheel, treated as a uniform circular disc, M′ that of the axle, then

$$\mathrm{K} = \tfrac{1}{2}\mathrm{M}\,a^2 + \tfrac{1}{2}\mathrm{M}'\,r^2.$$

The apparatus can usually be arranged so that the last term is very small.

The following method will give us the friction without an accurate knowledge of ω.

After the weight has fallen off let the wheel continue to run, and suppose it make n' turns before coming to rest. Then

$$\tfrac{1}{2}\mathrm{K}\,\omega^2 = \mathrm{F}\,n',$$
$$\therefore\ mgz = \tfrac{1}{2}mv^2 + \mathrm{F}\,(n + n').$$

This result is obvious, for, since the wheel has come to rest

again, the potential energy of the fallen weight has been used in producing kinetic energy $\frac{1}{2} mv^2$ in that weight, and in doing work $F(n + n')$ against friction in $(n + n')$ turns of the axle. Now we can usually arrange the experiment so that $\frac{1}{2} mv^2$ is very small compared with mgz, and when this is the case

$$F = \frac{mgz}{n + n'}.$$

Even if we cannot entirely neglect the kinetic energy of the weight, an approximate value of ω, and therefore of v, will enable us to calculate the term $\frac{1}{2} mv^2$ with sufficient accuracy.

Experiment.—Find the angular velocity generated by the effect of the given couples in measured intervals of time, and deduce the moment of interia of the fly-wheel.

E. Pendulum of any shape.

A simple pendulum consists of a mass m attached at one end of a string or weightless rod of length l, and allowed to vibrate about the other end. If such a pendulum be displaced a distance x measured along its path from its equilibrium position, then we know (see Maxwell, 'Matter and Motion,' art. cxix.) that it has potential energy measured by $mgx^2/2l$.

Moreover, if a mass m execute simple harmonic vibrations (n per second), the potential energy at a distance x is $2m\pi^2n^2x^2$, and these two expressions for the energy must be equal. Thus :—

$$2m\pi^2n^2 = mg/2l;$$

$$n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

A similar method of reasoning may be applied to the case in which the pendulum is not a simple one, but consists of a rigid body vibrating about a horizontal axis. For

take the plane of the paper as the plane of motion; let the axis cut this plane in O; then the forces acting on the body are its weight and the reaction through O. Now as the body oscillates, the point O, through which this reaction acts, remains fixed, and no work is done on the body by the reaction. The changes in the potential energy then depend only on the weight of the body which acts vertically through its centre of gravity and on the position of the centre of gravity. The potential energy is the same as it would be if the whole mass were concentrated at the centre of gravity.

Thus if M be the whole mass of the body, h the distance of its centre of gravity below O, and θ the angle through which a line through O is at any moment displaced, the value of the potential energy is

$$\tfrac{1}{2}g\,\theta^2 \mathrm{M} h,$$

for this is the expression we have found for the potential energy of a mass M oscillating at a distance h below a fixed point.

But taking the second expression for the potential energy given above, since the number n of vibrations per second is the same for each particle, we have

$$\text{Total potential energy} = \Sigma(2m\pi^2 n^2 x^2) = 2\pi^2 n^2 \Sigma(m l^2 \theta^2)$$
$$= 2\pi^2 n^2 \theta^2 \Sigma(m l^2) = 2\pi^2 n^2 \theta^2 \mathrm{M} k^2,$$

where $\mathrm{M}k^2$ expresses the result of finding the value, $\Sigma(ml^2)$, for all points.

$\mathrm{M}k^2$ is clearly the moment of inertia of the body, and k is known as its radius of gyration. We may sometimes conveniently denote the product $\mathrm{M}k^2$ by a single symbol K.[1]

Thus we have

$$\tfrac{1}{2}g\,\theta^2 \mathrm{M} h = 2\pi^2 n^2 \theta^2 \mathrm{K}$$
$$= 2\pi^2 n^2 \theta^2 \mathrm{M} k^2,$$

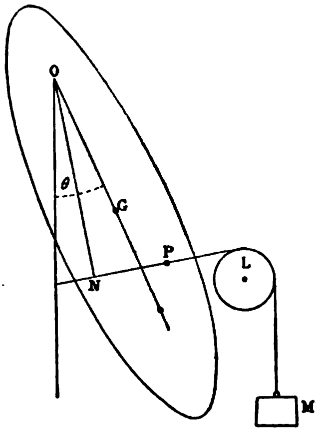
$$\therefore\; n = \frac{1}{2\pi}\sqrt{\frac{\mathrm{M}gh}{\mathrm{K}}} = \frac{1}{2\pi}\sqrt{\frac{gh}{k^2}}.$$

[1] See footnote, p. 166.

Now the value of K can be calculated for bodies of certain definite forms (see p. 167).

If K be known, we can use a rigid pendulum to calculate g; if, on the other hand, K be not known, we can use the above result to find it, provided we know g and can find n and $M\,h$.

FIG. xii.

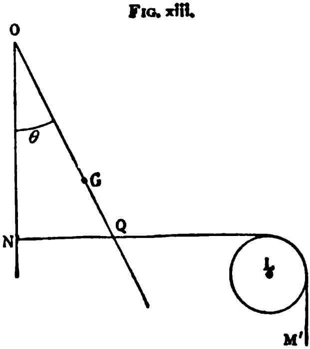


The following measurements will give us $M\,h$.

Attach a fine string to some point P of the pendulum (fig. xii), pass it over a good pulley L, and fasten a mass M' to the end. Then the tension of the string is $M'g$. Let O N be perpendicular to the direction of the string. Let θ be the angle between the displaced position of O G and the vertical. Taking moments about O, we have

$$M'.g.\,ON = M.g.\,OG\sin\theta$$
$$= Mgh\sin\theta;$$
$$\therefore\ Mh = M'.\,ON\operatorname{cosec}\theta.$$

FIG. xiii.



In practice it would be simplest to arrange the pulley so that the string is horizontal; then O N is vertical, and the equation to find K becomes,

$$K = \frac{Mgh}{4\pi^2 n^2}$$
$$= \frac{1}{4\pi^2 n^2} M'.g.\,ON\operatorname{cosec}\theta.$$

To find cosec θ mark with a plumb-line the vertical line through O before the body is displaced. This becomes

O G. Let the direction of the string, supposed horizontal, cut the displaced position of this line in Q (fig. xiii), then

$$\operatorname{cosec} \theta = \mathrm{OQ}/\mathrm{QN},$$

and

$$\mathrm{K} = \frac{1}{4\pi^2 n^2} \mathrm{M}' g \frac{\mathrm{ON} \cdot \mathrm{OQ}}{\mathrm{QN}}.$$

The following is another method of finding K.

Attach to the given body, so as to vibrate about the same axis, another body whose moment of inertia about the axis can be calculated. Let K′ be this moment of inertia, M′ the mass of the body, and h' the distance from O of its centre of gravity. Observe the time of vibration; let it be $1/n'$, then we have

$$n^2 = \frac{1}{4\pi^2} \frac{\mathrm{M} g h}{\mathrm{K}};$$

$$n'^2 = \frac{1}{4\pi^2} \frac{(\mathrm{M} h + \mathrm{M}' h') g}{\mathrm{K} + \mathrm{K}'}.$$

From these two equations we can eliminate $\mathrm{M}h$, and if M′, h', and K′ are known, can find K.

Experiment.—A rectangular bar of iron is made to vibrate about a knife-edge near one end at right angles to its length. Find the value of its moment of inertia, (1) by the first method; (2) by attaching to one end a sphere of lead.

F. Ballistic Pendulum. Measurement of Moment of Momentum and of Momentum.

If a moving body (*e.g.*, a moving iron ball or a hammer head) comes in contact with a heavy body (called for brevity a ballistic pendulum) having a definite position of equilibrium, but free to rotate about a horizontal axis, then (1) the momentum of the moving body is changed by the impact, and the change of momentum measures the impulse of the

blow delivered by the one body and received by the other; (2) the pendulum starts from its position of equilibrium with an angular velocity such that the moment of its momentum about the axis of rotation is equal to the moment of the impulse about the same axis, or, to put the case in more general terms, if the pendulum is already moving when the blow takes place, the change of moment of momentum about the axis of rotation is equal to the moment of the impulse about the axis. These two statements are derived directly from Newton's laws of motion (see p. 168). The pendulum will reach a position of instantaneous rest at the extremity of its first swing when the kinetic energy of its motion has been converted into the potential energy due to the lifting of the centre of mass of the pendulum against the forces of gravitation, allowance being made for work spent in overcoming the forces due to friction with the air and at the axis of rotation, the effects of which may usually be neglected. It will then swing back again and oscillate with gradually diminishing amplitude.

In computing the change of momentum of the impinging body we should require to know its velocity before and after impact as well as its mass. If the material of the surface be plastic, as lead or putty is, and the pendulum be, comparatively speaking, very heavy, the impinging body will be simply stopped by the blow, and the measure of the impulse then depends merely upon its initial velocity.

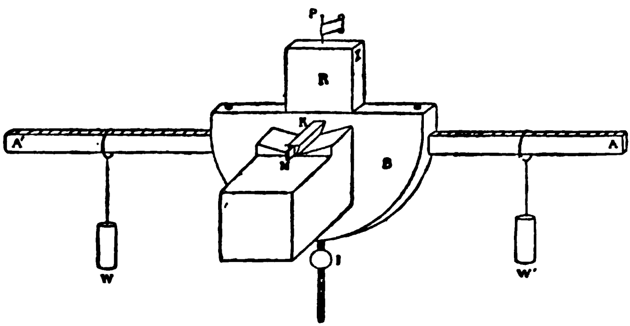
We could clearly compute the moment of the impulse if we could measure the angular velocity ω communicated to the pendulum provided we knew its moment of inertia κ; for the moment of momentum, to which the moment of the impulse is equivalent, is $\kappa\omega$; instead of measuring directly the angular velocity, we may deduce it from observations which are easier in practice—viz., the amplitude of

the first swing after receiving the blow, and the time vibration of the pendulum.

We shall describe a form of apparatus suitable for use ir a laboratory, in which the principles above indicated can b practically applied to the measurement of the moment c an impulse, and consequently to the calculation of th change of momentum produced by a blow. A somewhat similar form of apparatus has long been known under the name of the ballistic pendulum, and has been used to measure the initial momentum of a rifle bullet, and an apparatus based upon precisely similar dynamical principles is regularly used as a 'ballistic galvanometer needle' to measure transient electric currents. (See chap. xxi.)

The apparatus is represented in fig. xiv. A A′ is a long beam, tightly gripped in the two halves of a groove cut in a

FIG. xiv.



pair of thick boards, which are shaped into segments of circles somewhat greater than a semicircle, and which form when screwed together a substantial block B. The whole can swing on a knife-edge, fixed so that the axis of rotation coincides with the common axis of the semicircles. The top face of the beam passes through the axis, and the beam

is graduated on each side, from the line where the knife-edge meets it. Two weights of measured and equal mass are hung by knife-edge attachments (or simply by wire loops) from corresponding graduations on the two sides of the centre. These are used to alter the moment of inertia of the pendulum without altering its total mass or the position of its centre of gravity, so that in dealing with its oscillations the quantities denoted by M and h (p. 166) may be regarded as the same for all positions of the movable weights. At the top of the block B is firmly fixed a rectangular block of wood R to receive a horizontal blow at a marked point. To administer a horizontal blow a pendulum-bob, supported in the proper position by a V-shaped suspension can be used. The advantage of this arrangement is that the momentum of the bob can be calculated from its mass and its initial displacement. A vertical blow can be given (by a hammer or a falling mass) upon a stud driven into the horizontal face of the block B.

The knife-edge must be supported in a horizontal position by blocks on each side, the edge lying in a shallow V-groove. The whole pendulum is then symmetrical about the vertical plane through the middle of the beam and the vertical plane through the knife-edge.

The angular deflexion produced by a blow can be found roughly by graduations on the circular edge of the block B, or, more accurately, by reading, as with a mirror galvanometer, the displacement of the image of a scale viewed in a mirror M attached to the extremity of the knife-edge. A pin placed at the same distance behind the mirror as the image of the scale, and an opera-glass, will enable the experimenter to dispense with the darkening of the room.

To facilitate adjustments the apparatus above described should be completed by the addition of arrangements

P and I) corresponding respectively to the flag and inertia-bob of a balance, in order that the centre of gravity of the whole may be slightly moved horizontally or vertically, as may be found necessary.

The experiments which may be performed with the ballistic pendulum are as follows :—

(*a*) The observation of the time of vibration with the movable weights in two different positions, and the calculation from the observations of the moment of inertia of the pendulum.

(*b*) The observation by means of the mirror and scale of the amplitude of the first swing when a horizontal blow is struck by a pendulum-bob pulled aside through a measured vertical height, and the calculation of the momentum of the impinging bob. The variations of the effect caused by the ballistic pendulum not being quite at rest and by an alteration of the material upon which the blow is delivered can also be observed. The observations may be repeated with the movable weights at various distances.

(*c*) The observation of the deflexion due to the blow of a hammer or other impulse of unknown magnitude, with a view to its measurement.

(*d*) Observation of the permanent deflexion due to a force of known moment about the axis of rotation.

The theory of the working of the apparatus is as follows :—

(*a*) For the determination of the moment of inertia K, when the weights, each of mass W, are at a distance l from the centre. If τ is the observed time of vibration, K_0 the moment of inertia of the pendulum without any movable weights, M the mass of the pendulum, and h the distance of its centre of gravity below the axis,

$$\tau = 2\pi\sqrt{K/Mgh} = 2\pi\sqrt{(K_0+2Wl^2)/Mgh} \quad . \quad . \quad (1)$$

Let the weights be moved to a distance l', let the corre-

sponding time of vibration be τ'. Neither M, nor h, nor g is altered, hence

$$\tau' = 2\pi\sqrt{(K_0+2W\,l'^2)/Mgh}\,;$$

whence

$$K = 2W\,\tau^2(l'^2-l^2)/(\tau'^2-\tau^2) \quad . \quad . \quad . \quad (2)$$

(*b*) and (*c*) For the calculation of the moment of the impulse from the amplitude of the first swing, let ω be the initial angular velocity due to the impulse, then $K\,\omega$ is equal to the moment of the impulse. The initial kinetic energy is $\frac{1}{2}K\,\omega^2$. When the pendulum is in equilibrium, the distance of its centre of mass below the point of suspension is represented by h; when it has been deflected through an angle α, the centre of mass has been raised through a height $h\,(1-\cos\alpha)$, *i.e.* $2\,h\,\sin^2\frac{\alpha}{2}$, and the work done in raising it is $2\,Mgh\,\sin^2\frac{\alpha}{2}$, if α represents the amplitude of the first swing. The work done in the raising is the equivalent of the kinetic energy which has disappeared (neglecting the losses on account of friction). Hence we have

$$\tfrac{1}{2}K\,\omega^2 = 2\,Mgh\,\sin^2\frac{\alpha}{2},$$

and, making use of equation (1), we get

$$\omega = 4\pi\,\sin\frac{\alpha}{2}/\tau \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the moment of the impulse is $4\pi\,K\,\sin\frac{\alpha}{2}/\tau$. If the vertical distance of the point at which the blow is delivered above the knife-edge is b, we get the impulsive change of momentum of the moving body to be $4\pi\,K\,\sin\frac{\alpha}{2}/\tau b$.

(*d*) The permanent deflexion produced by a known mass, w, hung from the beam at distance λ, is merely the result of using the pendulum as if it were a balance, and

the theory is that of the balance. Hence, if θ is the deflexion produced,

$$w \times \lambda = \text{M}\, h \tan \theta,^{*}$$

whence

$$\text{M}\, h = w \lambda \cot \theta.$$

This equation enables us to determine the moment of inertia K from equation (1) without an observation of the time of vibration for an altered position of the movable weights. (Compare the corresponding use of the deflexion produced by a steady current in the ballistic galvanometer. Chap. xxi.)

We need only add a few practical details.

Measurement of Times of Oscillation.—There should be considerable distance between the two positions of the weights. On moving the weights to a new position, the position of equilibrium of the pendulum, as read by the mirror and scale, must be adjusted to be the same as before. The times may be taken with sufficient accuracy by timing, say, fifty vibrations with a stop-watch.

Measurement of First Swing.—To secure that the blow is horizontal the bob should be arranged to hang freely, just touching the block R when the pendulum is at rest. It is important that the pendulum should be quite at rest when the blow is delivered, and the position of equilibrium read before each observation. One observer should watch the image of the scale, while another lets the bob go. If the motion is too rapid for the scale reading to be satisfactorily taken, a piece of black thread may be tied round the scale, and gradually adjusted until its reflected image is just reached on repeating the observation. The scale reading of the thread can then be taken at leisure, and the differences for different successive observations can be estimated.

The ratio of the scale reading of the deflexion divided by the distance of the scale from the mirror (both ex-

pressed in the same measure) is the tangent of *twice* the angle of deflexion, and may for small angles be taken to be equal to the sine of the double angle. The sine of the half angle may accordingly be taken as one quarter of the ratio. The angular deflexion ought to be small in any case, as the law of isochronous vibrations does not apply with sufficient accuracy when the oscillations of a pendulum are of considerable amplitude.

Observations may be recorded in the following form :—

Distance of scale from mirror . . . . 60·3 cm.
Mass of sliding weights, each . . . . 2050 grammes

Distance of sliding weights from centre	11 in. = 27·94 cm.	22 in. = 55·88 cm.
Corresponding time of vibration	40 half-vibrations in 49 secs.	30 half-vibrations in 62·75 secs.
First swing after impulse	330 − 230 = 100 mm.	333 − 280 = 53 mm.

From time observations,

$$K_0 = 1{\cdot}602 \times 10^6 \text{ gm. cm.}^2$$
$$K_0 + K_{11} = 4{\cdot}808 \times 10^6 \text{ gm. cm.}^2$$
$$K_0 + K_{22} = 1{\cdot}442 \times 10^7 \text{ gm. cm.}^2$$

From first deflexion observation,

$$\text{Moment of impulse} = 1{\cdot}022 \times 10^6 \text{ gm. } \frac{\text{cm.}^2}{\text{sec.}}$$

From second deflexion observation,

$$\text{Moment of impulse} = 1{\cdot}016 \times 10^6 \text{ gm. } \frac{\text{cm.}^2}{\text{sec.}}$$

$$\text{Mean moment of impulse} = 1{\cdot}019 \times 10^6 \text{ gm. } \frac{\text{cm.}^2}{\text{sec.}}$$

Vertical distance of point of impact from axis = 17·8 cm.
Weight of bob, 320 gms.
Vertical height of fall required to generate the impulse

$$= \{1{\cdot}019 \times 10^6/17{\cdot}8 \times 320\}^2/981 \times 2 = 16{\cdot}3 \text{ cm.}$$

It will be evident that the same apparatus can be used to illustrate the logarithmic decrement of oscillations and some other interesting dynamical questions which we have not space to discuss.

G. Funicular Polygon. Graphic Method of Comparing Forces.

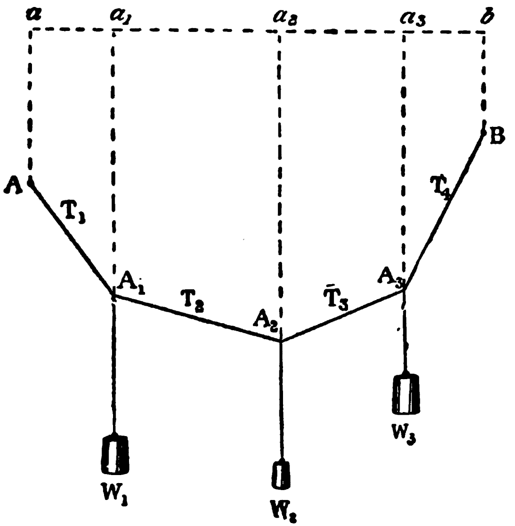
If three forces acting at a point are in equilibrium, they can be represented in *magnitude* and *direction* by the three sides of a triangle taken in order. This is the proposition known as the 'triangle of forces.' If the directions of the three forces are given, and if a triangle be constructed with its three sides respectively parallel to those directions, the magnitudes of the forces are proportional to the lengths of the respective sides, and the forces can be compared by measuring and comparing the lengths. We shall illustrate this method of comparing forces by applying it to the following special case, an example of what is well known as the funicular polygon.

Weights, W_1, W_2, W_3, *&c., the mass of one of which,* W_1, *is known, are hung from separate points,* A_1, A_2, A_3, *of a string. The ends of the string are made fast to two fixed points,* A *and* B. *Find the mass of* W_2 *and of* W_3, *and the tensions of the several portions of the string.*

We must first draw a scale diagram, A A_1 A_2 A_3 B (fig. xv), in which the directions of the forces, as indicated by the strings, are correctly given. For this purpose mount a graduated straight-edge accurately horizontal above the higher of the two points A, B, and set out on paper a straight line to represent the horizontal edge. Then with a T-square and rule, or a plumb-line, measure the vertical distances aA, $a_1 A_1$, &c., of the respective points A, A_1, A_2, A_3, B, below the straight-edge, and read also the horizontal distances $a a_1$, $a_1 a_2$, &c. Set out these horizontal and vertical distances in the diagram to any convenient scale, and join by straight lines the points representing A and A_1, A_1 and A_2, &c., respectively. These joining lines, together with the vertical, shew the directions of all the forces acting at the several points.

At each of the points A_1, A_2, A_3 three forces act, namely, two tensions and a weight. We next proceed to construct a triangle (fig. xvi) with its sides, taken in order, parallel to the three forces W_1, T_1, and T_2, acting at A_1. First set out $X_1 X_2$ vertical, and of a convenient length to represent accurately W_1, the known weight, in magnitude. The line $X_1 X_2$ is parallel to $A_1 a_1$. Next, by a parallel ruler, or by a

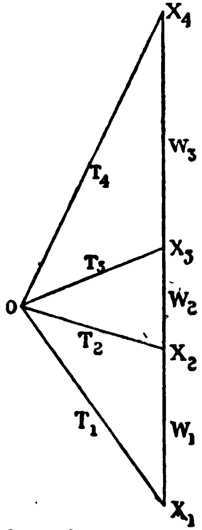
FIG. XV.



straight-edge and set square, draw a line $X_1 O$ *accurately* parallel to $A_1 A$, and then from the point X_2 draw $X_2 O$ accurately parallel to $A_1 A_2$. The point O is the intersection of the last two lines, and a triangle O X_1 X_2 has been drawn with its three sides, taken in order, parallel to W_1, T_1, and T_2 respectively, acting at the point A_1. Their lengths therefore represent those three forces in magnitude on the same scale as that on which $X_1 X_2$ represents W_1.

Next produce the vertical $X_1 X_2$, and through O draw OX_3 accurately parallel to $A_2 A_3$. Then, remembering that the force at A_2, due to the tension, is equal and opposite to the force at A_1, due to the tension of the same piece of the string, it is evident that $O X_3 X_2$ is a triangle with its sides representing in direction, and consequently in magnitude also, on the same scale as before, the three forces acting at A_2.

FIG. xvi.



Similarly we can construct the triangle $O X_4 X_3$, and the sides of the triangles which are comprised in the figure $O X_1 X_4$ represent in magnitude and direction all the forces acting. The magnitudes of these forces can then be compared by comparing the actual measured lengths of the respective lines. The lines to be measured in order to determine the magnitudes of the respective forces are indicated in fig. xvi.

We have neglected the weight of the portions of the string itself, and in practice this is quite justifiable with good sized weights. It is, however, not a difficult extension of the same method to compare the tensions at different points of a heavy chain hanging between two points, and find the weight of unit length by observing the shape in which the chain hangs when a known mass is hung on one link. The tensions at the two ends of the string can be found by the use of a spring balance.

SUMMARY OF THE GENERAL THEORY OF ELASTICITY.

The elastic properties of an isotropic homogeneous elastic body depend on two qualities of the body—viz. its compressibility and its rigidity. The compressibility determines the alteration in volume due to the action of external forces, the rigidity the alteration in form.

Compressibility and Elasticity of Volume.

Suppose we have a body whose volume is V, and that it is under a hydrostatic pressure P; let the pressure be changed to $P+p$, and the volume in consequence to $V-v$. Then v/V is the change in unit volume due to the increment of the pressure p, and $v/(Vp)$ is the change per unit volume due to unit increment of pressure.

This is called the compressibility of the body, which may be defined as the ratio of the cubical compression per unit volume to the pressure producing it. The reciprocal of the compressibility—viz. the value of Vp/v—is the elasticity of volume. We shall denote it by k.

Rigidity.

Any alteration of external form or of volume in a body is accompanied by stresses and strains throughout the body.

A stress which produces change of form only, without alteration of volume, is called a shearing stress.

Imagine one plane in the body to be kept fixed while all parallel planes are moved in the same direction parallel to themselves through spaces which are proportional to their distances from the fixed plane; the body is said to undergo a simple shear.

Suppose further that this simple shear is produced by the action of a force on a plane parallel to the fixed plane, and uniformly distributed over it; then the ratio of the force per unit of area to the shear produced is defined to be the rigidity of the body.

Let T be the measure of the force acting on each unit of area of the plane, and suppose a plane at a distance a from the fixed plane is moved through a distance c; then c/a is defined as the measure of the shear, and the rigidity of the body is $\mathrm{T}a/c$.

Let us call this n. It may be shewn mathematically that, if a circular cylinder of radius r and length l be held with one end fixed, the couple required to turn the other end through an angle θ is $n\dfrac{\pi r^4}{2l}\theta$.

Modulus of Torsion.

The couple required to twist one end of unit length of a wire through unit angle, the other end of the wire being kept fixed, is called the modulus of torsion of the wire.

Hence if τ be the modulus of torsion, the couple required to twist one end of a length l through an angle θ, the other end being kept fixed, is $\tau\theta/l$.

Relation between Modulus of Torsion and Rigidity.

We have given above two expressions for the couple required to twist one end of a length l of a wire of circular section through an angle θ, the other end being kept fixed; equating these two expressions we get for a wire of radius r,

$$n = \frac{2\tau}{\pi r^4}.$$

Young's Modulus.

If an elastic string or wire of length l be stretched by a weight W until its length is l', it is found that $\dfrac{l'-l}{l\mathrm{W}}$ is constant for that wire, provided that the wire is not strained beyond the limits of perfect elasticity; that is, the weight W must be such that, when it is removed, the wire will recover its original length.

If the cross section of the wire be of unit area, the ratio

of the stretching force to the extension per unit length is called Young's Modulus, for the material of which the wire is composed, so that if the cross section of the wire be ω sq. cm. and we denote Young's Modulus by E, we have

$$E = \frac{l W}{\omega (l' - l)}.$$

Relation between Young's Modulus and the Coefficients of Rigidity and Volume Elasticity.

We can shew from the theory of elasticity (see Thomson, *Ency. Brit.* Art. 'Elasticity'), that if E be Young's Modulus,

$$E = \frac{9 n k}{3k + n};$$

and hence

$$k = \frac{n E}{3 (3n - E)}.$$

Thus, knowing E and n, we can find k.

22. Young's Modulus.

To determine Young's Modulus for copper, two pieces of copper wire seven or eight metres in length are hung from the same support. One wire carries a scale of millimetres fixed to it so that the length of the scale is parallel to the wire. A vernier is fixed to the other wire,[1] by means of which the scale can be read to tenths of a millimetre. The wire is prolonged below the vernier, and a scale pan attached to it; in this weights can be placed. The wire to which the millimetre scale is attached should also carry a weight to keep it straight. Let us suppose that there is a weight of one kilogramme hanging from each wire.

Measure by means of a measuring tape or a piece of string the distance between the points of suspension of the

[1] We believe that we are indebted indirectly to the Laboratory of King's College, London, for this elegant method of reading the extension of a wire.

wires and the zero of the scale. Let this be 716·2 centimetres.

Now put into the pan a weight of 4 kilogrammes, and read the vernier. Let the reading be 2·56 centimetres.

The length of the wire down to the zero of the vernier is therefore 718·76 centimetres.

Now remove the 4 kilogramme weight from the pan. The vernier will rise relatively to the scale, and we shall obtain another reading of the length of the wire down to the zero of the vernier.[1] Let us suppose that the reading is 2·33 centimetres. The length of the wire to which the millimetre scale is attached is unaltered, so that the new length of the wire from which the 4 kilogramme weight has been removed is 718·53 centimetres.

Thus, 4 kilogrammes stretches the wire from 718·53 centimetres to 718·76 centimetres. The elongation, therefore, is 0·23 centimetre, and the ratio of the stretching force to the extension per unit length is

$$\frac{4 \times 718{\cdot}53}{{\cdot}23}, \text{ or } 12500 \text{ kilogrammes approximately.}$$

We require the value of Young's Modulus for the material of which the wire is composed. To find this we must divide the last result by the sectional area of the wire.

If, as is usual, we take one centimetre as the unit of length, the area must be expressed in square centimetres.

Thus, if the sectional area of the wire experimented on above be found to be 0·01 square centimetre (see § 3), the value of the modulus for copper is

$$\frac{12500}{{\cdot}01}, \text{ or } 1250000 \text{ kilogrammes per square centimetre.}$$

The modulus is clearly the weight which would double the length of a wire of unit area of section, could that be done without breaking it.

Thus, it would require a weight of 1,250,000 kilo-

[1] It is assumed that the zero of the vernier corresponds with its point of attachment to the wire.

grammes to double the length of a copper wire of one square centimetre section.

The two wires in the experiment are suspended from the same support. Thus, any yielding in the support produced by putting on weights below or any change of temperature affects both wires equally.

It is best to take the observations in the order given above, first with the additional weight on, then without it, for by that means we get rid of the effect of any permanent stretching produced by the weight.

The wire should not be loaded with more than half the weight required to break it. A copper wire of 0·01 sq. cm. section will break with a load of 60 kgs. Thus, a wire of 0·01 sq. cm. section may be loaded up to 30 kgs. The load required to break the wire varies directly as the cross-section.

To make a series of determinations, we should load the wire with less than half its breaking strain, and observe the length; then take some weights off—say 4 or 5 kgs. if the wire be of about 0·01 sq. cm. section, and observe again; then take off 4 or 5 kgs. more, and observe the length; and so on, till all the weights are removed.

The distance between the point of support and the zero of the millimetre scale, of course, remains the same throughout the experiment. The differences between the readings of the vernier give the elongations produced by the corresponding weights.

The cross-section of the wire may be determined by weighing a measured length, if we know, or can easily find, the specific gravity of the material of which the wire is made. For, if we divide the weight in grammes by the specific gravity, we get the volume in cubic centimetres, and dividing this by the length in centimetres, we have the area in square centimetres.

It may more readily be found by the use of Elliott's wire-gauge (see § 3).

Experiment.—Determine the modulus of elasticity for the material of the given wire

Enter results thus :—

Length of unstretched wire . . .	718·53 cm.
Extension per kilogramme (mean of 4 observations)	·0575 „
Cross-section	·01 sq. cm

Value of E 1,250,000 kilogrammes per. sq. cm.

Modulus of Torsion of a Wire.

If the wire contain l units of length, and the end be twisted through a unit angle, each unit of length is twisted through an angle $1/l$, and the couple required to do this is τ/l where τ is the modulus of torsion of the wire.

The couple required to twist unit length through an angle θ is $\tau\theta$, that required to twist a length l through an angle θ is $\tau\theta/l$.

Suppose a mass, whose moment of inertia is K, is fixed rigidly to the wire, which is then twisted, the mass will oscillate, and if t_1 sec. be the time of a complete oscillation, it can be shewn, in a manner similar to that of § E, that

$$t_1 = 2\pi\sqrt{\left(\frac{Kl}{\tau}\right)}.$$

To find τ, then, we require to measure t_1 and K.

K can be calculated if the body be one of certain determinate shapes.

If not, we may proceed thus : We can alter the moment of inertia of the system *without altering the force tending to bring the body, when displaced, back to its position of equilibrium*, either (1) by suspending additional masses of known shape, whose moment of inertia about the axis of rotation can be calculated, or (2) by altering the configuration of the mass with reference to the axis of rotation. Suppose that in one of these two ways the moment of inertia is changed

from K to K + k, where the change k in the moment of inertia can be calculated, although K cannot.

Observe the time of swing again. Let it be t_2.

Then $$t_2 = 2\pi\sqrt{\frac{(K+k)l}{\tau}}.$$

Thus $$\frac{t_1^2}{4\pi^2} = \frac{Kl}{\tau}, \qquad \frac{t_2^2}{4\pi^2} = \frac{(K+k)l}{\tau}.$$

Whence $$\frac{t_2^2 - t_1^2}{4\pi^2} = \frac{kl}{\tau}.$$

$$\therefore \ \tau = \frac{4\pi^2 kl}{t_2^2 - t_1^2}.$$

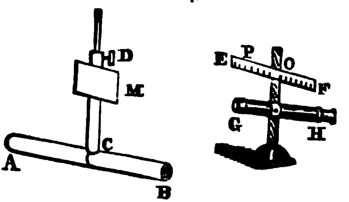
Thus τ can be expressed in terms of the observed quantities t_1, t_2 and l, and the quantity k which can be calculated.

We proceed to give the experimental details of the application of this method of finding the modulus of torsion of a wire by observing the times of vibration, t_1, t_2, when the moments of inertia of the suspended mass are K and K + k respectively. The change in the moment of inertia is produced on the plan numbered (2) above, by a very convenient piece of apparatus devised by Maxwell, and described in his paper on the Viscosity of Gases.

23. To find the Modulus of Torsion of a Wire by Maxwell's Vibration Needle.

The swinging body consists of a hollow cylindrical bar A B (fig. 14).

FIG. 14.



Sliding in this are four equal tubes which together just fill up the length of the bar; two of these are empty, the other two are filled with lead.

C D is a brass piece screwed into the bar, and M is a plane mirror fastened to it with cement. At D is a screw, by means of which

The scale will appear to cross the field of view of the telescope.

Note with a watch or chronometer the instant at which the middle point of the scale passes the cross-wire of the telescope, marking also the direction in which the scale appears to be moving. Let us suppose it is from left to right. It is of course impossible to see at the same time the cross-wire and scale and also the face of the chronometer; but the observation may be effected either as described in § 11 or as follows.

Let us suppose the chronometer ticks half-seconds.

Listen carefully for the sound of the tick next after the transit of the central division of the scale, and count six in time with the ticks, moving at the same time the eye from the telescope to the clock-face. Suppose that at the sixth tick the chronometer registers 10 h. 25 min. 31·5 sec., then the instant of transit was 3 sec. earlier, or 10 h. 25 min. 28·5 sec. Raise the eye quickly back to the telescope and watch for the next transit from left to right.

Again count six ticks, moving the eye to the chronometer, and let the time be 10 h. 26 min. 22 sec.

The time of the second transit is then 10 h. 26 min. 19 sec., and the time of a complete vibration is 50·5 sec.

But either observation may be wrong by ·5 sec., so that this result is only accurate to within 1 sec.

To obtain a more accurate result proceed exactly as in § 20.

It may happen that the time of vibration is so short that we have not time to perform all the necessary operations —namely, to move the eye from the telescope, look at the chronometer, note the result, and be ready for another transit before that transit occurs. In such a case we must observe every second or third transit instead of each one.

Again, we may find that 6 ticks do not give time to move the eye from the telescope to the chronometer-face. If this be so, we must take 8 or 10. Practice, however, soon renders the work more rapid.

Of course, if we always count the same number of ticks there is no need to subtract the 3 sec. from the chronometer reading; we are concerned only with the differences between the times of transit, and the 3 sec. affects all alike.

We may thus observe t_1, the time of vibration of the needle when the empty tubes are nearest the ends, the loaded tubes being in the middle; and in the same manner we may observe t_2, the time of vibration of the needle when the positions of the heavy and light tubes have been interchanged. Let the observed value of t_1 be 17·496 sec., and that of t_2, 25·263 sec.

To find the Value of k, the Increase in the Moment of Inertia.

We know that the moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis through its centre of gravity, together with the moment of inertia of the whole mass collected at its centre of gravity about the given axis (p. 44).

Thus, let m be the mass of a body whose moment of inertia about a certain axis is I; let a be the distance of the centre of gravity from that axis, and $\bar{I}$ the moment of inertia about a parallel axis through the centre of gravity.

Then $I = \bar{I} + ma^2$.

Moreover, the moment of inertia of a body is the sum of the moments of inertia of its parts (p. 44).

Now, let m_1 be the mass of each of the heavy tubes, and a the distance of the centre of each of them from the axis round which the whole is twisting when in the first position. Let I_1 be the moment of inertia of each of the heavy tubes about a parallel axis through its centre. Let m_2, I_2 have the same meaning for the empty tubes, and let b be the distance of the centre of each of these from the axis of rotation.

Let I be the moment of inertia of the empty case.

Then

$$K = I + 2I_1 + 2I_2 + 2m_1 a^2 + 2m_2 b^2. \quad . \quad . \quad (1).$$

In the second position, a is the distance from the axis of rotation of the centre of each of the masses m_2, b of that of the masses m_1.

To find the moment of inertia of the whole, therefore, we require simply to interchange a and b in equation (1), and this moment of inertia is $K+k$. Thus,

$$K+k=I+2I_1+2I_2+2m_1b^2+2m_2a^2. \quad . \quad . \quad . \quad (2).$$

from (1) and (2) $k=2(b^2-a^2)(m_1-m_2)$.

Thus, we do not need to know I, I_1 or I_2 to find k.

Now the length of each of the tubes is one-fourth of that of the whole bar A B. Calling this c, we have

$$a=\frac{c}{8} \qquad b=\frac{3c}{8},$$

and
$$k=\tfrac{1}{4}c^2(m_1-m_2).$$

To find m_1 and m_2, we require merely to determine by weighing the number of grammes which each contains. Our formula for τ (p. 191) becomes

$$\tau=\frac{\pi^2c^2(m_1-m_2)l}{t_2^2-t_1^2},$$

and it only remains to measure l. This can be done by means of the beam compass or a measuring tape.

We must, of course, measure from the point at which the upper end of the wire is attached, to the point at which it is clipped by the screw D.

The wire it will be found fits into a socket at the top of the apparatus C D. Be careful when fixing it initially to push it as far as possible into the socket; its position can then be recovered at any time.

Unloose the screw D and draw the wire from above, up through the tube which supports it, and measure its length in the ordinary manner.

The value of τ thus obtained gives the modulus of torsion for the particular specimen of wire. If the modulus of torsion for the material is required, we must make use of the addi-

tional law of torsional elasticity that the torsional couple in wires of the same material, differing only in area of section, is proportional to the fourth power of the radius of the wire. To find the value of the modulus of torsion of the material, the value of τ must be divided by $\frac{1}{2}\pi r^4$ where r is the radius in centimetres (p. 186).

Experiment.—Determine the modulus of torsion of the given wire.

Enter results thus:—

$t_1 = 5{\cdot}95$ sec.	$t_2 = 9{\cdot}75$ sec.
$m_1 = 351{\cdot}25$ gms	$m_2 = 60{\cdot}22$ gms.
$l = 57{\cdot}15$ cm.	$c = 45{\cdot}55$ cm.

$$\tau = 5{\cdot}67 \times 10^6.$$

CHAPTER VII.

MECHANICS OF LIQUIDS AND GASES.

Measurement of Fluid Pressure.

THE pressure at any point of a fluid is theoretically measured by the force exerted by the fluid upon a unit area including the point. The unit area must be so small that the pressure may be regarded as the same at every point of it, or, in other words, we must find the limiting value of the fraction obtained by dividing the force on an area enclosing the point by the numerical measure of the area, when the latter is made indefinitely small.

This theoretical method of measuring a pressure is not as a rule carried out in practice. On this system of measurement, however, it can be shewn that the pressure at any point of a fluid at rest under the action of gravity is uniform over any horizontal plane, and equal to the weight of a column of the fluid whose section is of unit area, and whose length is equal to the vertical height of the free surface of the heavy fluid above the point at which the pressure is required. The pressure is therefore numerically equal to the

weight of ρh units of mass of the fluid, where ρ is the mean density of the fluid, h the height of its free surface above the point at which the pressure is required.

This pressure expressed in absolute units will be $g\rho h$, where g is the numerical value of the acceleration of gravity.

If the fluid be a liquid, ρ will be practically constant for all heights; g is known for different places on the earth's surface.

The pressure will therefore be known if the height h be known and the kind of liquid used be specified.

This suggests the method generally employed in practice for measuring fluid pressures. The pressure is balanced by a pressure due to a column of heavy liquid—e.g. mercury, water, or sulphuric acid—and the height of the column necessary is quoted as the pressure, the liquid used being specified. Its density is known from tables when the temperature is given, and the theoretical value of the pressure in absolute units can be deduced at once by multiplying the height by g and by ρ, the density of the liquid at the temperature.

If there be a pressure Π on the free surface of the liquid used, this must be added to the result, and the pressure required is equal to $\Pi + g\rho h$.

Example.—The height of the barometer is 755 mm., the temperature being 15° C.: find the pressure of the atmosphere.

The pressure of the atmosphere is equivalent to the weight of a column of mercury 75·5 cm. high and 1 sq. cm. area, and $g = 981$ in C.G.S. units.

The density of mercury is equal to $13{\cdot}596\,(1 - {\cdot}00018 \times 15)$ gm. per c.c.

In the barometer there is practically no pressure on the free surface of the mercury, hence the pressure of the atmosphere

$$= 981 \times 13{\cdot}596\,(1 - .00018 \times 15) \times 75{\cdot}5 \text{ dynes per sq. cm.}$$

24. The Mercury Barometer.

Barometers are of various forms; the practical details ven here are intended to refer to the Fortin Standard

Barometer, in which the actual height of the column of mercury, from the surface of the mercury in the cistern, is measured directly by means of a scale and vernier placed alongside the tube. The scale is only graduated between twenty-seven and thirty-two inches, as the barometric height at any ordinary observatory or laboratory is never outside these limits.

To set and read the Barometer.

The barometer must first be made to hang freely, by loosening the three screws at the bottom of the frame, in order that the scale may be vertical.

The mercury in the cistern must be brought to the same level as the zero point of the scale. This zero point is indicated by a small ivory point; and the extremity of this point must first be made to coincide with the surface of the mercury.

This is attained by adjusting the bottom of the cistern by means of a screw which projects from the bottom of the barometer; raising this screw raises the mercury surface. On looking at the surface a reflexion of the pointer is seen. Raise the surface until the end of the pointer and its reflected image appear just to touch. Then the mercury surface and the zero of the scale are at the same level.

The upper surface of the mercury is somewhat convex. In taking a reading, the zero of the vernier must be brought to the same level as the top point of this upper surface.

Behind the barometer tube is placed a sheet of white paper, and by raising the vernier this can be seen, through the tube, between it and the upper surface of the mercury.

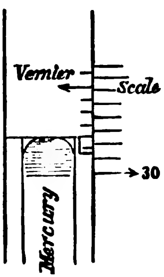
Lower the vernier until,[1] *looking horizontally*, it is just impossible to see the white paper between it and the top of the meniscus; then the zero of the vernier coincides with the top of the mercury column. To be able to make sure that the eye looks horizontally the vernier is provided with a brass piece on the opposite side of the tube, the lower edge

[1] See Frontispiece, fig. 2.

of which is on the same level as the lower edge of the vernier when the scale is vertical. By keeping the eye always in a line with these two edges we know that the line of sight is horizontal, and thus avoid error of parallax. Of course a glimpse of white may be obtained at the sides, owing to the curvature of the meniscus, as in the figure.

The scale is in inches, and is divided to twentieths. Twenty-five divisions of the vernier are equal to twenty-four of the scale; the instrument therefore reads to 500ths of an inch.

Fig. 15.



To read it rapidly; divide the reading of fractions of the inch on the scale by 2; the result is in tenths of an inch; multiply the vernier reading by 2; the result is in thousandths of an inch.

Thus suppose that the scale reading is 30 inches and three divisions. This is 30·15. The vernier reading is 13, and this is ·026 inch; the reading then is 30·176 inches.

If the scale is of brass and is graduated into inches which are 'correct' at 62° F., the corresponding length in millimetres on the same brass, 'correct' at 0° C., would be given by the annexed table. Thus 30·176 inches = 766·45 mm.

1 in. . .	= 25·392 mm.	6 in. . .	= 152·344 mm.
2 " . .	= 50·785 "	7 " . .	= 177·736 "
3 " . .	= 76·177 "	8 " . .	= 203·128 "
4 " . .	= 101·569 "	9 " . .	= 228·521 "
5 " . .	= 126·952 "	30 " . .	= 761·769 "

Correction of the Observed Height for Temperature, &c.

The height thus obtained requires several corrections.

(1) Mercury expands with a rise of temperature, and we must therefore reduce our observation to some standard temperature, in order to express the pressure in comparable measure. The temperature chosen is 0° C., and the co-

efficient of expansion of mercury is $\cdot000181$ per $1°$ C. Thus, if l be the observed height and t the temperature, the height of the equivalent column at $0°$ C. is $l(1-\cdot000181t)$. In applying this correction, it is very often sufficient to use the mean value, 760 mm. for l, in the small term $\cdot000181\,lt$.

Now $760 \times \cdot000181 = \cdot138$. Then we can get the corrected height with sufficient approximation by subtracting from the observed height $\cdot138 \times t$. Thus if the observed height be 766·45 mm. and the temperature $15°$, the true height, *so far as this correction only is concerned*, is

$$766\cdot45 - 15 \times \cdot138 = 766\cdot45 - 2\cdot07 = 764\cdot38 \text{ mm.}$$

(2) The same rise of temperature has caused the brass scale to expand, so that the apparent height of the column is on that account too short. To obtain the true height we must add to the observed height l, the quantity $l\beta t$, β being the coefficient of linear expansion of brass.[1]

Now $\beta = \cdot000019$. The complete correction then due to both causes will be $-(\cdot000181 - \cdot000019)\,lt$, and the true height is $l-(\cdot000181-\cdot000019)\,lt$ or $l-(\cdot000162)lt$.

If in the small term, $(\cdot000162)lt$, we take the mean value, 760 mm., for l, the true height is h, where $h = l - \cdot123t$. Thus in our case ($t = 15°$), $h = 766\cdot45 - 1\cdot85 = 764\cdot60$ mm.

(3) Owing to the capillary action between the glass of the tube and the mercury, the level of the mercury is depressed by a quantity which is roughly inversely proportional to the diameter of the tube. The depression is not practically of an appreciable amount unless the tube has a diameter less than a centimetre. In the instrument in the Cavendish Laboratory the tube is 5·58 mm. in radius, and in consequence the top of the meniscus is depressed by about ·02 mm.; we must therefore add this to the observed height, and we find that the corrected value of the height is 764·62 mm.

(4) Again, there is vapour of mercury in the tube, which

[1] The correction is made to $0°$ C. because millimetre graduation is generally made to be 'correct' at that temperature. If the scale correction is applied to the inches it must be computed from $62°$ F.

produces a pressure on the upper surface of the column. It is found that at temperature t this may be practically taken to be equivalent to $\cdot 002 \times t$ mm. of mercury. Thus, if the temperature be 15°, we must on this account add to the observed height ·03 mm., and we obtain as our corrected height 764·65 mm. This is the true height of the column of mercury at standard temperature, which gives a pressure equivalent to the pressure of the atmosphere at the place and time in question.

(5) Now the weight of this column is balanced against the pressure of the air. The weight of the column will depend on its position relatively to the earth. We must therefore determine the height of the column which at some standard position will weigh as much as our column. We take for that standard position sea-level in latitude 45°.

Let g_0 be the value of the acceleration due to gravity at this position, b_0 the height of a column weighing the same as our column b; g the acceleration due to gravity at the point of observation.

Then, since the weights of these two columns are the same, we have $b_0 g_0 = bg$, and therefore $b_0 = b\, g/g_0$.

Now it is known from the theory of the figure of the earth that if h is the height above the sea-level in metres and ϕ the latitude of the place of observation,

$$\frac{g}{g_0} = 1 - \cdot 0026 \cos 2\phi - \cdot 0000002h.$$

Hence

$$b_0 = b\,(1 - \cdot 0026 \cos 2\phi - \cdot 0000002h).$$

Experiment.—Read the height of the standard barometer, and correct to sea-level at 45° lat.

25. The Aneroid Barometer.

In the aneroid barometer at the Cavendish Laboratory each inch of the scale is divided into fiftieths, and there is a vernier,[1] twenty half-divisions of which equate with

[1] See Frontispiece, fig. 4.

twenty-one of the scale; the vernier reads, therefore, by estimation to thousandths of an inch. On the vernier each division must be counted as two, only the even divisions being marked.

The aneroid is set by comparison with a corrected mercury barometer, to give the true pressure at the time of the observation. If properly compensated for temperature, it would continue to give the true barometric height at any other station, even if the temperature changes.

To read the aneroid, set the zero of the vernier exactly opposite the end of the pointer, and read the inches and fiftieths on the scale up to the vernier zero.

Multiply the fractional divisions by 2; the result is in hundredths of an inch. Read the vernier, and again multiply by 2; the result is in thousandths of an inch.

(The numbers marked on the scale give tenths of an inch; those on the vernier thousandths.)

Thus the scale reading is between 30 and 31, the pointer standing between divisions 12 and 13. The scale reading, therefore, is 30·24. When the zero of the vernier is opposite the pointer there is a coincidence at division 8 of the vernier; the vernier reading is, therefore, ·016, and the exact height is 30·256.

To measure the height between two stations with the aneroid, take the reading at the two stations and subtract. The difference gives the pressure in inches of mercury of the column of air between the two.

Thus suppose that at a lower station the reading of the aneroid is 30·276, and the difference in pressure is that due to 0·020 inch of mercury; this is equivalent to 0·51 mm.

The specific gravity of mercury is 13·60; thus ·51 mm. mercury is equivalent to $·51 \times 13·60$ mm. of water at 4° C.

To find the true height of the column of air which is equivalent in pressure to this, we must divide by the specific gravity of air at the temperature and pressure of observation. This may be determined when the pressure and

temperature have been observed, by calculation from the data given in No. 36 of Lupton's 'Tables.'

If the difference of height is not great the pressure of the air between the two stations may, for this purpose, be taken to be the mean of the aneroid readings at the two stations, properly corrected by reference to the mercury standard. For the temperature, if there is any considerable difference between the thermometer readings at the two stations, some judgment must be used in order to get a mean result which shall fairly represent the average temperature of the air between the two. When these observations have been made, we are in a position to calculate the specific gravity of dry air under the given conditions. Since the atmosphere always contains more or less moisture, a correction must be applied. Since the specific gravity of aqueous vapour referred to air at the same temperature and pressure is $\frac{5}{8}$, the correction may be made by calculating what would be the specific gravity of the dry air if its pressure were diminished by an amount equivalent to three-eighths of the pressure of the water vapour it contains, as determined by observation of the dew-point or other hygrometric method. This correction is often so small as to produce no appreciable effect within the limits of accuracy of the pressure readings.

Thus if the mean of the pressure observations be 768 mm., and the estimated mean temperature 15° C., the specific gravity of dry air would be 0·001239, and if the observed pressure of aqueous vapour be 10 mm., the corrected specific gravity would be

$$\frac{768 - \frac{3}{8} \times 10}{768} \times \cdot 001239, \text{ or } \cdot 001233.$$

Hence the height of the column of air between the two stations is

$$\frac{\cdot 51 \times 13 \cdot 60}{\cdot 001233} \text{ mm., or } 563 \text{ cm.}$$

For a method of extending the application of barometric observations to the measurement of comparatively greater heights we may refer the reader to Maxwell's 'Heat,' chap. xiv.

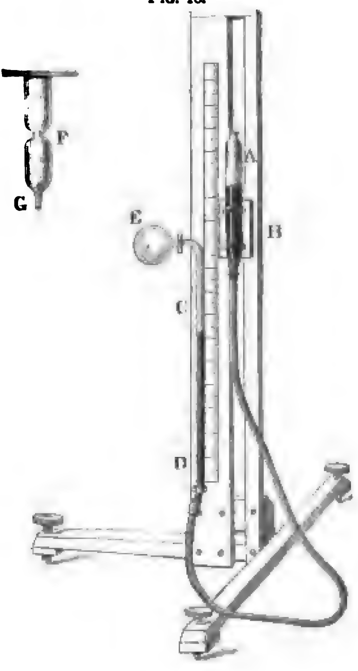
Experiment.—Read the aneroid and determine from your observation of the standard the correction to be applied to the aneroid to give the true reading.

Measure the height of the laboratory from the basement to the tower.

26. The Volumenometer.

The apparatus (fig. 16) consists of two glass tubes placed in vertical positions against a scale. The one tube (C D) is fixed, and has at the top an elbow with a screw by means of which a small flask E can be fastened on. [In another form of the apparatus the tube D C ends in a bulb F, which opens into a funnel-shaped space. The upper edges of the funnel are ground flat, and the whole can be closed so as to be air-tight by means of a ground glass plate and grease.]

FIG. 16.



The other glass tube is attached to a sliding piece movable along the vertical scale; the lower ends of the two tubes are connected by means of a piece of flexible india-rubber tubing; this and portions of the glass tubes contain

mercury, which so long as the end E is open stands at the same level in the two tubes. The instrument is supported on three levelling-screws, by means of which the scale can be set vertical. The whole apparatus should stand in a wooden tray, which serves to catch any mercury which may unavoidably be spilt. The following experiments may be made with it:—

(1) *To test Boyle's Law, viz., if* V *be the Volume and* p *the Pressure of a Mass of Gas at constant Temperature, then* Vp *is a constant.*

We shall require to know the area of the cross section of the tube C D. For this purpose suppose the flexible connection between the bottoms of the tubes is removed, and replaced by a short piece of tubing closed with a pinch-cock. Fill this tubing and the glass tube above it with mercury up to some convenient division of the scale, taking care that all the air-bubbles are removed; this can generally be done by tilting the apparatus or by means of an iron wire. The mercury should be clean and dry, otherwise it will stick to the glass. Now open the pinch-cock and allow some of the mercury to escape into a weighed beaker. When a convenient quantity has run out close the pinch-cock, and again read the level of the mercury on the scale; let the difference of the two levels be l centimetres, and let the area of the tube be a square centimetres. The volume of mercury which has run out, is la cubic centimetres, and if ρ is the density of mercury in grammes per c.c., its mass is $\rho\, l\, a$ grammes.

Weigh the mercury in the beaker; let its mass be m grammes; then

$$\rho\, l\, a = m,$$
$$\therefore\ a = m/\rho\, l.$$

The density of mercury is very approximately 13·59 grammes per c.c., and hence if we measure m and l we find

α, the area of the cross section. The above assumes the area to be constant throughout the length of the tube; if this condition is not sufficiently nearly satisfied the tube must be calibrated (see § 8).

When the value of α is known the connexion between the tubes at B and D may again be made and the apparatus filled with mercury, which can be poured into the open end of one of the tubes though a funnel; while this is being done the flask E should be removed, the end of the tube being left open, and the mercury should be poured in until it reaches nearly to the top of the tube D C.

Now screw on the flask or close the end of the tube with the glass plate. If this is done carefully the mercury will stand at the same level in the two tubes, and the air in the bulb will be at the same pressure as the air outside. Let the volume of the air be V cubic centimetres—we shall shew how to find V shortly—and let the height of the mercury barometer which measures the pressure be h centimetres. Read on the scale the level of the mercury in the tube D C. Let it be a centimetres. This is facilitated by having a vertical piece of looking-glass at the back of the tube; by placing your eye so that the mercury and its image appear at the same level errors of parallax are avoided. It is convenient to have the tube mounted so that a piece of looking-glass can be inserted between it and the frame and held pressed against the vertical stand in the proper position while making an observation; in some instruments the scales are engraved on looking-glass.

Now lower the sliding tube A B. The mercury falls in both tubes, but to a less amount in the tube D C than in the other. Read on the scale the level in each tube.

Let that in D C be b cm., and in A B, b' cm.

Then the volume of the enclosed air has increased from V to V $+ \alpha\ (a - b)$, while, since the difference in levels in

increases by v, which takes the place of $a(a - b)$ in the formula, and we have

$$\mathrm{V} = \frac{v\{h - (b - b')\}}{(b - b')}$$
$$= v\left\{\frac{h}{b - b'} - 1\right\}.$$

The method will give accurate results only in the case in which the volume of the solid is considerable; it should nearly fill the flask.

Experiments.

(1) Test Boyle's law, and measure the volume of the small flask attached to the volumenometer.

(2) Determine the density of the given glass beads.

Enter results thus:—

Area of cross section of tube 1·01 sq. cm.

Four observations of increase of volume and corresponding pressures, made and plotted on curve shewn.

Volume deduced from diagram	155 c.c.
Volume by calculation from one observation:—	
Division to which tube is filled, a	90 cm.
Division to which mercury falls, b	72 "
Level of mercury in sliding tube, b'	64 "
Height of barometer, h	76 "
$a - b$	18 cm.
$b - b'$	8 "
Volume	154·5 c.c.

H. Capillarity.

To Measure the Surface Tension of a Liquid by the height it rises in a Capillary Tube.

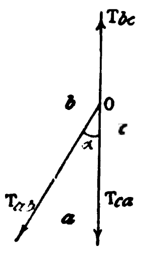
If a narrow tube is dipped into a liquid which wets it, the liquid rises in the tube and stands at a higher level than in the containing vessel. From this we infer that the particles of the liquid in the neighbourhood of the surface are in a different condition from those in the interior of its

mass, and, in consequence, possess a greater amount of potential energy (see Maxwell's 'Theory of Heat,' chap. xx).

The effect may be represented by supposing that the surface film of any liquid is under tension, so that if we draw any line across it we may conceive the portion of the film on one side of the line to act on the portion on the other side with a definite force. The amount of this force per unit of length is found to be a constant for the surface of separation of any two given fluids, and it may be shewn to be equal to the amount of surface energy per unit of area which the fluids possess.

If now we have three fluids meeting at a point, there will at that point be three definite forces—the tensions of the three surfaces of separation, and in order that there may be equilibrium the surfaces must meet at definite angles. Now let one of the substances c be a solid, and let a and b be the other two. Let T_{ab} (fig. xvii) represent the tension between the surfaces of a and b, and let this surface at O make an angle α with the surface of c. Then, resolving the forces at O parallel to the surface, we have for equilibrium

FIG. xvii.



$$T_{ab} \cos \alpha = T_{bc} - T_{ca}.$$

This equation determines α, the angle of capillarity.

If $T_{bc} - T_{ca}$ is greater than T_{ab}, no such angle as α can be found; the liquid is said to wet the surface of the solid, and will run all over it unless prevented by other forces, such as gravity. The system of two fluids and the solid tends to set itself, so that its whole energy is as small as possible.

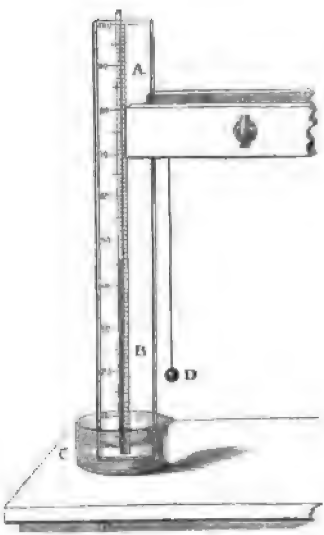
And since the surface energy of the water-air surface is less than that of the air-glass surface in the case of water in contact with glass, the water tends to cover the glass. If the glass surface be vertical the water as it creeps

up the surface gains potential energy, and equilibrium is reached when the gain of potential energy due to the rise of water is equal to the loss due to the diminution of air-glass surface.

To determine the surface tension of a liquid we require to know the density of the liquid, the diameter of the tube, the angle of contact, and the height the liquid rises.

Let the section of the tube be a circle of radius r. The circumference of this is $2\pi r$, and at each point of this circumference there is a force T per unit of length acting at an angle α with the vertical. The total vertical force is $2\pi r \,.\, T\cos\alpha$. If h be the height of the volume of liquid raised, measured from the flat surface of the liquid in the vessel to the bottom of the meniscus in the tube, and the weight of the very small portion forming the meniscus be neglected, then the weight of liquid raised is $\pi r^2 h \rho g$.

FIG. xviii.



$$\therefore\ 2\pi T r\cos\alpha = \pi\rho g r^2 h,$$

$$\therefore\ T = \tfrac{1}{2}\rho g r h \sec\alpha \text{ dynes per cm.}$$

In practice the method is only used with a liquid, such as water, which wets the glass, and then $\alpha = 0$, $\sec\alpha = 1$,

$$\therefore T = \tfrac{1}{2}\rho g r h \text{ dynes per cm.}$$

To perform the experiment a finely divided scale (A B, fig. xviii) must be placed in a vertical position, with one end dipping into the beaker C, which is to contain the liquid; the scale may most conveniently be of glass divided into millimetres and

some 20 cm. long. It may be adjusted to a vertical position by means of a plumb-line D.

The capillary tube is attached to the scale by two elastic bands; the scale should be illuminated from behind with a good light, which may be thrown on to it by a mirror if requisite.

The capillary tube is prepared by softening a piece of clean glass tubing in a blow-pipe flame, and drawing it out until the diameter is comparable with about half a millimetre. The ends of the tube should be sealed until it is wanted for use. When the scale and light have been arranged, fasten the tube to the scale in a vertical position so that it may dip into the water, and open the two ends; the water will rise in the tube. When the rise has ceased, dip the tube slightly further into the water and then raise it a little. This will ensure that the tube is wetted above the level of the water it contains.

Now read on the scale the height to which the water has risen; read also the position of the horizontal water-surface in the beaker. If there is any difficulty in doing this directly, it may be overcome by fastening a fine needle in a suitable clip and lowering it gently near the scale until it just touches the water; the level of the needle-point can then be found. The difference in these two readings gives the height h.

The height so found can, if required, be afterwards corrected for the meniscus by adding one-third of the radius of the tube.

We have next to measure the diameter of the tube; for this purpose it must be carefully cut in two close to the point to which the water rose. This may be done by holding the tube against the finger and gently drawing a fine file with a sharp edge across it. The tube is then mounted with a little wax on a suitable stand or clip so that the section is in the field of a good microscope with a micrometer scale in the eye piece. The value of the

divisions of the eye-piece micrometer must have previously been determined by viewing through the microscope a finely divided scale, and counting the number of divisions of the eye-piece micrometer which coincide with one division of the scale.

For this purpose a scale on glass, divided to half or quarter millimetres, is useful, or an ordinary stage micrometer having 100 divisions to the inch may be used. If, then, we find that a certain number of divisions—say 52—of the eye-piece micrometer coincide with 2 divisions of the scale, then 1 division of the micrometer is equivalent to $1/26 \times 100$ of 1 inch.

When the section of the tube is viewed through the microscope it will probably be seen that it is not circular; if it appears distinctly oval the results of the experiment will not be very satisfactory; otherwise by observing the diameter in several directions—say four—inclined at angles of about 45° to each other, and taking the mean, we shall obtain a result not far from the truth. It will usually happen that the diameter of the tube is not an exact whole number of divisions of the scale, but the divisions can be subdivided by eye to quarters, or even to tenths, and in this way a fairly accurate value for the diameter of a small tube may be found.

Experiment.—Determine by means of a capillary tube the surface tension of water.

Enter results thus:—

Values of h, $7{\cdot}39 - 1{\cdot}11 = 6{\cdot}28$ cm.
$7{\cdot}40 - 1{\cdot}11 = 6{\cdot}29$ „

$2r_1 = 5{\cdot}125 - {\cdot}71 = 4{\cdot}415$ divs. of micrometer.
$2r_2 = 3{\cdot}80 + {\cdot}07 = 3{\cdot}87$ „ „

Value of a division of micrometer scale = ·0123 cm.

$T = 78{\cdot}3$ dynes per cm.

I. Worthington's Capillary Multiplier.

If a substance which is wetted by a liquid is dipped into the liquid, the liquid is raised by the action of the surface tension, and there is a downward force on the substance equal to the weight of liquid raised. Thus, in the last experiment the force raising the column of liquid is $2\pi r \mathrm{T}$, and in opposition to this there must be a force acting in the downward direction on the tube. The apparent weight of the tube is increased by this amount.

If this increment of weight be determined, and the radius of the tube be known, we have another method of finding T. In the case of a narrow tube the apparent increase of weight would be very small; but suppose a flat strip of some substance—say a thin sheet of glass or of platinum foil—be immersed vertically in the fluid so that the lower edge of the strip is horizontal and level with the undisturbed surface, and let w be the weight of liquid raised, p the length of the strip with which it is in contact, then the surface tension is w/p. Now by making the strip sufficiently long p may be made considerable, and the apparent increase in weight large enough to be found with accuracy. It is necessary that the lower edge of the strip should be level with the undisturbed surface of the liquid, for if the strip dips beneath this there is a correction necessary on account of the buoyancy of the liquid, while if the base of the strip is raised above the free surface there is a traction to correct for, due to the adhesion of the liquid to the horizontal edge of the strip.

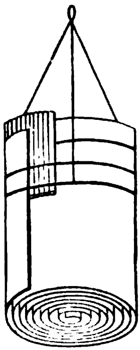
The method in some form or other is an old one. Recently Prof. A. M. Worthington has given an account of various improvements in carrying it out, which we proceed to describe in detail.

A strip of thin platinum foil, about 50 cm. long and some 6 or 8 cm. wide, is rolled into a spiral coil. The

successive convolutions of the coil are separated from each other by a number of glass beads, about 2 mm. in diameter, strung on platinum wire (see fig. xix).

The beads are made of combustion tubing, which is first cut into lengths of about 20 cm. A number of such lengths of the most uniform thickness are selected, and these are cut into pieces 2 cm. long. Of the beads thus formed a strip 50 cm. in length is put together by passing through each in turn in opposite directions the two ends of a fine platinum wire. The strip of beads is then laid on the foil, the length of the strip being parallel to that of the foil, and the whole is rolled into a spiral and secured with platinum wire; the convolutions of this spiral are thus 2 mm.—the diameter of the beads—apart, while the length of the strip which is in contact with the liquid is, taking both surfaces of the foil into account, 100 cm. The beads should be at least 3 cm. above the lower edge of the foil in order that the liquid which rises between the convolutions may not reach so high as to wet the glass. The whole is made of platinum and hard glass, in order that it may readily be cleaned by heating to a bright red in a Bunsen flame. The foil should not be more than ·0025 cm. thick.

FIG. xix.



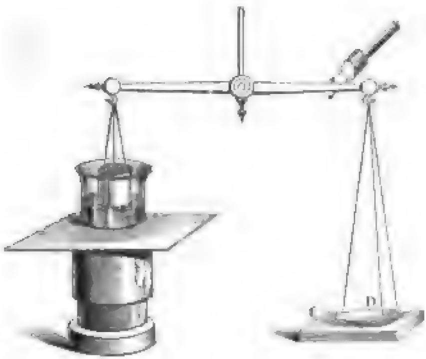
The coil is then suspended from one arm of a balance, and the suspending strings are adjusted so that its under surface may be horizontal (see fig. xx).

In order to prevent its being drawn too far below the surface of the liquid when in use, stops, as shewn in the figure, are fitted to the balance. These consist merely of two stout pins stuck in a cork and held in place by a clip. The coil is then carefully counterpoised, and a wooden block C is placed under the pan D in such a position that when the pan is held down on the block the beam may be

horizontal. Weights are then placed in D to hold it in this position.

The liquid is then placed in a beaker on a small table fitted with a vertical screw motion under the coil (Mr. Worthington made use of the end of an optical lantern with a card on the top), and the screw is turned, thus raising

FIG. xx.



the liquid until it wets the foil. Some of the liquid is drawn up between the convolutions, and the screw is adjusted until the under-surface of the coil when the beam is horizontal is level with the undisturbed surface of the water. This adjustment can be made with considerable accuracy, and Mr. Worthington has shewn that an error of 1 mm. will not cause an error of more than $\frac{1}{360}$ in the value of the surface tension of alcohol.

On removing the weights in D the coil is drawn as far below the liquid (some 4 or 5 mm.) as the stops will allow. C is then removed, and weights put into D to restore equilibrium. The difference between these weights and the weights originally used to counterbalance the coil gives the total downward pull due to the surface tension. On dividing this by the total length of the line of separation between the liquid and the foil—100 cm. if the strip of platinum be

50 cm. long—we get the value of the surface tension. The adjustment for level should be made more than once, and the observations of weight repeated.

Experiment.—Determine the surface tension of water by the capillary multiplier.

Enter the results thus:—

Length of strip in contact, given with instrument, 100 cm.

Weight to counterpoise strip	. . .	37·258 gm.
Weights with coil in water	. . .	44·918 „ 44·914 „ 44·901 „
Mean . .	44·911.	
Total downward force	. .	7·653 „

Surface tension $\frac{7\cdot653 \times 980}{100}$ dynes per cm. = 75 dynes per cm.

CHAPTER VIII.

ACOUSTICS.

Definitions, &c.

A MUSICAL note is the result of successive similar disturbances in the air, provided that they follow each other at regular intervals with sufficient rapidity. Similar disturbances following each other at regular equal intervals are said to be periodic. The interval of time between successive impulses of a periodic disturbance determines the pitch of the note produced—that is, its position in the musical scale. The pitch of a note is therefore generally expressed by the number of periodic disturbances per second required to produce it. This number is called the 'vibration number,' or 'frequency' of the note.

It generally happens that any apparatus for producing a note of given frequency produces at the same time notes of other frequencies. The result is a complex sound, equivalent to the combination of a series of simple sounds or tones.

The simple tones of which the complex sound may be regarded as consisting are called 'partial tones;' the gravest of these—that is, the one of lowest pitch—is called the 'fundamental tone' of the sounding body, and the others are called 'upper partials.' A note which has no upper partials is called a pure tone. By means of suitable resonators the different partial tones of a complex note may be made very clearly audible. For many musical instruments, as organ-pipes, string instruments, &c., the ratio of the vibration frequency of any upper partial tone to that of the fundamental tone is a simple integer, and the upper partials are then called 'harmonics;' for others, again, as for bells, tuning-forks, &c., the ratios are not integral, and the upper partials are said to be inharmonic.

27. To compare the Frequencies of two Tuning-forks of nearly Identical Pitch, and to tune two Forks to unison.

A tuning-fork mounted upon a resonator—a wooden box of suitable size—furnishes a very convenient means of obtaining a pure tone; the upper partials, which are generally heard when the fork is first sounded, are not reinforced by the sounding box, and rapidly become inaudible, while the fundamental tone is, comparatively speaking, permanent. When two forks which differ only slightly in pitch are set in vibration together, the effect upon the ear is an alternation of loud sound with comparative silence. These alternations are known as beats, and they frequently are sufficiently well marked and sufficiently slow for the interval of time between successive beats to be determined with considerable accuracy by counting the number occurring in a measured interval of time.

It is shewn in text-books on sound[1] that the number of beats in any interval can be inferred from the vibration num-

[1] Deschanel, *Natural Philosophy*, p. 813: Stone, *Elementary Lessons*, p. 72; Tyndall, *On Sound*, p. 261.

bers of the two notes sounded together, and that, if N be the number of beats per second, n, n' the frequencies of the two notes, n being the greater, then

$$N = n - n'.$$

We have, therefore, only to determine the number of beats per second in order to find the difference between the frequencies of the two notes. This may be an easy or a difficult matter according to the rapidity of the beats. If they are very slow, probably only few will occur during the time the forks are sounding, and the observer is liable to confuse the gradual subsidence of the sound with the diminution of intensity due to the beats. If, on the other hand, there are more than four beats per second, it becomes difficult to count them without considerable practice. The difficulty is of a kind similar to that discussed in § 11, and we may refer to that section for further details of the method of counting.

In order to determine which of the two forks is the higher in pitch, count the beats between them, and then lower the pitch of one of them by loading its prongs with small masses of sheet lead, or of wax (softened by turpentine), and observe the number of beats again. If the number of beats per second is now less than before, the loaded fork was originally the higher of the two; if the number of beats has been increased by the loading, it is probable that the loaded fork was originally the lower; but it is possible that the load has reduced the frequency of the higher fork to such an extent that it is now less than that of the unloaded second fork by a greater number than that of the second was originally less than that of the first It is safer, therefore, always to adjust the load so that its effect is to diminish the number of beats per second, that is, to bring the two forks nearer to unison; to do so it must have been placed on the fork which was originally of the higher pitch.

In order to adjust two forks to unison, we may lower the

pitch of the higher fork by weighting its prongs until the beats disappear; the difficulty, already mentioned, when very slow beats are observed occurs, however, in this case, and it is preferable to use a third auxiliary fork, and adjust its pitch until it makes, say, four beats a second with that one of the two forks which is to be regarded as the standard, noting whether it is above or below the standard. The second fork may then be loaded so that it also makes four beats a second with the auxiliary fork, taking care that it is made higher than the auxiliary fork if the standard fork is so. The second fork will then be accurately in unison with the standard—a state of things which will probably be shewn by the one, when sounded, setting the other in strong vibration, in consequence of the sympathetic resonance.

A tuning-fork may be permanently lowered in pitch by filing away the prongs near their bases; on the other hand, diminishing their weight by filing them away at their points raises the pitch. Such operations should, however, not be undertaken without consulting those who are responsible for the safe custody of the forks.

Experiment.—Compare the frequencies of the two given forks A and B by counting the beats between them. Determine which is the higher and load it until the two are in unison.

Enter results thus:—

Number of beats in 25 secs.	67
Number per sec.	2·7
„ „ (A loaded)	3·3
„ „ (B loaded)	2·1

B is the higher fork.

Number of beats per sec. between A and the auxiliary fork C	3·6
Number of beats per sec. between B (when loaded) and the auxiliary fork C	3·6

28. Determination of the Vibration Frequency of a Note by the Siren.

A siren is essentially an instrument for producing a musical note by a rapid succession of puffs of air. The simplest form of siren is a large circular cardboard disc, provided with perforations arranged in circles concentric with the disc. The puffs of air may be produced by blowing through a fine nozzle on to the circle of holes while the disc is maintained in rapid rotation. In order that the disturbances produced by the puffs of air passing through the holes may be periodic (see p. 218), the holes must be punched at equal distances from each other, and the disc must be driven at a uniform rate. If the pressure of the water-supply of the laboratory is sufficiently high, a small water-motor is a convenient engine for driving the disc, which must be mounted on an axle with a driving pulley. If the diameter of the disc is considerable, so that a large number of holes can be arranged in the circle, a rotation of the disc giving four revolutions per second is quite sufficient to produce a note of easily recognisable pitch. The revolutions in a given interval, say, one minute, can be counted, if a pointer be attached to the rim of the disc, and arranged so that it touches a tongue of paper fixed to the table once in every revolution. The number of taps on this paper in a given time is the number of revolutions of the disc. Suppose the number of taps in one minute is N, and the number of holes in the circle which is being blown is n, then the number of puffs of air produced per minute is $\text{N}\,n$, and hence the number per second is $\text{N}\,n/60$.

The disc is generally provided with a series of concentric rings of holes differing in the number of perforations in the circle, so that a variety of notes can be blown for the same rate of rotation of the disc.

In the more elaborate forms of the instrument a metal

disc, which is perforated with holes arranged in concentric circles, is mounted on a spindle so that it can revolve parallel and very near to the lid of a metallic box, which can be supplied by air from foot-bellows. The lid of this box is perforated in a manner corresponding to the revolving disc, but the holes in either opposing plate, instead of being bored perpendicularly through the metal, are made to run obliquely, so that those in the upper disc are inclined to those in the lower. When air is driven through the box it escapes through the holes, and in so doing drives the disc round. The disc may thus be maintained in a state of rotation, and if the pressure of the air be maintained constant the rotation will be uniform. In driving the siren a pressure-gauge, consisting of a U-tube containing water should be in connection with the tube conveying the air from the bellows to the instrument; the blowing should be so managed as to keep the pressure of wind as indicated by this gauge constant.

The number of revolutions of the spindle carrying the revolving disc is generally indicated on two dials—one showing revolutions up to a hundred, and the other the number of hundreds—by a special counting arrangement. This arrangement can be thrown in and out of gear at pleasure, by pushing in one direction or the opposite the knobs which will be found either in front or at the sides of the box which carries the dials.

The process of counting the revolution of the spindle is then as follows:—First read the dials, and while the rotation is being maintained constant by keeping the pressure constant, as indicated by the gauge, throw the counting apparatus into gear as the second hand of a watch passes the zero point; throw it out of gear after a minute has been completed, and read the dials again. The difference of readings gives the number oı revolutions of the spindle in one minute; dividing by 60 the number per second is obtained.

To obtain the number of puffs of air we have to multiply by the number of holes in the revolving circle. In the modification of the siren by Dove there is a series of circles of holes, which can be opened or shut by respectively pushing in or pulling out plugs in the side of the box. The number of holes in the circles opened or shut by the respective plugs is stamped on the head of the plugs themselves.

In Helmholtz's double siren [1] we have practically two siren discs working on the same spindle; the box of one of the sirens is fixed, while that of the other is capable of comparatively slow rotation. By shutting off all the holes of the one box this siren can be used exactly as a single one.

We are thus furnished with a means of producing a note of any pitch, within certain limits, and of counting at the same time the number of puffs of air which are required to produce it. The note produced by a siren is not by any means a pure tone; the upper partials are sometimes quite as loud as the fundamental tone.

To measure the vibration frequency of a note by means of the siren, the pressure of air from the bellows must be adjusted so that the siren is maintained at a constant rate of rotation, and giving out a note whose *fundamental tone* is in unison with that of the given note, one circle of holes alone being open. The condition of unison between the two notes may be attained by starting with the siren considerably below the necessary speed, and, sounding the note at same time, gradually increase the speed of the siren until beats are distinctly heard between the given note and the siren. As the speed of the siren is still further urged the beats become less rapid until they disappear; the blower should then keep the pressure so constant that the note of the siren remains in exact unison with the given note, and while this constancy is maintained a second observer should measure

[1] For a more detailed description of this instrument, see Tyndall's *Sound*, Lecture II.

the rate of rotation of the spindle. The beats which will be heard if the note of the siren is too high or too low serve to aid the blower in controlling the note of the siren. Suppose that the number of revolutions per minute is N, and the number of holes in the open circle n, then the vibration frequency of the note is $N n/60$.

The method of procedure with the simpler siren previously described is similar. The speed of rotation depends in that case, however, on the rate of driving of the engine; the experiment is therefore somewhat simpler, although the range of notes obtainable is rather more limited. The speed can be controlled and kept steady by subjecting the driving string to more or less friction by the hand covered with a leather glove.

Care should be taken not to mistake the beats between the given note and the first upper partial of the note of the siren, which are frequently very distinct, for the beats between the fundamental tones.

The result of a mistake of that kind is to get the vibration frequency of the note only half its true value, since the first upper partial of the siren is the octave of the fundamental tone. It requires a certain amount of musical perception to be able to distinguish between a note and its octave, but if the observer has any doubt about the matter he should drive the note of the siren an octave higher, and notice whether or not beats are again produced, and whether the two notes thus sounded appear more nearly identical than before.

The most convenient note to use for the purpose of this experiment is that given out by an organ-pipe belonging to the octave between the bass and middle c's. In quality it is not unlike the note of the siren, and it can be sounded for any required length of time. For a beginner a tuning-fork is much more difficult, as it is very different in quality from the siren note, and only continues to sound for a comparatively short time.

Q

If a beginner wishes to find the vibration frequency of a fork by the siren, he should first select an organ-pipe of the same pitch. This can be tested by noticing the resonance produced when the sounding fork is held over the embouchure of the pipe. Then determine the pitch of the note of the organ-pipe by means of the siren, and so deduce that of the fork.

Experiment.—Find the vibration frequency of the note of the given organ-pipe.

Enter results thus :—

Organ-pipe—Ut. 2

(1) By the Helmholtz siren:
Pressure in gauge of bellows, 5 inches.

Revolutions of spindle of siren per minute, 648.
Number of holes open, 12.
Frequency of note, 129.

(2) By Ladd's siren:
Speed of rotation of disc, 3·6 turns per sec.
Number of holes, 36.
Frequency of note, 130.

29. Determination of the Velocity of Sound in Air by Measurement of the Length of a Resonance Tube corresponding to a Fork of known Pitch.

If a vibrating tuning-fork be held immediately over the opening of a tube which is open at one end and closed at the other, and of suitable length, the column of air in the tube will vibrate in unison with the fork, and thus act as a resonator and reinforce its vibrations. The proper length of the tube may be determined experimentally.

If we regard the motion of the air in the tube as a succession of plane wave pulses sent from the fork and reflected at the closed end, we see that the condition for resonance is that the reflected pulse must reach the fork

again at a moment when the direction of its motion is the opposite of what it was when the pulse started. This will always be the case, and the resonance will in consequence be most powerful, if the time the pulse takes to travel to the end of the tube and back to the fork is exactly half the periodic time of the fork.

Now the pulse travels along the tube with the constant velocity of sound in air; the length of the tube must be, therefore, such that sound would travel twice that distance in a time equal to one half of the periodic time of the fork.

If n be the vibration frequency of the fork, $1/n$ is the time of a period, and if l be the required length of the resonance tube and v the velocity of sound, then

$$\frac{2l}{v}=\frac{1}{2n}$$

or

$$v=4ln. \qquad \qquad (1)$$

In words, the velocity of sound is equal to four times the product of the vibration frequency of a fork and the length of the resonance column corresponding to the fork.

This formula (1) is approximately but not strictly accurate. A correction is necessary for the open end of the pipe; this correction has been calculated theoretically, and shewn to be nearly equivalent to increasing the observed length of the resonance column by an amount equal to one half of its diameter.[1]

Introducing this correction, formula (1) becomes

$$v=4(l+r)n, \qquad \qquad (2)$$

where r is the radius of the resonance tube.

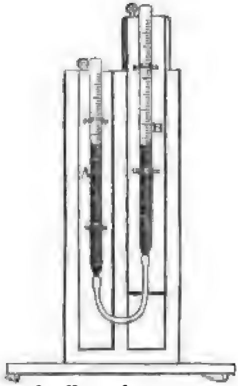
This furnishes a practical method of determining v.

It remains to describe how the length of the resonance tube may be adjusted and measured. The necessary capability of adjustment is best secured by two glass tubes as A, B, in fig. 17, fixed, with two paper millimetre scales

[1] See Lord Rayleigh's *Sound*, vol. ii. § 307 and Appendix A.

behind them, to two boards arranged to slide vertically up and down in a wooden frame; the tubes are drawn out at the bottom and connected by india-rubber tubing. The bottoms of the tubes and the india-rubber connection contain water, so that the length of the column available for resonance is determined by adjusting the height of the water. This is done by sliding the tubes up or down.

FIG. 17.



The position to be selected is the position of maximum resonance, that is, when the note of the fork is most strongly reinforced. The length of the column can then be read off on the paper scales. The mean of a large number of observations must be taken, for it will be noticed, on making the experiment, that as the length of the tube is continuously increased the resonance increases gradually to its maximum, and then gradually dies away. The exact position of maximum resonance is therefore rather difficult of determination, and can be best arrived at from a number of observations, some on either side of the true position.

From the explanation of the cause of the resonance of a tube which was given at the outset, it is easily seen that the note will be similarly reinforced if the fork has executed a complete vibration and a half, or in fact any odd number of half-vibrations instead of only one half-vibration. Thus, if the limits of adjustment of the level of the water in the tube be wide enough, a series of positions of maximum resonance may be found. The relation between the velocity of sound, the length of the tube, and the vibration frequency of the fork, is given by

$$l = \frac{2x+1}{4} \frac{v}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where x is some integer.

This gives a series of lengths of the resonance tube, any two consecutive ones differing by $v/2n$.

Now v/n is the wave-length in air of the note of the fork. So that with a tube of sufficient length, a series of positions of maximum resonance can be determined, the difference between successive positions being half the wave-length in air of the note of the fork.

Introducing the correction for the open end of the pipe, the formula (3) for determining the velocity of sound becomes

$$v = \frac{4n(l+r)}{2x+1}.$$

[The most suitable diameter of the tube for a 256 fork is about 5 centimetres; for higher forks the diameter should be less.]

Experiment.—Determine the lengths of the columns of air corresponding to successive positions of maximum resonance for the given fork and deduce the velocity of sound in air.

Enter results thus :—

Vibration frequency of fork, 256 per sec.

Lengths of resonance columns :

(1) Mean of twelve observations, 31 cm.
(2) " " " 97 "

Radius of tube, 2·5 cm.

Velocity of sound, from (1) 34,340 cm. per sec.
" " from (2) 34,000 cm. "

30. Verification of the Laws of Vibration of Strings. Determination of the Absolute Pitch of a Note by the Monochord.

The vibration of a string stretched between two points depends upon the reflection at either end of the wave motion transmitted along the string. If a succession of waves travel along the string, each wave will in turn be reflected at the one end and travel back along the string and be

reflected again at the other end ; the motion of any point of the string is, accordingly, the resultant of the motions due to waves travelling in both directions. Premising that a node is a point in the string at which the resultant effect of the incident and reflected waves is to produce no change of position, and that a loop is a point at which the change of position due to the same cause is a maximum, it is evident that if a string is to remain in a state of vibration the two ends of the string which are fixed to the supports must be nodes, and it follows that the modes of vibration of the string must be such that the distance between the two ends contains an exact multiple of half the length of a wave, as transmitted along a uniform string of indefinite length and without obstacles.

It is shewn in works on acoustics[1] that a wave of any length travels along such a string with a velocity v where $v=\sqrt{\text{T}/m}$, T being the stretching force of the string in dynes, and m the mass of a unit of its length expressed in grammes per centimetre.

If τ be the time of vibration of the note, and λ its wave length in centimetres, we have, just as in the case of air,

$$\lambda=v\tau.$$

If n be the vibration frequency of the note

$$n=\frac{1}{\tau},$$

hence

$$\lambda=\frac{v}{n}=\frac{1}{n}\sqrt{\frac{\text{T}}{m}}.$$

The distance l between the fixed ends of the string being an exact multiple of $\frac{\lambda}{2}$, we have

$$l=\frac{x}{2n}\sqrt{\frac{\text{T}}{m}},$$

where x is some integer.

[1] See Lord Rayleigh's *Sound*, vol. i. chap. vi. ; Thomson and Tait, *Elements of Natural Philosophy*, Appendix b, p. 284.

Whence

$$n=\frac{x}{2l}\sqrt{\frac{T}{m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It is this formula whose experimental verification we proceed to describe. The apparatus usually employed for the purpose is known as a monochord or sonometer, and consists of a long wooden box with a wire, fixed at one end and stretched between two bridges by a spring at the other, or by means of a weight hanging down over a pulley. The one bridge is fixed at the fixed end of the string; the other one is movable along a graduated scale, so that the length of the vibrating portion of the string can be read off at pleasure. The measurement of the stretching force T, either by the hanging weight or by the stretching of a spring attached to the end of the box, is rendered difficult in consequence of the friction of the bridge, and therefore requires some care. The pulley itself may be used instead of the bridge if care be taken about the measurement of length. For a fine brass or steel wire a stretching force equivalent to the weight of from 10 to 20 kilogrammes may be employed. This must be expressed in dynes by the multiplication of the number of grammes by 981.

It is convenient to have two strings stretched on the same box, one of which can be simply tuned into unison with the adjustable string at its maximum length by an ordinary tuning-key, and used to give a reference note. The tuning can be done by ear after some practice. When the strings are accurately tuned to unison, the one vibrating will set the other in strong vibration also; this property may be used as a test of the accuracy of tuning. We shall call the second the auxiliary string.

It is advisable to use metallic strings, as the pitch of the note they give changes less from time to time than is the case with gut strings.

Referring to the formula (1), we see that the note as

there defined may be any one of a whole series, since x may have any integral value. We get different notes on putting x equal to 1, 2, 3 . . . . successively.

These notes may in fact all be sounded on the same string at the same time, their vibration numbers being n, $2n$, $3n$, $4n$ . . . . and their wave-lengths $2l$, l, $2l/3$, $2l/4$ . . . respectively. The lowest of these is called the fundamental note of the string, and the others harmonics. These may be shewn to exist when the string is bowed, by damping the string—touching it lightly with the finger—at suitable points. Thus, to shew the existence of the first harmonic whose wave-length is l, bow the string at one quarter of its length from one end, and touch it lightly at the middle point. The fundamental note will be stopped, and the octave will be heard, thus agreeing in pitch with the first of the series of harmonics given above.

To obtain the second harmonic bow the string about one-sixth of its length from the end, and touch it lightly with the finger at one-third of its length. This stops all vibrations which have not a node at one third of the length, and hence the lowest note heard will be the second harmonic, which will be found to be at an interval of a fifth from the first harmonic or of an octave and a fifth from the fundamental tone. We may proceed in this way for any of the series of harmonics, remembering that when the string is damped at any point only those notes will sound that have a node there, and on the other hand, there cannot be a node at the place where the string is bowed; hence the place for bowing and the place for damping must not be in corresponding positions in different similar sections of the wave-curve; if they were in such corresponding positions the damping would suppress the vibration of the string altogether.

The intervals here mentioned may be estimated by ear, or compared with similar intervals sounded on the piano or harmonium.

We shall now confine our attention to the fundamental note of the string. Putting $x=1$ in formula (1) we get

$$n=\frac{1}{2l}\sqrt{\frac{T}{m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

We have first to verify that the vibration number of the note varies inversely as the length of the string when the tension is constant. This may be done by sliding the movable bridge until the note sounded is at a definite interval from the note of the auxiliary string, with which it was previously in unison. Suppose it to be the octave, then the length of the adjustable string will be found to be one half of its original length; if a fifth, the ratio of its new length to its original length will be 2/3, and so on; in every case the ratio of the present and original lengths of the string will be the inverse ratio of the interval.

In a similar manner we may verify that the vibration frequency varies as the square root of the tension. By loading the scale pan hung from the pulley, until the octave is reached, the load will be found to be increased in the ratio of 4 : 1, and when the fifth is obtained the load will be to the original load in the ratio of 9 : 4.

It yet remains to verify that the vibration frequency varies inversely as the square root of m, the mass per unit of length of the string. For this purpose the string must be taken off and a known length weighed. It must then be replaced by another string of different material or thickness, the weight of a known length of which has also been determined. Compare then the length of the two strings required to give the same note, that is, so that each is in turn in unison with the auxiliary string. It will be found that these lengths are inversely proportional to the square root of the masses per unit of length, and having already proved that the lengths are inversely proportional to the vibration frequencies, we can infer that the vibration frequencies are

inversely proportional to the square roots of the masses per unit of length.

We can also use the monochord to determine the pitch of a note, that of a fork for instance. The string has first to be tuned, by adjusting the length, or the tension, until it is in unison with the fork. A little practice will enable the observer to do this, and when unison has been obtained the fork will throw the string into strong vibration when sounded in the neighbourhood. Care must be taken to make sure that the fork is in unison with the fundamental note and not one of the harmonics. The length of the string can then be measured in centimetres, and the stretching force in dynes, and by marking two points on the wire and weighing an equal length of exactly similar wire, the mass per unit of length can be determined. Then substituting in formula (2) we get n.

This method of determining the pitch of a fork is not susceptible of very great accuracy in consequence of the variation in the pitch of the note of the string, due to alterations of temperature and other causes.

Experiment.—Verify the laws of vibration of a string with the given wire and determine the pitch of the given fork.

Enter results thus :—

Length of wire sounding in unison with the given fork, 63·5 cm.
Stretching force (50 lbs.), 22,680 grammes weight $= 22680 \times 981$ dynes.
Mass of 25 cm. of wire, ·670 grammes.
Vibration frequency of fork, 227 per sec.

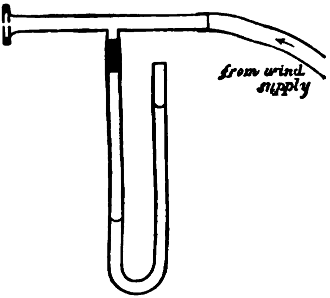
31. Determination of the Wave-length of a high Note in Air by means of a Sensitive Flame. (Lord Rayleigh, Acoustical Observations, *Phil. Mag.*, March, 1879.)

For this experiment a note of very high pitch is required. Probably a very high organ-pipe or whistle might

be employed, but a simple and convenient arrangement, the same in principle as a 'bird-call,' consists of two small parallel metallic discs, fixed so as to be a short distance—a millimetre more or less—apart, and perforated, each with a small circular hole the one behind the other. This pair of discs is then fixed on to the end of a supply-tube, and air blown through the holes by means of a loaded gas-bag or bellows. It is convenient to connect a manometer with the supply-tube, close to the whistle, in order to regulate the supply of air from the reservoir, and thus maintain a note of constant pitch.

Fig. 18 shews a section of this part of the apparatus. It is very easily constructed. The one disc can be fixed to the tube of glass or metal by sealing wax, and the other adjusted and kept in its place with soft wax.

FIG. 18.



A sensitive gas flame 'flares' when a note of sufficiently high pitch is sounded in its neighbourhood; thus a hiss, or the shaking of a bunch of keys is generally effective. To obtain a sensitive flame, a pin-hole steatite burner may be employed; it must be supplied with gas at a high pressure (9 or 10 inches of water) from a gas holder. The ordinary gas supply of a town, which gives only about 1 inch pressure, is of no use for the purpose.

The tap—best an india-rubber tube with pinch-cock—which regulates the flame, must be turned on until the flame is burning steadily (it will generally be some 18 inches high), but just on the point of flaring. The sound of the 'bird-call,' described above, will then, if it be high enough,

make the flame flare, but it will recover its steadiness when the sound ceases.

In order to determine the wave-length of a note by this apparatus, a board is placed so that the sound is reflected perpendicularly from its surface. Placing the nozzle of the burner in the line from the source of sound perpendicular to the board, and moving the burner to and fro along this line, a series of positions can be found in which the effect of the sound upon the flame is a minimum.

The positions are well-defined, and their distances from the board can be measured by taking the distances between the board and the orifice of the burner with a pair of compasses, and referring them to a graduated scale. These positions correspond to the nodal points formed by the joint action of the incident vibration and the vibration reflected from the surface of the board. The distance between consecutive positions corresponds accordingly to half a wave-length of the incident vibration. The wave-length of the note sounded is, therefore, twice the distance between consecutive positions of minimum effect upon the flame.

The distances of as many successive positions as can be accurately observed should be taken. Each observation should be repeated three or four times and the mean taken.

Instead of the sensitive flame, an india-rubber tube leading to the ear may be employed, and positions of silence determined. It must be remembered, however, in this case that the position of silence for the ear corresponds to a position of minimum pressure-variation at the orifice of the tube—that is to say, to a loop and not to a node. The distances of these positions of silence from the wall are, therefore, *odd* multiples of quarter-wave-lengths instead of even multiples, as when the sensitive flame is used.

Experiment.—Determine the wave-length of the given note by means of a sensitive flame.

Enter results thus:—

No. of position of minimum effect, reckoning from the board	Actual observations of the distance in mm. of the nozzle from the board.	Mean of Observations	Half-Wave-Length deduced in Millimetres.
1	16½, 16½, 16, 16	16·25	16·25
2	31, 31½, 32½, 31, 32	31·5	15·75
3	47, 47½, 46½, 47, 45½	46·75	15·6
4	62, 62½, 64, 60½, 62½	62·25	15·6
5	78½, 78½	78·5	15·5

Mean wave-length = 31·2 mm.

CHAPTER IX.

THERMOMETRY AND EXPANSION.

THE temperature of a body may be defined as its thermal condition, considered with reference to its power of communicating heat to or receiving heat from other bodies. This definition gives no direction as to how the temperature of a body is to be measured numerically. We may amplify it by saying that if, when a body A is placed in contact with another body B, heat passes from A to B, the body A is at a higher temperature than B; but this extension only indicates the order in which a scale of temperatures should be arranged.

In order to *measure* temperature we may select one of the effects produced by an accession of heat in a particular instrument, and estimate the range of temperature through which that instrument is raised or lowered when placed in contact with the body whose temperature is to be measured by measuring the amount of the effect produced. This is the method practically adopted. The instrument which is

so used is called a thermometer, and the branch of the science of heat which treats of the application of such instruments is called thermometry.

A continuous accession of heat produces continuous alteration in many of the physical properties of bodies, and any one of them might have been selected as the basis of a system of thermometry. Attempts, which have met with more or less success, have been made to utilise several of these continuous alterations for the purpose. The change of volume of various liquids enclosed in glass vessels; the change in pressure of a gas when the volume is kept constant, or the change in volume when the pressure is kept constant; the change in the electrical resistance of a wire; the change in the electromotive force in a thermo-electric circuit; the change in length of a metallic bar; the change in the pressure of the vapour of a liquid; change of shape of a spiral composed of strips of different metals, as in Bréguet's thermometer, have all been thus employed.

Of all these methods of forming a system of thermometry, the one first mentioned is by far the most frequently employed. It owes its general acceptance to the fact that the change of volume of a liquid in a glass vessel is very easily measured with great accuracy. Moreover, if it were not for certain slow-working changes of very small magnitude in the volume of the glass envelope, of which we shall speak later, the indication of such an instrument would practically depend upon the temperature and upon nothing else. The liquids which have been employed are mercury, alcohol, and ether. Mercury can easily be obtained pure, and remains a liquid, with a vapour-pressure less than the ordinary atmospheric pressure for a wide range of temperatures, including those most frequently occurring in practice. Ether has a larger coefficient of expansion, but can only be used for a small range of low temperatures. The thermometers most generally in use are accordingly filled with mercury, and the expansion of mercury in a glass vessel has thus been

adopted as the effect of heat to be employed as the basis of the numerical measurement of temperature.

A mercury thermometer consists of a stem, a glass tube of very fine and uniform bore, having a cylindrical or spherical bulb blown at the end. The bulb and part of the tube are filled with mercury, and the top of the stem is hermetically sealed, when the bulb is so heated that the whole instrument is filled with the liquid. When the mercury cools and contracts, the space above it is left empty. The numerical measurement is introduced by marking upon the stem the points reached by the mercury when the thermometer is maintained successively at each of two temperatures which can be shewn to be constant, and dividing the length of the stem between the two marks into a certain number of equal parts. These two fixed temperatures are usually the temperature of melting ice, and the temperature of steam which issues from water boiling under a standard pressure of 760 mm. They have been experimentally shewn to be constant, and can always be obtained by simple apparatus (see § 33).

The two marks referred to are called the freezing and the boiling point respectively, and the distance between them on the stem is divided into 100 parts for the centigrade thermometer, and 180 for the Fahrenheit, each part being called a degree.

On the former the freezing point is marked 0°, and on the latter 32°. The remarks which follow, when inapplicable to both kinds, may be held to refer to the centigrade thermometer.

It should first be noticed that this system, which supplies the definition of the numerical measure of temperature, is completely arbitrary. A number of degrees of temperature corresponds to a certain percentage of the total expansion of mercury in a glass vessel between 0° and 100°. Two quantities of mercury will doubtless expand by the same fraction of their volume for any given range of temperature,

and thus two mercury thermometers, similarly graduated, may be expected to give identical indications at the same temperature, provided each tube is of uniform bore, and the expansion of the glass, as referred to the corresponding expansion of the mercury, is uniform for each instrument. This is in general sufficiently nearly the case for two thermometers which have been very recently graduated. But a thermometer filled with any other liquid, and agreeing with a mercury thermometer at two points, cannot be expected to, and does not in fact, agree with it for temperatures other than those denoted by the two points. If it did it would shew that the rate of expansion of its liquid in glass was uniform for successive intervals of temperature, as defined by the mercury thermometer, and this is generally not the case.

Even the conditions necessary for two mercury thermometers to give identical indications at the same temperature are not, as a rule, satisfied. In the first place, the bore of a thermometer is not generally uniform. The variation may, indeed, be allowed for by calibration (see § 8), so that we may correct the indications for want of uniformity of bore; the determination of the corrections in this way is a somewhat tedious operation. Moreover, the volume of the glass envelope undergoes a slow secular change. A thermometer bulb, when blown and allowed to cool, goes on contracting long after the glass has attained its normal temperature, the contraction not being quite complete even after the lapse of years. If the bulb be again heated, the same phenomenon of slow contraction is repeated, so that, after a thermometer is filled, the bulb gradually shrinks, forcing the mercury higher up the tube. If the thermometer has been already graduated, the effect of this slow contraction will appear as a gradual rise of the freezing point.

In some thermometers the error in the freezing point due to this cause amounts to more than half a degree, and the error will affect the readings of all temperatures

between 0° and 100° by nearly the same amount. The instrument should, therefore, not be graduated until some considerable time after being filled; but even when this precaution is taken the change in the zero point is not completely eliminated, but only considerably diminished. A corresponding small change of the zero point is set up whenever the thermometer is raised to the boiling point.

The reading of a mercury thermometer does not, therefore, give an indication of temperature which will be clearly understood by persons who do not measure temperatures by that particular thermometer. To ensure the reading being comparable with those of other instruments, the tube must have been calibrated, and the fixed points quite recently re-determined, and the readings thus corrected; or, adopting another and more usual method, the individual thermometer in question may be compared experimentally with some instrument generally accepted as a standard. A set of such are kept at the Kew Observatory; they have been very carefully made and calibrated, and their fixed points are repeatedly determined, and a standard scale is thus established. With one or more of these standards any thermometer can be compared by immersing them in water which is kept well stirred, and taking simultaneous readings of the two at successive intervals of temperature. In this way a table of corrections is formed for the thermometer which is tested, and its indications can be referred to the Kew standard by means of the table. However, the secular contraction of the bulb may still be going on; but to allow for any contraction subsequent to the Kew comparison, it is sufficient to ascertain if there has been any change in the freezing point, and in that case consider that an equal change has taken place for every temperature, and that, therefore, each correction on the table is changed by that amount.

A specimen table of Kew corrections is appended as an

example of the way in which this method of referring thermometers to a common standard is worked.

THER. FORM. D.

KEW OBSERVATORY.—Certificate of Examination.

CENTIGRADE THERMOMETER.—No. 79915 K.O. 8894.

by J. Hicks, London.

(VERIFIED UNMOUNTED AND IN A VERTICAL POSITION.)

Corrections to be applied to the Scale Readings, determined by comparison with the Standard Instruments at the Kew Observatory.

At 0°	−0°·1
5	−0·1
10	−0·1
15	−0·1
20	−0·2
25	−0·2
30	−0·2
35	−0·4

Note—I.—When the sign of the Correction is +, the quantity is to be *added* to the observed reading, and when − to be *subtracted* from it.

II.—Mercurial Thermometers are liable, through age, to read too high; this instrument ought, therefore, at some future date, to be again tested at the melting point of ice, and if its reading at that point be found different from the one now given, an appropriate correction should be applied to all the above points.

KEW OBSERVATORY,

December 1880. G. M. Whipple,

SUPERINTENDENT.

MST. 500—5 78.

So far we have dealt with the principles of the method of measuring temperatures within the range included between the freezing and boiling points of water. In order to extend the measurement beyond these limits various plans have

been adopted. The mercury thermometer is sometimes used, its stem beyond the limits being divided into degrees equal *in length* to those within the limits. A thermometer divided in this way can be used for temperatures down to $-40°$, and up to $350°$ C.; but, unfortunately, the difference in the expansion of different specimens of glass is such that at the higher temperatures two thermometers, similarly graduated, may differ by as much as ten degrees, and hence the mercury thermometer thus used does not give a satisfactory standard. Two air thermometers, on the other hand, when properly corrected for the expansion of the glass, always give the same readings, and thus the air thermometer has come to be recognised as the temperature standard for high and low temperatures. It is referred to the mercury standard for the freezing and boiling points and intermediate temperatures; thus the higher temperatures are expressed in centigrade degrees by a species of extrapolation, using the formula for the expansion of a permanent gas as determined by observations within the limits of the mercury thermometric standard.

Other methods of extrapolation from a formula verified by comparison, either with the mercury or air thermometer, have sometimes been employed with more or less success, in order to determine temperatures so high that the air thermometer is unsuitable, such as, for instance, the temperature of a furnace. In the case of Siemens' resistance pyrometer, a formula is obtained by experiments at low temperatures, expressing the relation between the resistance of a platinum wire and its temperature; the temperature of the furnace is then deduced from an observation of the resistance of the platinum on the supposition that the formula holds, although the temperature is a long way outside the limits of verification. The temperature obtained in some manner, generally analogous to this, is often expressed as so many degrees centigrade or Fahrenheit. It is evident that numbers obtained by different methods may

be widely different, as all are arbitrary. At present it is a matter of congratulation if two different instruments on the same principle give comparable results; and, until some more scientific, or rather, less arbitrary, method of measuring temperatures is introduced, the precise numbers quoted for such temperatures as those of melting silver or platinum must remain understood only with reference to the particular system of extrapolation adopted to extend the range of numbers from those properly included in the range of the mercury thermometer, namely, those between the freezing and boiling points of water.

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32. Construction of a Water Thermometer.

The method of filling a thermometer is given in full in Garnett's 'Heat,' §§ 10–18, also in Deschanel's[1] or Ganot's 'Natural Philosophy,' and Maxwell's 'Heat.'

In this case water is to be used instead of mercury.

One or two points may be noticed:—

(1) The tube and bulb have not always a cup at the top as in Garnett (fig. 1). When this is the case, a piece of wide glass tubing must be drawn out to serve as a funnel, and joined by means of clean india-rubber to the tube of the thermometer.

(2) It would be difficult to seal the glass tube when full of water, unless it has been previously prepared for closing. After the bulb has been filled, but before it is again heated to the high temperature, the upper part of the tube is softened in a blow-pipe flame, and drawn out so as to leave a fine neck in the tube. Then the bulb is heated until the liquid rises above this neck, and when this is the case the tube is sealed by applying a small blow-pipe flame at the thinnest part.

At the moment of sealing the source of heat must be removed from the bulb, otherwise the liquid will continue to expand, owing to the rise of temperature, and will burst

[1] Deschanel, *Natural Philosophy*, p. 245, etc.

the bulb. The safest way of heating the bulb is to put it in a bath of liquid—melted paraffin, for example, or water if the thermometer be not required for use near the boiling point —and apply heat to the bath until the liquid in the thermometer reaches beyond the neck. Remove the source of heat from the bath and seal off the tube as the level of the water sinks past the narrow neck.

(3) The water used for filling the thermometer should be distilled water from which the dissolved air has been driven by long-continued boiling. This precaution is essential, as otherwise bubbles of air separate from the water in the bulb and stem after sealing, and this often renders the thermometer useless until it has been unsealed and the air removed and the tube re-sealed.

We proceed to shew how to use the thermometer to determine the variation of volume of the water.

We require, for this purpose, to know the volume of any given length of the tube and the whole volume of water contained in the thermometer.

To find the Volume of any Length of the Tube.

Before filling the thermometer, introduce into the tube a small pellet of mercury and measure its length, which should be from 10 to 20 cm. Then warm the bulb and force the mercury out into a beaker, of which the weight is known. Weigh the beaker and mercury, and get by subtraction the weight of the mercury. Now, we may take the density of mercury to be 13·6. If, then, we divide the mass in grammes by this number, we get the volume in cubic centimetres.

We thus find the volume of a known length—that of the pellet of mercury—of the tube, and from this can determine the volume of any required length. For greater accuracy it is necessary to measure the length of the same pellet of mercury at different parts of the tube, thus calibrating the tube (see § 8).

To find the Volume of the Water which is contained in the Thermometer.

Weigh the bulb and tube when empty, then weigh it again

when filled, before sealing off. The difference in the weights gives the number of grammes of water in the bulb and tube, and hence the number of cubic centimetres of water in the two can be calculated.

It may be more convenient to seal off before weighing, but in this case great care must be taken not to lose any of the glass in the act of sealing, and to put the piece of glass which is drawn off on the balance with the tube.

Let us suppose the volume of 1 cm. length of the tube is ·01 c.c., and that the volume of the water contained is 4·487 c.c.

After sealing the tube as already described, immerse it in a bath of water at the temperature of the room, noting that temperature by means of a thermometer; suppose it to be 15° C.

Make a series of marks on the tube at a known distance above the level of the water in it; let us say at each centimetre.

Now raise the temperature of the bath until the level of the water in the tube rises to these marks successively, and note the successive temperatures as indicated by the other thermometer. In this way determine the temperatures corresponding to the successive steps in the expansion of the water estimated in fractions of the original volume. Set out in a diagram points representing these temperatures and the corresponding volumes in the manner suggested in chap. iii., pp. 50, 51. A continuous curve can then be drawn passing through the series of points, and the curve so drawn will represent approximately the true course of the variation of volume with temperature, of the water relatively to glass. From it, the mean coefficient of relative expansion for any interval can be determined by dividing the change of volume during the interval by the change of temperature, and the true relative coefficient at any temperature can be inferred from the tangent of the angle which the tangent to the curve (at the point corresponding to that temperature)

makes with the temperature-axis after the same manner as velocity is inferred in § B. chap. v.* p. 147.

Experiment.—Determine by means of a water thermometer the thermal expansion of water relative to glass.

Enter results thus :—

Length of pellet of mercury	15·3 cm.
Weight of do.	2·082 gm.
Vol. of 1 cm. of tube	·01 c.c.
Vol. water initially 4·487 c.c.	Temp. 15°
Vol. finally 4·587 c.c.	Temp. 70°
Mean coeff. of expansion = ·000405 per 1°.	

33. Thermometer Testing.

By this we mean determining the indications of the thermometer which correspond to the freezing point of water and to its boiling point under a pressure of 760 mm.

The first observation is made by placing the thermometer so that its bulb and stem up to the zero are surrounded with pounded ice. The ice must be very finely pounded and *well washed* to make quite sure that there is no trace of salt mixed with it. This precaution is very important, as it is not unusual to find a certain amount of salt with the ice, and a very small amount will considerably reduce the temperature.

The ice should be contained in a copper or glass funnel in order that the water may run off as it forms. The thermometer should be supported in a clip, lest when the ice melts it should fall and break.

The boiling point at the atmospheric pressure for the time being may be determined by means of the hypsometer, an instrument described in any book on physics.[1]

The thermometer to be tested must be passed through the cork at the top of the hypsometer, and there fixed

[1] Garnett, *Heat*, § 12, &c. Deschanel, *Natural Philosophy*, p. 248, &c.

so that the $100°$ graduation is just above the cork. One aperture at the bottom of the cover of the hypsometer is to allow the steam issuing from the boiling water to escape; to the other aperture is attached by an india-rubber tube a pressure gauge, which consists of a U-shaped glass tube containing some coloured liquid. The object of this is to make sure that the pressure of the steam within the hypsometer is not greater than the atmospheric pressure.

The water in the hypsometer must be made to boil and the thermometer kept in the steam until its indication becomes stationary. The temperature is then read.

In each of these operations, in order to make certain of avoiding an error of parallax in reading (i.e. an error due to the fact that since the object to be read and the scale on which to read it are in different planes, the reading will be different according as the eye looks perpendicularly on the stem or not), the thermometer must be read over the edge of a card or by a telescope at the same height as the graduation to be read. If, then, the thermometer be vertical, the line of sight being horizontal will be perpendicular to it. (It must be remembered in estimating a fraction of a division of the thermometer that in the telescope the image of the scale is inverted.)

We thus determine the boiling point at the atmospheric pressure for the time being. We have still to correct for the difference between that pressure and the standard pressure of 760 mm. To do this the height of the barometer must be read and expressed in millimetres. We obtain from a table shewing the boiling point for different pressures, the fact that the difference in the temperature of the boiling point of $1°$ corresponds to a difference of pressure of 26·8 mm. We can, therefore, calculate the effect of the difference of pressure in our case.

Suppose the observed boiling point reading is 99·5, and the height of the barometer 752 mm. We may assume that, for small differences of pressure from the standard pressure,

the difference in the boiling point is proportional to the difference of pressure; hence

$$\frac{760-752}{26\cdot8}=\frac{\text{required correction}}{1^\circ};$$

$$\therefore \text{ the required correction} = \frac{8^\circ}{26\cdot8} = \cdot3^\circ.$$

And therefore the corrected boiling point would read 99·8° on the thermometer.

The correction is to be added to the apparent boiling-point reading if the atmospheric pressure is below the standard, and *vice versâ*.

The difference of temperature of the two boiling points depends only on the difference of pressure. Also an increase of pressure of 1 mm. of mercury produces an alteration of the temperature of the boiling point of 0·0373° C., or an increase of temperature of the boiling point of 1° corresponds to a pressure of 26·8 mm. of mercury.

Now the specific gravity of mercury referred to water is 13·6, that of dry air at 760 mm. pressure, and 15° C. temperature is ·001225. Thus the pressure due to 1 mm. of mercury is equal to that due to $\frac{13\cdot6}{\cdot001225}$ mm., or 11·102 metres of dry air.

But a rise in temperature of 1° corresponds to an increase in pressure of 26·8 mm. mercury; that is, to an increase of pressure due to 11·102 × 26·8 metres of dry air.

Thus, the boiling point alters by 1° C. for an alteration of pressure equal to that due to a column of dry air at 15° C. and of 297·5 metres in height.

Experiments.

(1) Determine the freezing and boiling points of the given thermometer.

Enter results thus:—

Thermometer, Hicks, No. 14459.

Freezing point −0°·1.

Boiling point 99°·8.

The following additional experiments may be performed with the hypsometer.

(2) Put some salt into the hypsometer and observe the boiling point again.

(3) Tie some cotton wick round the bulb of the thermometer, and let the end drop into the solution. Vide Garnett, § 13.

(The cotton wick should be freed from grease by being boiled in a very dilute solution of caustic potash and well washed.)

(4) Remove the water, clean the thermometer, and repeat the observation with a given liquid.

Boiling point of alcohol is		79°.
" "	ether	37°.
" "	turpentine	130°.

(5) Clean the thermometer and hypsometer, and remove the apparatus to a room in the basement, and observe the temperature of the boiling point of water.

Take the apparatus up to the top of the building and repeat, and from the two observations determine the height of the building.

34. Boiling Point of a Liquid.

A liquid is usually said to boil at a temperature t when the pressure of its vapour at this temperature is equal to the external pressure p. But if the sides of the vessel be smooth and the liquid be quite free from dissolved air, or if it contain salts in solution, it will generally not boil till its temperature is higher than t.

Suppose the liquid to boil at $t^\circ+\tau$, then the vapour rising up at this temperature will exert a pressure greater than the external pressure p. Consequently it will expand till its pressure falls to p, its temperature at the same time falling till it reaches the corresponding temperature t.[1]

Hence the temperature of the vapour over a boiling liquid under a given pressure p, is a constant quantity under all

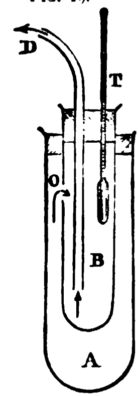
[1] Maxwell, *Heat*, pp. 25 and 289.

circumstances, and is called the boiling point of the liquid under the pressure p.

The hypsometer will serve to determine the boiling point of a liquid. In many cases, however, when the quantity of liquid obtainable is small, the apparatus described below is more convenient.

The liquid is put into the outer glass tube (A). The inner tube (B), made of brass, is then restored to its place, as in fig. 17, and the whole placed on a sand bath and heated by a Bunsen burner.

FIG. 19.



When the liquid boils, the vapour will enter by the aperture O into the tube B, and will leave B by the glass tube D, which should be connected by a short piece of india-rubber tube with a condenser, to prevent the vapour entering the room.

As the boiling continues, the thermometer will rise at first, but afterwards remain stationary. *Enter this reading*, and also the *height of the barometer* at the same time.

35. Fusing Point of a Solid.

The method to be adopted in order to determine the fusing point of a solid must depend on several considerations, as—

(1) Whether the temperature can be registered on a mercury thermometer; i.e. does it lie between $-40°$ C. and $+350°$ C.?

(2) Does the solid pass directly from the solid to the liquid state, or is there an intermediate viscous condition? If so, the melting point may be taken as somewhere between the temperature of the liquid and solid condition, but cannot be considered as a definite temperature.

(3) Whether or not the substance is a good conductor of heat. If it be, the temperature of a vessel containing the substance in part solid will be very nearly constant if kept

properly stirred. This is the case with ice and the fusible metals and alloys. For bodies which are bad conductors a method has to be adopted as occasion requires. We give as an instance the following, which is available in the case of paraffin wax.

The thermometer, when dipped into the melted paraffin, is wetted by the liquid, and when taken out is in consequence covered with a very thin and perfectly transparent film of liquid paraffin. This film cools, and on solidifying assumes a frosted appearance which extends rapidly all over the part of the thermometer that has been immersed. If the bulb of the thermometer is sufficiently small for us to neglect the difference of temperature between the interior and exterior portions of the mercury, the observation of the thermometer at the instant when this frosted appearance comes over the bulb may be taken as the melting point of paraffin. The only error likely to be introduced is that mentioned above, viz. that the temperature of the paraffin is not the mean temperature of the thermometer bulb. This can be rendered smaller and smaller by taking the liquid at temperatures approaching more and more nearly to the melting point as thus determined, and its direction can be reversed if we allow the paraffin to solidify on the bulb and then heat the bulb in a beaker of water and note the temperature at the instant when the film becomes transparent. The mean of this temperature and that deduced from the previous experiment will be the melting point.

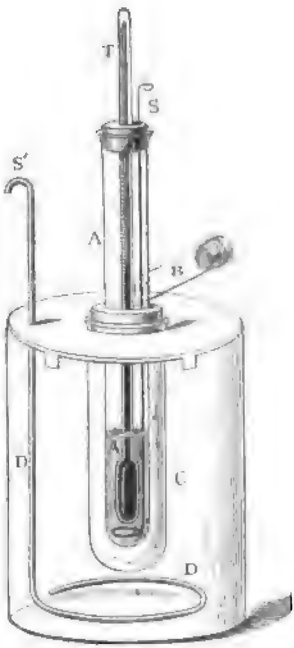
The following is another method of obtaining the fusing point of a solid such as paraffin. Draw out a fine glass tube a millimetre or so in bore, fill it with melted paraffin, and allow the paraffin to solidify. Fasten the glass tube to a thermometer by an elastic band or otherwise, the part of the tube filled with paraffin being close alongside the thermometer bulb. Immerse the thermometer and paraffin in beaker of water and heat it gently, keeping the water well -ed. When the paraffin melts it becomes transparent, and

the temperature at which this change takes place can be noted with considerable accuracy. Raise the temperature a little above the melting point, and allow the bath to cool slowly. When the paraffin solidifies it becomes opaque. By alternately heating and cooling within narrow limits, a series of values of the melting point which only differ very little can be obtained; the mean of these may be taken as the melting point of paraffin.

K. Effect of Dissolved Salts on the Freezing Point.

For the more accurate determination of the freezing point of a solution, and of the effect of dissolved salts in altering the freezing point, the following apparatus, described by Beekman, may be used.

FIG. xxi.



The glass tube A (fig. xxi) contains a delicate thermometer T, a stirrer of stout platinum wire, and the liquid to be experimented on. The salt whose effect it is wished to study can be introduced by a side tube B, sealed on to A, or more simply through a glass tube passing through the cork which closes the upper end of A. The tube A is placed inside a wider tube C, passing through a cork in the open end of C. This tube merely serves as an air-jacket. C passes through the lid of a wide glass vessel D, which contains water or a freezing mixture, the temperature of which should be some 5° below the freezing point of the liquid in A. The bath also contains

a stirrer. A weighed quantity of liquid is placed in A and the whole allowed to cool slowly, being kept at the same time well stirred. The liquid in A is probably thus cooled below its freezing point, freezing then takes place, and the thermometer rises suddenly to the melting point as the solid separates out.

The freezing point of the *solvent*—water, or whatever it may be that separates out on freezing—is thus determined. A known quantity of the substance whose effect is required is introduced through the side tube B, and the experiment is repeated. The effect of the salt in modifying the freezing point of the solvent is thus found.

It has been shewn by Raoult and others that, for a large number of substances, when a mass of the substance, p grammes, is dissolved in a solvent, the mass of the solution being w, then the product of the molecular weight of the substance multiplied by the depression of the freezing point is proportional to p/w, so that, if m be the molecular weight of the salt, Δ the depression of the freezing point, and k a constant, then for a large class of salts

$$m\,\Delta = k\,p/w.$$

This law may be verified by finding the depression of the freezing point produced by the addition of various amounts of the same salt, and then by comparing the depression produced by dissolving equal quantities of different salts so as to form solutions of equal volumes. The quantity k is the product of the molecular weight of the salt, the depression of the freezing point produced by the solution of one gramme of salt and the mass of the solution.

The results are generally stated on the supposition that m grammes of salt are dissolved per litre of the solution. We then have

$$p = m, \quad w = \text{mass of one litre of solution};$$

$$\therefore\ m\,\Delta = k\,\frac{m}{w}\,; \quad \therefore\ k = w\,\Delta.$$

Δ is the depression produced when m grammes of salt (m being the molecular weight) are dissolved in 1 litre of

the solvent. Thus, according to the law, Δ should be constant. It is found to have the value $1{\cdot}8°$ C. approximately.

According to Raoult there is a relation between the constant k, the absolute temperature of solidification T, and the latent heat L. By supposing a small quantity of the liquid taken round a thermodynamic cycle at the temperature of solidification, he shews that $k = 2T^2/L$. This result may be verified if the latent heat of fusion of the substance be known.

Experiment.—Shew that the lowering of the freezing point of a solvent due to the presence of a salt is proportional to the mass of salt dissolved, and inversely proportional to its molecular weight.

COEFFICIENTS OF EXPANSION.

For any ordinary substance, with the exception of water, the changes of volume for equal increments of temperature are so nearly equal that the expansion may be calculated from a coefficient approximately constant for each substance, which may be defined as follows :—

Definition.—A coefficient of expansion by heat may be defined as the ratio of the change of a volume, area, or length per degree of temperature to the value of that volume, area, or length at zero centigrade.

In solids and liquids the expansion is so small that in practice we may generally use, instead of the value of the quantity at zero, its value at the lower of the two temperatures observed in the experiment.

For solid bodies we have the coefficients of linear, superficial, and cubical expansion depending on the alteration of length, breadth, or thickness (linear), of surface (superficial), and of volume (cubical) respectively.

Let α, β, γ be these three respectively, and suppose the body to be isotropic, i.e. to have similar properties in all directions round any given point; then it can be shewn that $\beta = 2\,\alpha$, $\gamma = 3\,\alpha$.

For consider a rectangle the sides of which are a and b. When the temperature is raised by t° the sides increase respectively by $a\,\alpha\,t$ and $b\,\alpha\,t$, so that their new values are $a(1+\alpha\,t)$ and $b(1+\alpha\,t)$. Thus the area is $a\,b(1+\alpha\,t)^2$, or, since α is very small, $a\,b(1+2\alpha\,t)$. But if β be the coefficient of superficial expansion, the new area is $a\,b(1+\beta\,t)$. Thus we have $\beta=2\,\alpha$.

In a similar way considering the expansion of a cube we may shew that $\gamma=3\,\alpha$.

For liquid bodies we have to deal only with the coefficient of cubical expansion.

Any measurement of expansion is attended with considerable difficulty.

A liquid requires to be contained in some vessel, and thus we have to consider the alteration in volume of the vessel as well as that of the liquid itself. In the case of a solid, any cause which changes the temperature of the body to be measured probably changes that of the measuring apparatus and causes it to expand also. Our measurements will therefore give the expansion of one substance relatively to another. Thus, we should find, mercury and most liquids expand considerably as compared with glass, while the metals expand greatly in comparison with wood or stone.

Methods, it is true, have been devised for determining the absolute expansion either of a liquid or a solid, but these are too complicated for an elementary course.

We shall explain how to determine (1) by means of reading microscopes, the coefficient of linear expansion of any solid which can be obtained in the form of a long rod, and (2), by means of the weight thermometer, the coefficient of expansion of a liquid and also that of cubical expansion of a solid.

In the case of a gas we may consider either the alteration of volume under constant pressure or the alteration of pressure at constant volume. We shall describe experimental methods of measuring these two.

36. Coefficient of Linear Expansion of a Rod.

We require to measure the length of a rod, or the distance between two marks on it, at two known temperatures, say 15° C. and 100° C.

The highest degree of accuracy requires complicated apparatus. The following method is simple, and will give very fair results.

A thick straight rod is taken, about 50 cm. in length, and a glass tube of 4 or 5 cm. bore and somewhat greater length than the rod. The tube is closed with a cork at each end, and through each cork a small piece of glass tubing is passed, and also a thermometer. Two fine scratches are made on the rod, one close to each end, at right angles to its length, and two other scratches, one across each of the former, parallel to the length. The glass tube is clamped in a horizontal position and the rod placed inside it, resting on two pieces of cork or wood in such a manner that the scratches are on the upper surface and can be seen through the glass. The whole should rest on a large stone slab—the window-sill serves admirably.

The piece of glass tubing in one of the corks is connected with a boiler from which steam can be passed into the tube, the other communicates with an arrangement for condensing the waste steam.

A pair of reading microscopes are then brought to view the cross-marks on the rod, and are clamped securely to the stone. The microscopes, described in § 5, should be placed so that they slide parallel to the length of the rod; this can be done by eye with sufficient accuracy for the purpose.

If microscopes mounted as in § 5 are not available, a pair with micrometer eye-pieces, or with micrometer scales in the eye-pieces, may be used.

For convenience of focussing on the rod which is in the glass tube, the microscopes must not be of too high a power. Their supports should be clamped down to the stone at

points directly behind or in front of the position of the microscopes themselves, to avoid the error due to the expansion of the metal slides of the microscopes, owing to change of temperature during the experiment.

Call the microscopes A and B; let A be the left-hand one of the two, and suppose the scale reads from left to right. Turn each microscope-tube round its axis until one of the cross-wires in the eye-piece is at right angles to the length of the rod, and set the microscope by means of the screw until this cross-wire passes through the centre of the cross on the rod.

Read the temperature, and the scale and screw-head of each microscope, repeating several times. Let the mean result of the readings be

Temp.	A	B
15° C. . . .	5·106 cm.	4·738 cm.

Now allow the steam to pass through for some time; the marks on the copper rod will appear to move under the microscopes, and after a time will come to rest again.

Follow them with the cross-wires of the microscopes and read again. Let the mean of the readings be

Temp.	A	B
100° C. . . .	5·074 cm.	4·780 cm.

Then the length of the rod has apparently increased by $5{\cdot}106-5{\cdot}074+4{\cdot}780-4{\cdot}738$, or ·074 cm.

The steam will condense on the glass of the tube which surrounds the rod, and a drop may form just over the cross and hide it from view. If this be the case, heat from a small spirit flame or Bunsen burner must be applied to the glass in the neighbourhood of the drop, thus raising the temperature locally and causing evaporation there.

Of course the heating of the rod and tube produces some alteration in the temperature of the stone slab and causes it to expand slightly, thus producing error. This will be very slight, and for our purpose negligible, for the rise of

temperature will be small and the coefficient of expansion of the stone is also small.

We have thus obtained the increase of length of the rod due to the rise of temperature of 85°. We require also its original length.

To find this, remove the rod and tube and replace them by a scale of centimetres, bringing it into focus. Bring the cross-wires over two divisions of the scale, say 10 and 60, and let the readings be

A	B
4·576 cm.	5·213 cm.

Then clearly the length of the rod at 15° is

$$50-(5{\cdot}106-4{\cdot}576)+(4{\cdot}738-5{\cdot}213),$$

or

$$48{\cdot}995 \text{ cm.}$$

To find the coefficient of expansion we require to know the length at 0° C.; this will differ so little from the above that we may use either with all the accuracy we need, and the required coefficient is $\dfrac{\cdot 074}{85 \times 48{\cdot}995}$, or ·0000178.

Experiment.—Determine the coefficient of expansion of the given rod.

Enter results thus :—

Increase of length of rod between 15° and 100°	·074 cm.
Length at 15°	48·995 cm.
Coefficient	·0000178

37. The Weight Thermometer.

The weight thermometer,[1] consists of a glass tube closed at one end, drawn out to a fine neck, which is bent so that it can easily dip into a vessel of liquid.

It is used (1) to determine the coefficient of expansion of a liquid relatively to glass; (2) to determine the coefficient of expansion of a solid, that of the liquid being known.

[1] Garnett, *Heat*, §§ 80, 84. Deschanel, *Natural Philosophy*, p. 283.

For (1) we first fill the thermometer with the liquid and determine the weight of liquid inside, when the whole is at some known low temperature, e.g. that of the room or that of melting ice. We then raise the thermometer and liquid to some higher temperature, that of boiling water, suppose. Part of the liquid escapes from the open end. The weight of that which remains inside is then determined, and from these two weights, and the known difference between the temperatures at which they respectively fill the thermometer, we can calculate the coefficient of expansion of the liquid relatively to the glass.

Our first operation will be to weigh the empty glass tube, which must be perfectly clean and dry. Let its weight be 5·621 grammes.

We now require to fill it.

For this purpose it is heated gently in a Bunsen burner or spirit lamp, being held during the operation in a test-tube holder. Its neck is then dipped under the surface of the liquid whose coefficient of expansion is required—glycerine, suppose—and the tube allowed to cool. The pressure of the external air forces some of the glycerine into the tube. As soon as the liquid ceases to run in, the operation is repeated, and so on until the tube is nearly full. It is then held with its orifice under the glycerine, and heated until the fluid in the tube boils. The air which remained in is carried out with the glycerine vapour and the tube left filled with hot glycerine and its vapour.

The flame is removed and the thermometer again cooled down, when the vapour inside condenses and more liquid is forced in by the external air pressure. If a bubble of air is still left inside, the operation of heating and cooling must be repeated until the bubble is sufficiently small to be got rid of by tilting the thermometer so that it floats up into the neck.

There is another plan which may sometimes be adopted with advantage for partially filling the thermometer.

Place it, with its beak dipping into the glycerine, under the receiver of an air-pump and exhaust. The air is drawn both out of the thermometer and the receiver. Re-admit the air into the receiver. Its pressure on the surface of the glycerine forces the liquid into the tube. It is difficult, however, by this method to get rid of the last trace of air.

Suppose the thermometer is filled; it is now probably considerably hotter than the rest of the room. Hold it with its beak still below the surface of the glycerine and bring up to it a beaker of cold water, so as to surround with water the body of the tube and as much as possible of the neck. This of course must not be done too suddenly lest the glass should crack.

Let the thermometer rest in the beaker of water—its orifice still being below the surface of the glycerine—and stir the water about, noting its temperature with an ordinary thermometer.

At first the temperature of the water may rise a little; after a time it will become steady, and the tube may be removed. Let the observed temperature be 15° C. We have now got the weight thermometer filled with glycerine at a temperature of 15° C.

Weigh the tube and glycerine; let the weight be 16·843 grammes. The weight of glycerine inside then is 16·843 −5·621, or 11·222 grammes.

It is advisable to arrange some clamps and supports to hold the tube conveniently while it is cooling in the beaker of water.

Instead of using water and cooling the thermometer to its temperature, we may use ice and cool it down to a temperature of 0° C. If we do this we must, as soon as the tube is taken out of the ice, place it inside a small beaker of which we know the weight, for the temperature will at once begin to rise and some of the glycerine will be driven out. Thus we should lose some of the liquid before we could complete the weighing.

Our next operation is to find the weight of liquid which the tube will hold at 100° C. To do this we place it in a beaker of boiling water, setting at the same time a receptacle to catch the glycerine which is forced out. When the water has been boiling freely for some time take out the tube, let it cool, and then weigh it. Subtracting the weight of the glass, let the weight of the glycerine be 10·765 grammes. Thus 10·765 grammes of glycerine at 100° C. apparently occupy the same volume—that of the thermometer—as 11·222 grammes did at 15° C.

The apparent expansion for an increase of temperature of 85° (from 15°−100°) is therefore ·0425. The mean apparent expansion per 1° C. throughout that range is, therefore, ·0425/85, or ·00050.[1]

This is only the coefficient of expansion relatively to glass, for the glass bulb expands and occupies a greater volume at 100° C. than at 15° C.

To find the true coefficient of expansion we must remember that the apparent coefficient is the true coefficient diminished by that of the glass—had the glass at 100° been of the same volume as at 15° more glycerine would have been expelled. The coefficient of expansion of glass may be taken as ·000026. Thus the true coefficient of expansion of the glycerine is ·000526.

To obtain the temperature when we take the tube from the bath of boiling water, we may use a thermometer, or, remembering that water boils at 100° C. for a barometric pressure of 760 mm. of mercury, while an increasing pressure of 26·8 mm. of mercury raises the boiling point by 1° C., we may deduce the temperature of the boiling water from a knowledge of the barometric pressure.

It is better, if possible, to raise the temperature of the weight thermometer to the boiling point by immersing it in

[1] A very convenient form of weight thermometer for accurate measurement consists of a small flask with drawn-out neck provided with a tubular collar ground to fit the neck. See Shaw, *Practical Work at Cav. Lab.*, p. 13.

the steam rising from boiling water, as in the hypsometer. A suitable arrangement is not difficult to make if the laboratory can furnish a hypsometer somewhat wider than the usual ones, with a good wide opening in the top of the cover.

(2) To obtain the coefficient of expansion of a piece of metal—iron, for example—relatively to glycerine, we take a bar of the metal whose volume is obtained from a knowledge of its weight and specific gravity, and place it in the tube before the neck is drawn out.

The bar should be bent so as only to touch the tube at a few points, otherwise it will be impossible to fill the tube with the glycerine.

The tube is filled after having been weighed when empty, and the weight of glycerine in it at a known temperature is determined. Let the temperature be 0° C. It is then raised to say 100° C. and the weight of the glycerine within again determined. The difference between these two gives the weight of glycerine expelled.

Let us suppose we know the specific gravity of glycerine; we can obtain the volume of the glycerine originally in the tube by dividing its weight by its density. Let us call this v_1. We can also find the volume of the glycerine expelled; let this be v, and let v_2 be the volume of the iron, at the lower temperature, v, the volume of the thermometer, t, the change in temperature, α, the coefficient of expansion of the glycerine, β, the coefficient of expansion of the metal, γ, the coefficient of expansion of the glass.

Then $$\mathrm{v}=\mathrm{v}_1+\mathrm{v}_2.$$

When the temperature has risen $t°$ the volume of glycerine is $\mathrm{v}_1(1+\alpha t)$ and that of the metal is $\mathrm{v}_2(1+\beta t)$; thus the whole volume of glycerine and iron will be $\mathrm{v}_1(1+\alpha t)+\mathrm{v}_2(1+\beta t)$. The volume of the glass is $\mathrm{v}(1+\gamma t)$.

The difference between these must clearly give the volume of glycerine which has escaped, or v.

Thus $$\mathrm{v}_1(1+\alpha t)+\mathrm{v}_2(1+\beta t)-\mathrm{v}(1+\gamma t)=v.$$

But $$\mathrm{v}=\mathrm{v}_1+\mathrm{v}_2.$$

Thus $$\mathrm{v}_1(\alpha-\gamma)t+\mathrm{v}_2(\beta-\gamma)t=v.$$

$v_1(a-\gamma)t$ is the volume of glycerine which would have been expelled if the volume of the tube had been v_1; that is to say, if the tube had been such as to be filled entirely with the glycerine which was contained in it at the first weighing. This can be calculated from the knowledge of the weight and specific gravity of the glycerine and of the value of the coefficient of expansion of the glycerine relatively to the glass. Subtract this from the volume actually expelled. The difference is the increase in volume of the metal relatively to glass for the rise in temperature in question. Divide the result by the volume of the metal and the rise in temperature; we get the coefficient of relative expansion of the metal.

Thus, let the original weight of glycerine be 11·222 gms., then the amount which would be expelled, due to the rise of temperature of the glycerine only, will be ·457 gramme, since the coefficient of expansion of glycerine relative to glass is ·0005. Suppose that we find that ·513 gramme is expelled. The difference, ·056 gramme, is due to the expansion of the metal. Taking the specific gravity of glycerine as 1·30, the volume of this would be ·043 c.c. Suppose that the original volume of the metal was 5 c.c. and the rise of temperature 100° C., the coefficient of expansion is given by dividing ·043 by 500, and is, therefore, ·000086.

Experiments.—Determine the coefficient of expansion of the given liquid and of cubical expansion of the given solid.

Enter results thus :—

Weight of empty tube	5·06 gms.
Weight of tube full at 15°·5 . . .	11·58 „
„ „ „ 100°·6 . . .	11·32 „
Weight of liquid at 15·5	6·52 „
Weight expelled	·26 „
Coefficient of expansion relative to glass .	·000488
„ „ „ of glass . .	·000026
True coefficient of expansion . . .	·000514

Similarly for the second experiment,

38. The Constant Volume Air Thermometer. Determination of the Coefficient of Increase of Pressure per degree of Temperature of a Gas at constant Volume.

The air is contained in a closed flask or bulb, which can be heated to any required temperature. From this a tube, after being bent twice at right angles, passes vertically downwards to a reservoir of mercury, into one end of which a plunger is fitted. A second and longer vertical tube is also screwed into this reservoir. On the tube connecting the bulb with the reservoir is a mark, which should be as near the bulb as it can conveniently be.

By means of the plunger the level of the mercury in this tube is adjusted until it coincides with the mark, the bulb being kept at 0° C. by immersion in melting ice. The mercury at the same time moves in the other tube, and the difference of level of the two columns is measured by means of the kathetometer or of scales placed behind the tubes.

Let this difference be 5·62 cm., and, suppose the height of the barometer to be 75·38 cm., then the pressure on the enclosed gas is that due to a column of mercury 81 cm. in height.

It is of the greatest importance that the air in the bulb should be free from moisture. The bulb must, therefore, have been thoroughly dried and filled with dry air by the use of the three-way cock, drying tubes, and air-pump, as already described, (§ 16). In Jolly's air-thermometer the three-way cock is permanently attached to the tube which connects the bulb with the reservoir.

The bulb is next immersed in a vessel of water which is made to boil, or, better still, in the steam from boiling water. The mercury is thus forced down the tube connected with the bulb, but by means of the plunger it is forced back until it is level again with the mark. At the same time it rises considerably in the other tube. When the water boils and the conditions have become steady, the

difference of level in the two tubes is again noted. Suppose we find it to be 34·92 cm., and that the barometer has remained unchanged.

The air is now under a pressure due to 110·3 cm. of mercury, its volume remaining the same. The increase of pressure, therefore, is that due to 29·3 cm., and the coefficient of increase per degree centigrade is

$$\frac{29\cdot3}{81 \times 100}, \text{ or } \cdot00362.$$

In this case it is important that the lower temperature should be 0° C., for to determine the coefficient we have to divide by the pressure at 0° C., and the difference between this and the pressure at the temperature of the room, say 15°, is too great to be neglected, as in the case of a solid or liquid.

If greater accuracy be required, allowance must be made for the expansion of the glass envelope, and for that portion of the air in the connecting tube which is not at the temperature of the bath.

The same apparatus can be used to determine the coefficient of increase of volume at constant pressure per degree of temperature.

In this case make the first observation as before, noting at the same time the height at which the mercury stands in the marked tube. Now heat the bulb. The air will expand and drive the mercury down the one tube and up the other, thus increasing at the same time the volume of the air and the pressure to which it is subject. By withdrawing the plunger the mercury is allowed to sink in both tubes. It must, however, sink faster in the one open to the external air, and after a time a condition will be reached in which the difference between the levels in the two is the same as it was originally. The air in the bulb is under the same pressure as previously, but its temperature has been raised to 100° C. and its volume altered. Observe the level of the mercury in the tube connected with the bulb. If

the bore of this tube be known, the change of level will give the increase of volume; hence, knowing the original volume, the coefficient of expansion per degree of temperature can be found.

Owing to the large amount of expansion produced in a gas by a rise of temperature of 100° C., a tube of large bore is required.

The method, however, as here described will not lead to very accurate results, for it is almost impossible to insure that the air in the bulb and that in the tube should be all at the same high temperature. In the first method, on the other hand, the portion of tube occupied by air can be made very small, so as easily to be jacketed along with the bulb and kept at an uniform high temperature.

The method is open to the objection that the air in contact with the mercury, and therefore the mercury itself, is at a different temperature in the two parts of the experiment. The density of the mercury, therefore, is different and the increment of pressure is not strictly proportional to the difference of level. This error will be but small.

We have described the experiment as if air was the gas experimented with. Any other gas which does not attack the mercury may be used.

Experiment.—Determine for the given gas the coefficient of the increase of pressure per degree of temperature at constant volume.

Enter results thus:—

Temperature of gas	Difference of level of mercury
0° C.	5·62 cm.
100° C.	34·92 cm.
Barometer	75·38 cm.
Coefficient of expansion	·00362

L. The Constant-pressure Air Thermometer. Determination of the Coefficient of Increase of Volume per degree of temperature of a Gas at constant pressure.

The measurement described in the latter part of the last section can be more accurately made in the following manner :—

A glass bulb some 5 to 8 cm. in diameter opens into a short glass tube, which ends in a fine point. The bulb is weighed. Suppose the weight to be w grammes. It is then filled with dry air, as in the last section, and placed in a hypsometer (§33), with its open end projecting through the cork at the top. The water in the hypsometer is heated, and after a time, when the bulb and air it contains have reached the temperature of the steam, the point is sealed off. If great accuracy is aimed at the bulb should, while the heating is in process, be connected with drying-tubes through a piece of indiarubber tubing. This will prevent the ingress of moisture during the heating.

The temperature of the steam will be known if the height of the barometer during the experiment be read. Let it be $t_1°$.

We have thus obtained a mass of air which at a temperature of $t_1°$ and at a pressure given by the barometric reading fills the bulb. Now cool down the bulb, and immerse it in some liquid of known density. When under the surface of the liquid break off the point of the tube, carefully preserving the broken fragments of glass. Since the bulb has cooled down the pressure inside has been reduced, and the atmospheric pressure forces the liquid inside.

The air in the bulb contracts. Adjust the bulb so that the surface of the liquid inside is level with that of the liquid in the vessel, and leave it for a time to take the temperature of this liquid. Let this be $t_2°$.

The pressure inside the bulb is that due to the enclosed

air together with the vapour pressure of the liquid used at $t_2°$, and the sum of the two is equal to the atmospheric pressure. If, then, the vapour pressure of the liquid be appreciable, the pressure due to the air inside is not the same as at the time of sealing. For this reason the liquid used is generally mercury, which has a very small vapour pressure. We may, however, employ water without serious error, and correct the result for the vapour pressure of the water. One method of doing this is as follows:—Note the vapour pressure of water at $t_2°$; let it be equal to d centimetres of water pressure. If $t_2°$ be 15°, d will be about 17 cm. Then depress the bulb in the water, keeping the point down, until the level of the water in the bulb is d cm. below that outside; in this position the pressure in the bulb exceeds that of the external air by that due to a column of water d cm. in height. But the pressure of the water vapour in the bulb is that due to d cm. of water; thus the pressure of the air in the bulb is the atmospheric pressure.

Thus the air in the bulb at the volume it occupies in this position and at a temperature of $t_2°$ will expand when heated to $t_1°$ so as to fill the bulb.

To determine the volume of air in the bulb, close the open end of the tube with the finger or with wax and lift the bulb out of the water; dry the outside of the bulb and weigh again, taking care to include the small fragments broken off. Let the weight be w_1 grammes. Then fill the bulb completely with water by treating it as in §37, and weigh again. Let the weight be w_2 grammes.

Then w_2-w gives the mass of water which fills the bulb, or, taking the density of water as unity, the number of cubic centimetres in the bulb; while w_1-w gives the number of cubic centimetres of water which were in the bulb when taken from the water-bath. The difference w_2-w_1 is therefore the number of cubic centimetres of air which were in the bulb at a temperature of $t_2°$ when taken out of the water-bath; and this volume of air expands at constant

pressure to $w_2 - w$ at $t_1°$. If, then, a be the coefficient of expansion at constant pressure, and v_0 the volume of the same mass of air at 0° C., we have

$$\frac{w_2 - w_1}{1 + a\,t_2} = v_0 = \frac{w_2 - w}{1 + a\,t_1}.$$

And from this equation we can find a.

On reducing we have

$$a = \frac{w_1 - w}{t_1(w_2 - w_1) - t_2(w_2 - w)}.$$

The calculation is a good deal simplified if t_2 is zero, for then $w_2 - w_1$ is the volume of air at 0° C., which expands, on being heated to $t_1°$, to $w_2 - w$. Thus $w_2 - w = (w_2 - w_1)(1 + a\,t_1)$.

This may be attained by using water cooled down to zero as the liquid in which the bulb is immersed, and this course has the additional advantage that the correction for vapour pressure is thereby greatly reduced, the vapour pressure of water at 0° being ·46 cm. of mercury, or about 6·4 cm. of water, and the error committed by entirely neglecting the correction will be only about $\frac{1}{200}$.

Experiment.—Determine coefficient of expansion of air at constant pressure.

Enter results thus:—

Height of barometer . . .	754·6 m.
Temperature of steam, t_1 . .	99·8°
Weight of empty bulb, w . .	16·54
Temperature of water-bath, t_2 .	15°
Weight of partly filled bulb, w_1 .	34·06
Weight of bulb when full, w_2 .	93·22
Coefficient of expansion . .	·00371

CHAPTER X.

CALORIMETRY.

By Calorimetry we mean the measurement of quantities of heat. There are three different units of heat which are employed to express the results: (1) the amount of heat required to raise the temperature of unit mass of water from 0°C. to 1°C.; (2) the amount of heat required to melt unit mass of ice; (3) the amount of heat required to convert unit mass of water at 100° into steam at the same temperature. Experiments will be detailed below (§ 39) by which the last two units may be expressed in terms of the first, which is generally regarded as the normal standard. Calorimetric measurements are deduced generally from one of the following observations: (1) the range of temperature through which a known quantity of water is raised, (2) the quantity of ice melted, (3) the quantity of water evaporated or condensed; or from combinations of these. The results obtained from the first observation are usually expressed in terms of the normal unit on the assumption that the quantity of heat required to raise a quantity of water through one degree is the same, whatever be the position of the degree in the thermometric scale. This assumption is very nearly justified by experiment. As a matter of fact, the quantity of heat required to raise unit mass of water from 99° C. to 100° C. is said to be 1·016 nornal units.

The results of the second and third observations mentioned above give the quantities of heat directly in terms of the second and third units respectively, and may therefore be expressed in terms of normal units when the relations between the various units have once been established.

39. The Method of Mixture.

Specific Heat.

In this method a known mass of the material of which the specific heat is required is heated to a known temperature, and then immersed in a known mass of water also at a known temperature. A delicate thermometer is immersed in the water, and the rise of temperature produced by the hot body is thereby noted. The quantity of heat required to produce a rise of temperature of 1° in the calorimeter itself, with the stirrer and thermometer, is ascertained by a preliminary experiment. We can now find an expression for the quantity of heat which has been given up by the hot body, and this expression will involve the specific heat of the body. This heat has raised the temperature of a known mass of water, together with the calorimeter, stirrer, and thermometer, through a known number of degrees, and another expression for its value can therefore be found, which will involve only known quantities. Equating these two expressions for the same quantity of heat, we can determine the specific heat of the material. Let M be the mass of the hot body, T its temperature, and C its specific heat; let m be the mass of the water, t its temperature initially, and θ be the common temperature of the water and body after the latter has been immersed and the temperature become steady; let m_1 be the quantity of heat required to raise the temperature of the calorimeter, stirrer, and thermometer 1°. This is numerically the same as the 'water equivalent' of the calorimeter. We shall explain shortly how to determine it experimentally.

The specific heat of a substance is the ratio of the quantity of heat required to raise the temperature of a given mass of the substance 1° to the quantity of heat required to raise the temperature of an equal mass of water 1°. If we adopt as the unit of heat the quantity of heat required to raise the temperature of 1 gramme of water 1°, then it

follows that the specific heat of a substance is numerically equal to the number of units of heat required to raise the temperature of 1 gramme of that substance through 1°.

The mass M is cooled from $T°$ to $\theta°$. The quantity of heat evolved by this is therefore

$$M C (T-\theta),$$

assuming that the specific heat is the same throughout the range. The water in the calorimeter, the calorimeter itself, the stirrer, and the thermometer are raised from $t°$ to $\theta°$; the heat necessary for this is

$$m(\theta-t)+m_1(\theta-t),$$

for m_1 is the heat required to raise the calorimeter, stirrer, and thermometer 1°, and the unit of heat raises 1 gramme of water 1°.

But since all the heat which leaves the hot body passes into the water, calorimeter, &c., these two quantities of heat are equal.

Hence

$$M C (T-\theta) = (m+m_1)(\theta-t)$$

$$\therefore\ C = \frac{(m+m_1)(\theta-t)}{M(T-\theta)} \quad . \quad . \quad . \quad . \quad (1)$$

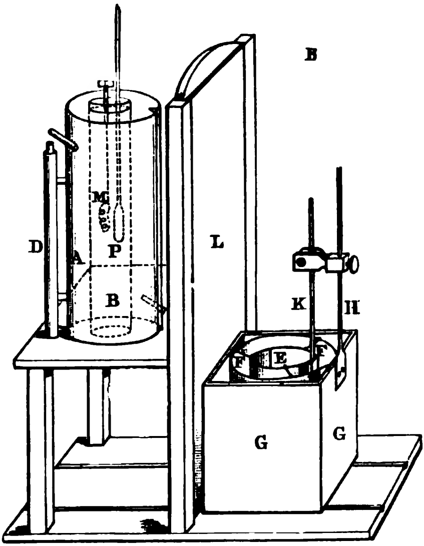
The reason for the name 'water equivalent' is now apparent, for the value found for m_1 has to be added to the mass of water in the calorimeter. We may work the problem as if no heat were absorbed by the calorimeter if we suppose the quantity of water in it to be increased by m_1 grammes. The quantity m_1 is really the 'capacity for heat' of the calorimeter, stirrer, and thermometer.

We proceed to describe the apparatus, and give the practical details of the experiments.

The body to be experimented on should have considerable surface for its mass; thus, a piece of wire, or of thin sheet, rolled into a lump is a convenient form. Weigh it,

and suspend it by means of a fine thread in the heater. This consists of a cylinder, A (fig. 20), of sheet copper, closed at both ends, but with an open tube, B, running down through the middle. Two small tubes pass through the outer casing of the cylinder; one is connected with the boiler, and through this steam can be sent; the other communicates with a condenser to remove the waste steam.

FIG. 20.



The cylinder can turn round a vertical axis, D, which is secured to a horizontal board, and the board closes the bottom end of the central tube. A circular hole is cut in the board, and by turning the cylinder round the axis the end of the tube can be brought over this hole. The upper end of the tube is closed with a cork, which is pierced with two holes; through the one a thermometer, P, is fixed. and through the other passes the string which holds the mass M. The thermometer bulb should be placed as close as possible to M.

The steam from the boiler is now allowed to flow through the outer casing, raising the temperature of the mass M; the cylinder is placed in such a position that the

lower end of the tube in which M hangs is covered by the board. The temperature in the enclosed space will rise gradually, and it will be some time before it becomes steady, After some considerable interval it will be found that the thermometer reading does not alter, the mercury remaining stationary somewhere near 100°.

Note the reading; this is the value of T in the above equation (1).

While waiting for the body to become heated the operation of finding the water equivalent of the calorimeter may be proceeded with.

The calorimeter consists of a copper vessel, E, which is hung by silk threads inside a larger copper vessel, F. The outside of the small vessel and the inside of the large one should be polished, to reduce the loss of heat by radiation.

This larger vessel is placed inside a wooden box, G, to the bottom of which slides are fixed. These slides run in grooves in the wooden baseboard of the apparatus, and the box can be pushed easily under the board to which the heater is attached, being just small enough to slide under it. When the box is thus pushed into position the calorimeter is under the hole in the board which has already been mentioned; and if the cylinder be turned so that its inner tube may come over this hole, the heated body can be dropped directly into the calorimeter. L is a sliding screen, which serves to protect the calorimeter from the direct radiation of the heater, and which must be raised when it is required to push the calorimeter under the heater.

A brass rod, H, is attached to the back of the box G, and carries a clip in which a delicate thermometer, K, is fixed. The thermometer bulb is in the calorimeter, a horizontal section of which is a circle with a small square attached to it; the thermometer is placed in the square part, and is thus protected from injury by the mass M when it is immersed, or by the stirrer. The stirrer is a perforated disc of copper, with a vertical stem. A wooden cover with a slot in it,

through which the stirrer and thermometer pass, fits over the box G. There is a long vertical indentation in the heater A, and the upper part of the thermometer can fit into this when the box G is pushed into position under the heater. Care must be taken to adjust the clip and thermometer so that they will come into this indentation.

In determining the water equivalent it is important that the experiment should be conducted under conditions as nearly as possible the same as those which hold when the specific heat itself is being found.

Let us suppose that it has been found, either from a rough experiment or by calculation from an approximate knowledge of the specific heat of the substance, that if the calorimeter be rather more than half full of water the hot body will raise its temperature by about 4°. Then, in determining the water equivalent, we must endeavour to produce a rise in temperature of about 4°, starting from the same temperature as we intend to start from in the determination of the specific heat.

Weigh the calorimeter. Fill it rather more than half full of water, and weigh it again. Let m' be the increase in mass observed; this will be the mass of water in the calorimeter; let t' be the temperature of the water. The experiment is performed by adding hot water at a known temperature to this and observing the rise in temperature. If the hot water be poured in from a beaker or open vessel its temperature will fall considerably before it comes in contact with the water in the calorimeter. To avoid this there is provided a copper vessel with an outer jacket. The inner vessel can be filled with hot water, and the jacket prevents it from cooling rapidly. A copper tube with a stopcock passes out from the bottom of the vessel, and is bent vertically downwards at its open end. This tube can pass through the slot in the covering of the wooden box G close down to the surface of the water in the calorimeter. A thermometer inserted in a cork in the top of the vessel

serves to read the temperature of the hot water. For the present purpose this may be about 30°. It is not advisable that it should be much higher.

Turn the tap of the hot-water vessel, and let some water run into a beaker or other vessel; this brings the tube and tap to the same temperature as the water that will be used. Turn the tap off, and place the calorimeter, which should be in the wooden box, with the thermometer and stirrer in position, underneath the tube, and then turn the tap again, and allow the hot water to run into the calorimeter rather slowly. The temperature of the water in the calorimeter rises. When it has gone up about 3° stop the hot water from flowing. Stir the water in the calorimeter well; the temperature will continue to rise, probably about 1° more; note the highest point which the mercury in the thermometer attains. Let the temperature be θ'. Note the temperature of the hot water just before and just after it has been allowed to flow into the calorimeter; the two will differ very little; let the mean be T'. This may be taken as the temperature of the hot water. Weigh the calorimeter again; let the increase in mass be M' grammes. This is the mass of hot water which has been allowed to flow in, and which has been cooled from T' to θ'. The heat given out is

$$M'(T'-\theta').$$

It has raised the temperature of the calorimeter, stirrer, &c., and a mass m' of water from t' to θ'. The heat required to do this is

$$m'(\theta'-t')+m_1(\theta'-t'),$$

and this must be equal to the heat given out by the hot water in cooling, m_1 being, as before, the required water equivalent.

Hence

$$m'(\theta'-t')+m_1(\theta'-t')=M'(T'-\theta')$$

and

$$m_1=\frac{M'(T'-\theta')}{\theta'-t'}-m'.$$

In doing this part of the experiment it is important that the apparatus should be under the same conditions as when determining the specific heat. The measurements should be made, as we have said, with the calorimeter in the box, and the initial and final temperatures should be as nearly as may be the same in the two experiments. The error arising from loss by radiation will be diminished if the experiment be adjusted so that the final temperature is as much above that of the room as the initial temperature was below it.

Having found the water equivalent of the calorimeter we proceed to determine the specific heat of the substance. The mass of the empty calorimeter is known; fill the calorimeter with water from one-half to two-thirds full; weigh it, and thus determine m, the mass of the water. Replace the calorimeter in the wooden box on the slides of the apparatus, and take the temperature of the water two or three times to see if it has become steady; the final reading will be the value of t. Note also the temperature of the thermometer P; when it is steady raise the slide L, and push the box G under the heater, turning the latter round the axis D until the tube B is over the hole in the stand. Then by loosening the string which supports it drop the mass M into the calorimeter. Draw the box back into its original position, and note the temperature with the thermometer K, keeping the water well stirred all the time, but being careful not to raise the substance out of the water. When the mercury column has risen to its greatest height and is just beginning to recede read the temperature. This gives the value of θ, the common temperature of the substance and the water.

Thus all the quantities in the equation for the specific heat have been determined, and we have only to make the substitution in order to find the value.

The same apparatus may be used to determine the specific heat of a liquid, either by putting the liquid into a very thin vessel, suspending it in the heater, and proceeding in the same way, allowing, of course, for the heat emitted by the

vessel, or by using the liquid instead of water in the calorimeter, and taking for the mass M a substance of known specific heat. Thus C would be known, and if m be the mass of the liquid, c its specific heat, we should have

$$\text{M C}(\text{T}-\theta) = m\,c\,(\theta - t) + m_1(\theta - t).$$

Hence

$$c = \frac{\text{M C}(\text{T}-\theta)}{m(\theta - t)} - \frac{m_1}{m},$$

t, θ, and T having the same meaning as above.

Experiment.—Determine by the method of mixture the specific heat of the given substance, allowing for the heat absorbed by the calorimeter &c.

Enter results thus :—

Name and weight of solid. Copper	32·3 gms.
Temp. of solid in the heater . .	99·5 C.
Weight of water	65·4 gms.
Initial temperature of water .	12·0 C.
Common temp.	15·7 C.
Water equivalent of calorimeter &c.	2·0

Specific Heat = ·092.

Latent Heat of Water.

DEFINITION.—The number of units of heat required to convert one gramme of ice at 0° C. into water, without altering its temperature, is called the latent heat of water.

A weighed quantity of water at a known temperature is contained in the calorimeter. Some pieces of ice are then dropped in and the fall of temperature noted. When the ice is all melted the water is weighed again, and the increase gives the mass of ice put in. From these data, knowing the water equivalent of the calorimeter, we can calculate the latent heat of the water.

The ice must be in rather small pieces, so as to allow it to melt quickly. It must also be as dry as possible. We may attain this by breaking the ice into fragments and putting it piece by piece into the calorimeter, brushing off

from each piece as it is put in all traces of moisture with a brush or piece of flannel.

The ice may be lifted by means of a pair of crucible tongs with their points wrapped in flannel. These should have been left in the ice for some little time previously, to acquire the temperature of 0° C.

Another method is to put the ice into a small basket of fine copper gauze and leave it to drain for a few moments, while the ice is stirred about with a glass rod, previously cooled down to 0° C. by being placed in ice. The basket is put into the calorimeter with the ice. The water equivalent of the basket must be allowed for, being determined from its mass and specific heat.

Care must be taken not to put so much ice into the water that it cannot all be melted.

The formula from which the latent heat is found is obtained as follows: Let M be the mass of water initially, τ its temperature; let m be the mass of ice put in, which is given by the increase in mass of the calorimeter and contents during the experiment; let θ be the temperature when all the ice is melted, m_1 the water equivalent of the calorimeter, and L the latent heat.

Then the heat given out by the water, calorimeter, etc., in cooling from τ to θ is

$$(\mathrm{M}+m_1)(\tau-\theta).$$

This has melted a mass m of ice at 0° C., and raised the temperature of the water formed from 0° to θ°.

The heat required for this is

$$m\mathrm{L}+m\theta,$$

$$\therefore m\mathrm{L}+m\theta=(\mathrm{M}+m_1)(\tau-\theta),$$

$$\therefore \mathrm{L}=\frac{\mathrm{M}+m_1}{m}(\tau-\theta)-\theta.$$

The temperature of the water used should be raised above that of the room before introducing the ice, and noted just before the ice is immersed. It is well to take a quantity

of ice such that the temperature of the water at the end of the experiment may be as much below that of the room as it was above it initially. We may calculate this approximately, taking the latent heat of ice as 80.

Thus suppose we have 45 grammes of water at 20°, and that the temperature of the room is 15°. Then the water is to be cooled down to 10°, or through 10°.

Thus the heat absorbed from water will be 450 units.

Let us suppose we have x grammes of ice. This is melted, and the heat absorbed thereby is $80 \times x$. It is also raised in temperature from 0° to 10°, and the heat absorbed is $x \times 10$.

$$\therefore 80x + 10x = 450.$$

$$x = \frac{450}{90} = 5.$$

Thus we should require about 5 grammes of ice.

(If in practice we did not know the latent heat of the substance experimented upon at all, we should for this purpose determine it approximately, then use our approximate result to determine the right quantity of the substance to employ in the more accurate experiment.)

Experiment.—Determine the latent heat of ice.

Enter results thus :—

Quantity of water . . .	47 gms.
Temp. water	20°
Mass of ice	5 gms.
Common temp. . . .	10°
Water equivalent of calorimeter	3·5

Latent heat of water, 79.

Latent Heat of Steam.

DEFINITION.—The heat required to convert a gramme of water at 100° C. into steam without altering its temperature is called the latent heat of steam at 100° C.

Steam from a boiler is passed in to a weighed quantity of water at a known temperature for a short time, and the

rise of temperature noted. The contents of the calorimeter are again weighed, and the increase in the weight of water gives the steam which has passed in. From these data we can calculate the latent heat of the steam by means of a formula resembling that of the last section.

Let M be the mass of water in the calorimeter, m_1 the water equivalent, τ the temperature initially, θ the common temperature after a mass m of steam has been passed in, L the latent heat of steam.

The amount of heat given out by the steam in condensing to water, which is then cooled from 100° to $\theta°$, is

$$Lm + m\,(100-\theta).$$

The heat required to raise the calorimeter with the water from τ to θ is

$$(M+m_1)\,(\theta-\tau),$$

and these two quantities of heat are equal.

Hence

$$L = \frac{(M+m_1)\,(\theta-\tau)}{m} - (100-\theta).$$

In practice various precautions are necessary.

The steam coming directly from the boiler carries with it a large quantity of water, and moreover, in its passage through the various tubes some steam is condensed. Thus water would enter the calorimeter with the steam, and produce considerable error in the result. This is avoided by surrounding all the tubes with jackets and drying the steam. To dry the steam a closed cylindrical vessel is employed, with two tubes entering it at the top and bottom, and a hole at the top, which can be closed by a cork carrying a thermometer. Inside this is a spiral of thin copper tubing; the spiral emerges at the top where a glass nozzle is attached by india-rubber tubing, and terminates at the bottom in a stop-cock.

The continuation of the stop-cock and the tube at the top of the cylinder are attached by india-rubber tubing to the

boiler; the tube at the bottom is connected with a condenser. Thus, on putting the top of the cylinder into connection with the boiler, a current of steam passes through the copper cylinder, raising it and the spiral inside to the temperature of 100°.

If now we put the lower end of the spiral into communication with the boiler, the steam passes through the spiral, emerging through the nozzle. The spiral being kept hot at 100°, the steam inside it is freed from moisture and emerges from the nozzle in a dry state.

The nozzle is connected with the spiral by means of a short piece of india-rubber tubing. This should be surrounded with cotton wool; the cylindrical heater is placed inside a wooden box, and surrounded with wool, or felt, or some other non-conducting substance.

Sometimes it is more convenient to use the boiler itself to dry the steam; in this case the copper spiral is placed inside the boiler, from which one end emerges. The other end of the spiral inside the boiler is open above the level of the water. The steam, before emerging from the boiler, has to circulate through the spiral, and this dries it thoroughly.

The calorimeter may conveniently take the form of a flask, or pear-shaped vessel, of thin copper, supported by silk threads inside another copper vessel. Its water equivalent must be determined in the same way as has been described in the section on specific heat (p. 276). In doing this, however, it must be remembered that the steam will probably raise the water to a temperature considerably higher than is the case in the determination of the specific heat of a metal. In like manner the temperature of the hot water used in finding the water equivalent should be considerably higher than that which was found most suitable in the previous experiments; it may with advantage be some 60° to 70°. Now water at this high temperature may cool considerably in being poured into the calorimeter, and care must be used to prevent loss of heat from this as far as possible.

In allowing the steam to pass into the calorimeter the following method may be adopted:

See that the steam passes freely from the nozzle, and note the temperature of the water in the calorimeter; pinch the india-rubber tube connecting the nozzle with the calorimeter for an instant, and immerse one end of the nozzle under the water, then allow the steam to flow until the temperature has risen about 20°. Raise the nozzle until its end is just above the level of the water in the calorimeter; again pinch the india-rubber tubing, stopping the flow of steam, and remove the calorimeter; note the highest point to which the temperature rises; this will be the value of θ, the common temperature.

By pinching the tube as described above, the steam is prevented from blowing over the outer surface of the calorimeter. If, on the other hand, the tube be pinched and the flow stopped while the nozzle is under the water, the steam in the nozzle at the moment will be condensed, and the atmospheric pressure will drive some water up into the nozzle, and this will produce error. If the calorimeter is small there is some danger that the steam from the nozzle may flow directly on to the thermometer, and thus raise its temperature more than that of the surrounding water. This may be avoided by the use of a calorimeter of sufficient size. Another method of avoiding this error, and one which will lead to more accurate results, is the following, which has, however, the disadvantage of requiring more elaborate apparatus.

The calorimeter contains a spiral tube of thin copper, ending in a closed vessel of the same material. This is completely surrounded by water, and the dry steam is passed through it instead of into the water. The water in the calorimeter is kept well stirred, and the heat given out by the steam in condensing is transmitted through the copper spiral and vessel to the water. The rise of temperature is noted as before, and when the temperature reaches its highest point,

that is taken as the common temperature of the water, spiral, and calorimeter. The heat absorbed by the spiral and vessel is determined with the water equivalent; the quantity of water in the spiral at the end gives the mass of steam condensed. (See Regnault's paper on the 'Latent Heat of Steam.' *Mémoires de l'Academie*, T. XXI.)

The calculation is proceeded with in the usual way.

Experiment.—Determine the latent heat of steam.

Enter the results as below:—

Weight of water in calorimeter	221·3 gms.
Temp.	14°·5 C.
Weight of steam let in	10·4 gms.
Temp. of steam given by thermometer in heater	100°
Common temp. of mixture	41° C.
Water equivalent of cal.	10·9
Latent heat of steam	532·7

40. The Method of Cooling. To determine the Specific Heat of a Liquid.

A known weight of the liquid is put into a copper vessel with a thermometer. This is hung by means of silk threads, like the calorimeter, inside another copper vessel which is closed by a lid with a cork in it supporting the thermometer. The exterior vessel is kept in a large bath of water at a known temperature, the bath being kept well stirred. It is intended to be maintained at the temperature of the room throughout the experiment; the bath is simply to ensure this. A small stirrer should pass through the cork which holds the thermometer, to keep the liquid well stirred. The outer surface of the inner vessel and the inner surface of the outer should be coated with lampblack.

The liquid is heated up to, say, 70° or 80°, and then put into the calorimeter.

Allow the liquid to cool, and note the intervals taken by it to cool, through, say, each successive degree. If the

rate of cooling is too rapid to allow this to be done, note the intervals for each 5° or 10°, and calculate from these observations the mean rate of cooling for the range experimented on, say from 70° to 30°.

Suppose we find that, on the average, it cools 3° in a minute. Then, if the liquid weigh 25 grammes and its specific heat be c, the quantity of heat which leaves it in one minute is $25 \times 3 \times c$.

Now empty the liquid out from the calorimeter and perform a similar experiment with water instead. The water should fill the calorimeter to the same level, and be raised to the same temperature as the liquid previously used.

Let us now suppose that there are 32 grammes of water, and that the temperature of the water falls through ·9 of a degree in one minute; thus the quantity of heat which escapes from the water per minute is $32 \times \cdot 9$ units.

The quantity of heat radiated from one surface at a given temperature to another at a constant lower temperature depends solely on the nature and material of the surfaces and the temperature of the warmer surface.[1]

In the two experiments described above, the surfaces are of the same nature; thus the rate at which heat escapes must be the same for the two experiments at the same temperatures,

$$\therefore\ 25 \times 3 \times c = 32 \times \cdot 9,$$
$$c = \cdot 384.$$

We can get the result required from the observations more quickly thus:–

Observe the time it takes the temperature to fall, say, from 60° to 55° in the two cases; let it be t_1 minutes and t_2 minutes respectively.

Then the fall of temperature per minute in the two cases respectively is $5/t_1$ and $5/t_2$.

The amount of heat which is transferred in the first case

[1] See Garnett, *Heat*, ch. ix. Deschanel, *Natural Philosophy*. p. 399, &c.

is $5c\text{M}_1/t_1$, and in the second it is $5\text{M}_2/t_2$, M_1, M_2 being the masses of the liquid and the water respectively. Thus

$$\frac{5c\,\text{M}_1}{t_1}=\frac{5\text{M}_2}{t_2}$$

and

$$c=\frac{\text{M}_2}{\text{M}_1}\,\frac{t_1}{t_2}.$$

The effect of the vessel has hitherto been entirely neglected. Let k be its specific heat and m its mass, then in the first case the heat lost is

$$5\,(km+c\text{M}_1)\,/t_1,$$

in the second it is

$$5\,(k\,m+\text{M}_2)\,/t_2.$$

Thus

$$c=\frac{\text{M}_2}{\text{M}_1}\,\frac{t_1}{t_2}+\frac{km}{\text{M}_1}\left\{\frac{t_1}{t_2}-1\right\}.$$

Instead of calculating the quantity km, we may find by experiment the water equivalent of the vessel and thermometer and use it instead of km.

Experiment.—Determine the specific heat of the given liquid.

Enter results thus :—

Weight of calorimeter . . .	15·13 gms.
Weight of water	10·94 "
Weight of liquid	13·20 "

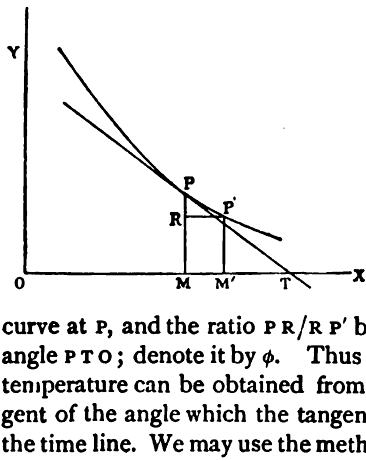
Range of Temperature	Time of cooling of Liquid	Time of cooling of Water	Specific heat uncorrected
70 – 65	115 secs.	130 secs.	·733
65 – 60	125 "	140 "	·734
60 – 55	150 "	170 "	·733
55 – 50	167 "	190 "	·736

Mean specific heat (uncorrected for calorimeter)	= ·734
Correction for calorimeter	= – ·013
Specific heat of liquid .	= ·721

M. Method of Cooling. Graphic Method of Calculation.

We may also determine the rate of cooling of a body by a graphical construction in the following manner:—Observe the temperature of the body at equal intervals of time, say every 30″, and then plot a curve, taking the time for abscissa and the temperatures for ordinates. The curve will take the form of that given in fig. xxii. Let P M, P′M′ be ordinates at two times represented by O M and O M′; draw P′ R parallel to O M. Then in the interval M M′ the temperature falls by P R; the average rate of change of temperature during that interval is P R/R P′. When the time is sufficiently small, P P′ coincides with P T, the tangent to the curve at P, and the ratio P R/R P′ becomes the tangent of the angle P T O; denote it by ϕ. Thus the rate of cooling at any temperature can be obtained from the curve, being the tangent of the angle which the tangent to the curve makes with the time line. We may use the method to determine the radiation between two lamp-black surfaces, one of which is kept at a constant temperature while the other cools down. In any such experiment we must recollect there is very great loss of heat by convection, which we cannot avoid, so that the numbers obtained are not a true measure of the radiation. We take as the two surfaces those of the calorimeter, already described, and its enclosing vessel. The latter being in a large vessel of water remains constant in temperature. The calorimeter may take the form of a narrow rectangular vessel, having considerable surface for its volume. Let the surface be measured, and let it be A sq. cm. Place a weighed quantity of water in the calorimeter, and let M be

FIG. xxii.



the mass of water together with the water equivalent of the calorimeter. The calorimeter should have a closely fitting cover, with two holes for the stirrer and thermometer respectively, and the outer case should also be covered. Determine the temperature at equal intervals of time, keeping the water well stirred, and by plotting the results find the rate of fall of temperature. In drawing the curve it may be more convenient to change the scale, and to represent n seconds by one horizontal division and m degrees Centigrade by one vertical division. In that case

$$\tan \phi = \frac{n}{m} \times \text{Rate of fall of temperature};$$

$$\therefore \text{ Rate of fall of temperature} = \frac{m}{n} \tan \phi.$$

Thus the heat lost per second by the water and calorimeter in cooling is $\text{M} \times \frac{m}{n} \tan \phi$, water grm degrees per second. And if R is the excess of the radiation per unit area emitted by the hot calorimeter over that received from the enclosure,

$$\text{R} \cdot \text{A} = \text{M} \frac{m}{n} \tan \phi,$$

$$\therefore \text{R} = \frac{\text{M}}{\text{A}} \frac{m}{n} \tan \phi.$$

We may also find the radiation-difference for a difference of temperature of 1° by dividing R by the excess of the temperature of the calorimeter over that of the enclosure, and thus test Newton's law of cooling.

Experiment.—Plot a curve of cooling for the given calorimeter, and determine from your results the radiation per unit area between the surfaces at various temperatures.

Enter results thus:—

Temperature of outer bath	15°
Area of calorimeter, A	130·3 sq. cm.
Mass of water + water equivalent, M	86·8 gm.
m . . . 1	n . . . 24

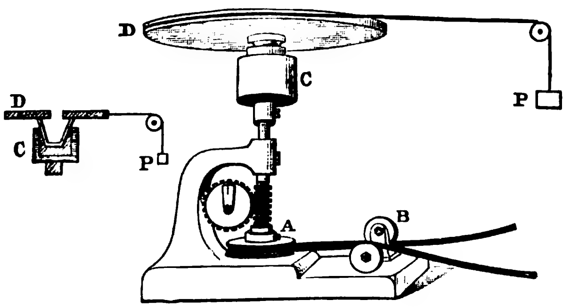
$$\text{R} = {\cdot}0276 \tan \phi.$$

Temperature of Calorimeter	Tan ϕ	R	Difference of Temperature	Value of R per 1°
90°	·94	·02596	75°	·000346
80°	·65	·01794	65°	276
70°	·54	·01491	55°	271
60°	·44	·01215	45°	270
50°	·33	·00912	35°	260

N. Determination of the Mechanical Equivalent of Heat.

The apparatus (fig. xxiii) consists of a strong casting, supporting a vertical spindle which works in bearings, and

FIG. xxiii.



which can be driven by a large hand-wheel. There is a driving-pulley A on the axle, and near it two small pulleys B for guiding the driving-cord to the hand-wheel. The cord must pass over the top of the lower wheel to the bottom of the hand-wheel, and under the higher wheel to the top of the hand-wheel.

Above the driving-pulley is fixed a screw, which gears into a cog-wheel having 100 teeth, so that the cog-wheel advances one tooth for each revolution of the axle. An index is fixed so that we can tell when 100 revolutions have been completed. At the top of the spindle is fixed a cast-iron cup C, lined with cork. The cup is shown in section in the figure. Into the hollow in the cork there fits

tightly a thin brass vessel in the shape of a hollow truncated cone, and within this again fits another brass vessel of a similar shape. The last vessel is provided with two pegs, which fit into a horizontal wooden wheel D, so that when the wheel is turned the vessel is turned also. A string is wound round the edge of the wooden wheel, passes over a smooth pulley, and then supports a weight P. If the apparatus is left to itself, this weight P will fall and turn the wheel and inner cup round. But if, by means of the hand-wheel, we cause the spindle to rotate in the opposite direction, it will be possible, by turning at the right speed, to keep the weight P supported so that it does not fall. The two brass cups now rub one against the other, and heat is produced. We must now calculate the work spent on friction in each revolution of the spindle when the weight P is just supported.

Let r cm. be the radius of the wheel,
P grammes the mass of the weight,
g=981 cm. per sec. per sec.=acceleration of gravity.

When the weight is supported the tension of the string is Pg dynes.

The work spent on friction is the same whether the outer cup is in motion and the inner one at rest, or whether the inner cup is in motion and the outer one at rest. In this case the work done each revolution would be $Pg \times 2\pi r$ ergs, since $2\pi r$ would be the distance through which P would have to fall in order to turn the wooden wheel through one revolution. If the spindle makes n revolutions, the whole work spent on friction is

$$W = n \,.\, Pg \,.\, 2\pi r \text{ ergs.}$$

Now let m be the mass of the two brass vessels, M the mass of water placed inside the inner vessel, c the specific heat of brass. Then the brass and water together are equivalent to $M + cm$ grammes of water. If during n turns of the spindle the temperature is raised by θ degrees, the

number of 'water-gramme-degrees' communicated to the water and brass is given by

$$H = \theta (M + m c).$$

Let J be the mechanical equivalent of heat, i.e. the work in ergs that must be spent in order to produce one 'water-gramme-degree' of heat. Then in producing H 'water-gramme-degrees' we must expend J H ergs.

$$\therefore J H = W,$$

$$\text{or } J = \frac{W}{H} = \frac{n . P g . 2 \pi r}{\theta (M + m . c)}.$$

Practical Details.—Fill the inner vessel with water up to about 1·5 cm. of the top.

It will be advantageous to cool the water and vessels to a temperature of about 10° C. lower than that of the room. Work the apparatus till the temperature has risen by about 20° C., so that it is about 10° C. above the temperature of the room at the end of the experiment.

If the water be not cooled, a correction must be made as follows:—

When the wheel stops, note the temperature of the water, and also note the time t during which the wheel was being turned. Determine the fall of temperature (when the apparatus is at rest) which takes place during a time t. Let it be ϕ. Then correct for the loss by radiation and convection by writing $\theta + \frac{1}{2} \phi$ instead of θ in the formula.

The formula would then stand

$$J = \frac{n . P g . 2 \pi r}{(\theta + \frac{1}{2} \phi) (M + m . c)}.$$

But it is much better, if possible, to make the temperature at the end as much above that of the room as it was below it at the beginning, for in this case no correction is necessary.

If the temperature of the water is *above* the temperature

of the room on starting the experiment, the correction for loss by radiation, &c., may be made as follows :—

Let the rate of falling of temperature at the initial and final temperatures be observed. Take the mean of these rates, and multiply this by the time the experiment has lasted This product must be used instead of $\frac{1}{2}\phi$ above.

Two observers are required, one to turn the hand-wheel, and the other to note the revolutions of the cog-wheel and the temperature.

Make a note of the *time* of the beginning of the experiment, and also the time at which each successive 100 turns of the spindle are completed. This will be a great check on accuracy of counting.

Stir the water all the time, by moving the stirrer gently up and down. Do not splash. Place two or three (not six or seven) drops of oil on the inside of the outer vessel, and place the inner vessel in it before the oil has run down to the bottom of the vessel.

Hang a sensitive thermometer from a clip, so as to pass through the hole in the centre of the wooden wheel, and so as to have its bulb not quite touching the bottom of the vessel.

Place weights symmetrically on the wooden wheel so as to produce enough friction to raise the weight P when the wheel is worked at a convenient speed.

On starting, the cones slip with much greater difficulty than when once started.

The following plan is convenient :—Fasten a string to P, and attach the other end to a weight Q, which rests on the floor. On starting, P will not be sufficient to keep the inner cup from revolving, and Q will come into play; as soon as the statical friction has been overcome Q will fall to the ground again, and the driving-wheel must then be so manipulated that the string PQ is always slack. Great care must be taken that the string supporting P is always a tangent to the wheel.

The mass of P should be about 200 grammes.

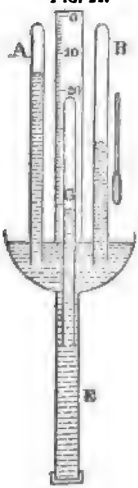
CHAPTER XI.

PRESSURE OF VAPOUR AND HYGROMETRY.

41. Dalton's Experiment on the Pressure of Mixed Gases.

To shew that the Maximum Pressure produced by a Vapour in a given Space depends on the Temperature and not on the Presence of Air or other Vapours in that Space.

FIG. 21.



The apparatus and experiment are described in Garnett's 'Heat.'

A, B, G, fig. 21, are three barometer tubes. A and B are to be filled with mercury and inverted over the cistern of mercury D E. G contains some air above the mercury.

We require, first, to explain how to fill the tubes with mercury.

They must first be cleaned by washing out with dilute acid, and then dried by being repeatedly exhausted with the air-pump and filled with air that has passed through chloride of calcium tubes. This can be done by means of a three-way cock, as already described (§ 16).

Having cleaned and dried a tube, we may proceed to fill it.

For this purpose it is connected with a double-necked receiver which contains enough mercury to fill the tube, the other neck of the receiver being connected with the air-pump, and the tube and receiver are exhausted by working the air-pump. Then by raising the end of the tube to which the receiver is attached and tilting the receiver the mercury is allowed to flow into the empty tube from the receiver. We are thus able to fill the tube with mercury free from air without its being necessary to boil the mercury.

The three tubes should be filled in this way and inverted

over the mercury cistern. A convenient arrangement for the latter is a hemispherical iron basin screwed on to the end of a piece of iron tubing, the lower end of the tubing being closed.

Connect the open end of G by means of a bent piece of small-sized glass tubing with the drying tubes, and allow a small quantity of dry air to flow in. The amount of air introduced should be such as to cause the mercury in G to rise to about half the height that it reaches in A and B. The quantity can be regulated by pinching the india-rubber tube which connects G with the drying tubes.

Adjust in a vertical position behind the three tubes a scale of millimetres, and hang up close to them a thermometer. Place a telescope at some distance off, so as to read on the millimetre scale the height at which the mercury columns stand and also the thermometer. The tube G should be so placed that it can be depressed into the iron tubing below the cistern.

Mark the height at which the mercury stands in G by means of a piece of gummed paper fastened to the tube.

Read on the millimetre scale the heights of A, B, and G, above the level of the mercury in the cistern.

Suppose the readings are—

A	B	G
765	765	524

Introduce, by the aid of a pipette with a bent nozzle, a little ether into B and G, putting into each tube just so much that a small quantity of the liquid rests above the mercury.

The mercury in B will fall. The amount of fall will depend on the temperature. Let us suppose that the new reading in B is 354 mm., then the mercury has fallen through $765 - 354$ mm.; thus the ether exerts a pressure equivalent to that of 411 mm. of mercury.

The mercury in G will fall also, but not by so much as that in B, for the pressure in G is the pressure of the ether

vapour together with that of the contained air; and as the mercury falls, the volume of the contained air increases and its pressure consequently decreases.[1]

Now lower the tube G in the cistern until the level of the mercury in G just comes back again to the paper mark. The volume of the contained air is now the same as before, therefore so also is its pressure. The depression of the mercury column in G below its original height is due therefore to the pressure of the ether vapour. Now read the height of G on the scale; it will be found to be about 113 mm. The column in G, therefore, has been depressed through 524—113 mm., or 411 mm. Thus B and G are depressed through equal amounts provided that the volume of air in G is allowed to remain the same.

The assumption has been made that the temperature remains constant during the experiment. This will not be far from the truth in the laboratory, provided that the readings are taken from a distance so as to avoid the heating effects of the body; if necessary, a correction must be applied for a change in temperature.

Having made these measurements, depress B into the iron tube; it will be found that the consequence is simply to increase the amount of condensed liquid above the surface of B without altering the height of that surface.

The difference between the heights of the columns in A and B gives in millimetres of mercury the maximum pressure which can be exerted by ether vapour at the temperature of the laboratory.

Experiment.—Determine the maximum pressure exerted by the vapour of ether at the temperature of the laboratory, and shew that it is independent of the presence of air.

Enter results thus:—

Height of mercury in A . . . 765 mm.

[1] The presence of the air in G retards the evaporation of the ether; a considerable time must therefore be allowed for the mercury to arrive at its final level.

Height of mercury in B—		
initially	765	mm.
after introduction of ether . .	354	"
Pressure of ether vapour	411	"
Height of mercury in G—		
initially	524	"
after introduction of ether . .	113	"
Pressure of ether vapour	411	"
Temperature 15°·5 throughout.		

The volumenometer described in § 26 will afford us another means of testing Dalton's law. Introduce a small quantity of water or other liquid into the bulb E (fig. 16), and screw it on. As the water evaporates the pressure will increase and the level of the mercury change. When it has become steady read the level in both tubes, and note the height of the barometer. Alter the position of the tube A and take another reading, and thus obtain a series of corresponding values of volume and pressure. Let us suppose the volume of the flask is known, so that v, the actual volume occupied by the air, can be found. Allowance must be made for the volume occupied by the water, which of course changes slightly; this is easily done by weighing the flask empty, then with the water, at the beginning and end of the experiment. These last two will differ, but very slightly, owing to the evaporation. From the mean of the two weights and the weight of the empty flask we can obtain the average volume of the water, which will be sufficient for our present purpose.

Write down the reciprocals of the observed values of v, and then plot a curve with these reciprocals as abscissæ and the observed pressures as ordinates. If Dalton's law is true, or has the same actual error at all pressures, the curve will be found to be a straight line, as A B in fig. xxiv, cutting the axis of V in B. Let P M be any ordinate, and through O draw O Q parallel to A B, cutting P M in Q. Let O B $= p_0$, then

$$\text{OM} = 1/\text{V},\ \text{PM} = p,$$
$$\text{QM} = p - p_0.$$

Now, from the figure,

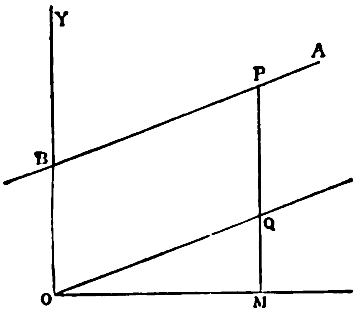
$$\frac{\text{QM}}{\text{OM}} = \tan \text{QOM} = k, \text{ say, } k \text{ being some constant;}$$

$$\therefore\ (p - p_0)\,\text{V} = \text{constant}.$$

Thus if we diminish the observed pressure by a constant quantity p_0, the product of the difference and the volume is constant.

The observed pressure p is therefore the sum of a con-

FIG. xxiv.



stant pressure p_0 and a pressure p_1, which satisfies Boyle's law—i.e. the actual pressure is that due to the air obeying Boyle's law together with a constant pressure, that of the aqueous vapour saturating the space at the given temperature. On varying the temperature the same law will be found to hold, but the pressure p_0 will be different for different temperatures; and if Dalton's law is true, the values of p_0 for different temperatures will correspond exactly with those given in Regnault's table of saturation-pressures of aqueous vapour.

In carrying out the experiment it is very important that the temperature should be constant, as the pressure of the vapour changes greatly with temperature. Time must in each case be given for the air to become saturated.

Experiment.—Verify Dalton's law.

HYGROMETRY.

Pressure of Aqueous Vapour.[1]—The determination of the amount of water contained in the atmosphere as vapour is a problem of great importance, especially to meteorology. There are several ways in which we may attempt to make the determination, and the result of the experiment may also be variously expressed. The quantity of water which can be contained in air at a given temperature is limited by the condition that the pressure[1] of the vapour (considered independently of the pressure of the atmosphere containing it) cannot exceed a certain amount, which is definite for a definite temperature, and which for temperatures usually occurring, viz. between $-10°$ C. and $+30°$ C., lies between 2 mm. of mercury and 31·5 mm. Dalton concluded, from experiments of his own, that this maximum pressure, which water vapour could exert when in the atmosphere, was the same as that which the vapour could exert if the air were removed, and indeed that the dry air and the vapour pressed the sides of the vessel containing them with a pressure entirely independent one of the other, the sum of the two being the resultant pressure of the damp air (see the previous experiment, § 41). This law of Dalton's has been shewn by Regnault to be true, within small limits of error, at different temperatures for saturated air, that is, for air which contains as much vapour as possible; and it is now

[1] In the first edition of this work the words 'pressure' and 'tension' were used, in accordance with custom, as synonymous. In this edition it is intended to use the term 'pressure' only in referring to aqueous vapour.

a generally accepted principle, not only for the vapour of water and air, but for all gases and vapours which do not act chemically upon one another, and accordingly one of the most usual methods of expressing the state of the air with respect to the moisture it contains is to quote the pressure exerted by the moisture at the time of the observation. Let this be denoted by e; then by saying that the pressure of aqueous vapour in the atmosphere is e, we mean that if we enclose a quantity of the air without altering its pressure, we shall reduce its pressure by e, if we remove from it, by any means, the whole of its water without altering its volume. The quantity we have denoted by e is often called the pressure of aqueous vapour in the air.

Relative Humidity.—From what has gone before, it will be understood that when the temperature of the air is known we can find by means of a table of pressures of water vapour in vacuo the maximum pressure which water vapour can exert in the atmosphere. This may be called the saturation pressure for that temperature. Let the temperature be t and the saturation pressure e_t, then if the actual pressure at the time be e, the so-called fraction of saturation will be $\frac{e}{e_t}$ and the percentage of saturation will be $\frac{100\,e}{e_t}$. This is known as the *relative humidity*.

Dew Point.—If we suppose a mass of moist air to be enclosed in a perfectly flexible envelope, which prevents its mixing with the surrounding air but exerts no additional pressure upon it, and suppose this enclosed air to be gradually diminished in temperature, a little consideration will shew that if both the dry air and vapour are subject to the same laws of contraction from diminution of temperature under constant pressure,[1] the dry air and vapour will contract by the same fraction of their volume, but the pressure of *each* will be

[1] The condition here stated has been proved by the experiments of Regnault, Herwig, and others, to be very nearly fulfilled in the case of water vapour.

always the same as it was originally, the sum of the two being always equal to the atmospheric pressure on the outside of the envelope.

If, then, the pressure of aqueous vapour in the original air was e, we shall by continual cooling arrive at a temperature—let us call it τ—at which e is the saturation pressure; and if we cool the air below that we must get some of the moisture deposited as a cloud or as dew. This temperature is therefore known as the *dew point.*

If we then determine the dew point to be τ, we can find e, the pressure of aqueous vapour in the air at the time, by looking out in the table of pressures e, the saturation pressure at τ, and we have by the foregoing reasoning

$$e = e_\tau$$

42. The Chemical Method of Determining the Density of Aqueous Vapour in the Air.

It is not easy to arrange experiments to determine directly, with sufficient accuracy, the diminution in pressure of a mass of air when all moisture shall have been abstracted without alteration of volume, but we may attack the problem indirectly. Let us suppose that we determine the weight in grammes of the moisture which is contained in a cubic metre of the air as we find it at the temperature t and with a barometric pressure H.

Then this weight is properly called the actual density of the aqueous vapour in the air at the time, in grammes per cubic metre. Let this be denoted by d, and let us denote by δ the specific gravity of the aqueous vapour referred to air at the same pressure e and the same temperature t, and moreover let w be the density of air at 0° C. and 760 mm. pressure expressed in grammes per cubic metre. Then the density of air at the pressure e and temperature t, also expressed in grammes per cubic metre, is equal to $\dfrac{e w}{760(1 + a t)}$.

where $a =$ coefficient of expansion of gases per degree centigrade, and therefore

$$d = \frac{\delta e w}{760(1 + a t)},$$

$$\text{or } e = \frac{760(1 + a t)}{\delta w} d.$$

Now w is known to be 1293 and $a = \cdot 00366$;

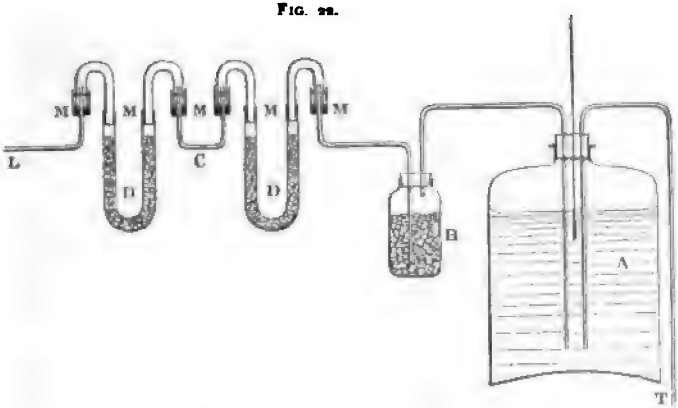
$$\therefore e = \frac{760(1 + \cdot 00366\, t)}{1293\, \delta} d. \quad . \quad . \quad . \quad . \quad (1)$$

If, therefore, we know the value of δ for the conditions of the air under experiment, we can calculate the tension of the vapour when we know its actual density. Now, for water vapour which is not near its point of saturation δ is equal to ·622 for all temperatures and pressures. It would be always constant and equal to ·622 if the vapour followed the gaseous laws up to saturation pressure. That is however, not strictly the case, and yet Regnault has shewn by a series of experiments on saturated air that the formula $e = \frac{760(1 + \cdot 00366\, t)}{1293 \times \cdot 622} d$ suffices to give accurately the pressure when d is known, even for air which is saturated, or nearly so, with vapour.

We have still to shew how to determine d. This can be done if we cause, by means of an aspirator, a known volume of air to pass over some substance which will entirely absorb from the air the moisture and nothing else, and determine the increase of weight thus produced. Such a substance is sulphuric acid with a specific gravity of 1·84. To facilitate the absorption, the sulphuric acid is allowed to soak into small fragments of pumice contained in a U-tube. The pumice should be first broken into fragments about the size of a pea, then treated with sulphuric acid and heated to redness, to decompose any chlorides, &c., which may be contained in it. The U-tubes may then be filled with the fragments, and the strong sulphuric acid poured on till the

pumice is saturated; but there must not be so much acid that the air, in passing through, has to bubble, as this would entail a finite difference of pressure on the two sides before the air could pass.

FIG. 22.



Phosphoric anhydride may be used instead of sulphuric acid, but in that case the tubes must be kept horizontal. Chloride of calcium is not sufficiently trustworthy to be used in these experiments as a complete absorbent of moisture.

The arrangement of the apparatus, the whole of which can be put together in any laboratory, will be understood by the fig. 22. As aspirator we may use any large bottle, A, having, besides a thermometer, two tubes passing airtight through its cork and down to the bottom of the bottle. One of these tubes is bent as a syphon and allows the water to run out, the flow being regulated by the pinch-cock T; the other tube is for the air to enter the aspirator; its opening being at the bottom of the vessel, the flow of air is maintained constant and independent of the level of the water in the bottle.

The vessel B, filled with fragments of freshly fused chloride of calcium, is provided with two tubes through an

airtight cork, one, connected with the aspirator, passing just through, and the other, connected with the drying tube D, to the bottom of the vessel. This serves as a valve to prevent any moisture reaching the tubes from the aspirator. The most convenient way of connecting up drying tubes is by means of mercury cups, consisting of short glass tubes with a cork bottom perforated for a narrow tube; over this passes one limb of an inverted U-tube, the other limb of which is secured to one limb of the drying tube either by an india-rubber washer with paraffin or, still better, by being thickened and ground as a stopper. A glance at the figure will shew the arrangement. The drying tubes can then be removed and replaced with facility, and a perfectly airtight connection is ensured. The space in the little cups, M, M, M, M, between the narrow tubes and the limbs of the inverted U's is closed by mercury. Care must be taken to close the ends of the inverted U's with small bungs during weighing, and to see that no globules of mercury are adhering to the glass. The connecting tubes C between the drying tubes should be of glass and as short as possible.

Two drying tubes must be used, and weighed separately before and after the experiment; the first will, when in good order, entirely absorb the moisture, but if the air is passed with too great rapidity, or if the acid has become too dilute by continued use, the second tube will make the fact apparent. A thermometer, X, to determine the temperature of the air passing into the tubes is also necessary.

To take an observation, the tubes are weighed and placed in position, the vessel A filled with water, the syphon tube filled, and the tube at the end of the drying tubes closed by means of a pinch-tap. Then, on opening the tap at T, no water should flow out; if any does there is some leak in the apparatus which must be made tight before proceeding further. When assured that any air supplied to the aspirator will pass through the drying tubes, the observation may be begun. The water is run out slowly

(at about the rate of 1 litre in ten minutes) into a litre flask, and when the latter is filled up to the scratch on the neck it is removed and weighed, its place being taken by another flask, which can go on filling during the weighing of the first. This is repeated until the aspirator is empty, when, the weight of the empty flasks being ascertained, the total weight of water thus replaced by air can be found. The height H of the barometer must be determined at the beginning and end of the experiment. During the observation the thermometer X must be read every ten minutes, and the mean of the readings taken as the temperature t of the entering air; the thermometer in the aspirator must be read at the end of the experiment; let the reading be t'. If the aspirator A is but small, it can be refilled and the experiment repeated, and we may of course determine, once for all, the volume of water which can be run out of the aspirator when filled up to a certain mark in the manner thus described; but as a exercise it is better to re-determine it for each experiment.

From the weight of water run out, with the assistance of Table 32 (Lupton, p. 28) we can determine the volume V of air taking the place of the water in the aspirator, V being measured in cubic metres. This air is evidently saturated with water at the temperature t'; its pressure is the barometric pressure, and therefore the pressure of the dry air in it is $H - e_{t'}$, $e_{t'}$ being the saturation pressure at t'. When it entered the drying tubes this air had a pressure $H - e$, and its temperature was t, e being the pressure whose value we are seeking. The volume of the air was, therefore,

$$\frac{H - e_{t'}}{H - e} \cdot \frac{1 + at}{1 + at'} V \qquad . \quad . \quad . \quad . \quad (2)$$

Hence, if w be the increase of weight of the drying tubes in grammes, we shall have for d the actual density of the moisture in the air;

$$d = \frac{w}{\dfrac{H - e_{t'}}{H - e} \cdot \dfrac{1 + at}{1 + at'} V} \qquad . \quad . \quad . \quad . \quad (3)$$

X

We thus obtain the quantity d; substituting its value from equation (1) above, we get

$$\frac{1293 \times \cdot 622}{760(1+at)}e = \frac{(\text{H}-e)(1+at')w}{(\text{H}-e_t)(1+at)\text{V}},$$

or

$$\frac{e}{\text{H}-e} = \frac{760}{1293 \times \cdot 622} \cdot \frac{1+at'}{\text{H}-e_t} \cdot \frac{w}{\text{V}} \qquad (4)$$

Experiment.—Determine the density of the aqueous vapour in the air, and also its pressure.

Enter results thus :—

Temperature of air	21°·7
Temperature of aspirator	21°·5
Volume of aspirator	36061 cc.
Gain of weight of tube (1)	·5655 gm.
" " " (2)	·0011 gm.
Total	·5666 gm.

$e = 16{\cdot}08.$

43. Dines's Hygrometer. Wet and Dry Bulb Thermometers.

Dines's Hygrometer is an instrument for directly determining the dew-point, i.e. the temperature at which the air in the neighbourhood of the instrument is completely saturated with aqueous vapour. It consists of a thermometer placed horizontally, so that its stem is visible while its bulb is enclosed in a box of thin copper through which cold water can be passed from a reservoir attached to the instrument by turning the tap at the back. The tap is full on when the side marked O is upward, and shut off when that marked S is upward. The bulb of the thermometer is placed close to the top of the box which encloses it, and the top of the box is formed of a plate of blackened glass, ground very thin indeed, in order, as far as possible, to avoid any difference of temperature between the upper and under

surfaces, and so to ensure that the temperature of the thermometer shall be the same as that of the upper surface of the glass.

The temperature of the box is cooled very gradually by allowing water, previously cooled by adding ice, to pass very slowly from the reservoir along the tube. As soon as the surface of the glass is at a temperature below that of the dew point, a deposit of dew can be observed on it. This can be easily noticed by placing the instrument so that the glass surface reflects the light of the sky, and accordingly presents a uniform appearance which is at once disturbed by a deposit of dew. The temperature t, say, at which this occurs is of course below the dew-point. The film of moisture is then allowed to evaporate, and when all has disappeared the temperature is again read—let it be t'. This must be accordingly above the dew-point. Now allow the water to flow only drop by drop, cooling the surface very slowly indeed, and observe the same phenomena again, until t and t' are not more than one or two tenths of a degree apart. Then we know that the dew-point lies between them, and by taking the mean of the two obtain an accuracy sufficient for practical purposes. The fall of temperature can in some cases be made so slow that a fugitive deposit forms and disappears at the same temperature, in which case the temperature of the dew-point is indicated by the thermometer as accurately as the variation of the quantity to be observed permits.

It is important that the observer should be as far as possible from the glass surface during the observation, in order to avoid a premature deposit of moisture. To this end a telescope must be mounted so as to read the thermometer at a distance, placing a mirror to reflect the scale of the thermometer to the telescope.

We may thus determine the dew-point, but the usual object of a hygrometric observation is to determine the pressure of aqueous vapour in the air at the time of observing.

We may suppose the air in the neighbourhood of the depositing surface to be reduced to such a state that it will deposit moisture, by altering its temperature merely, without altering its pressure, and accordingly without altering the pressure of aqueous vapour contained in it. We have, therefore, only to look out in a table the saturation pressure of aqueous vapour at the temperatnre of the dew-point and we obtain at once the quantity desired, viz. the pressure of vapour in the air before it was cooled.

We may compare the result thus obtained with that given by the wet and dry bulb thermometers. In this case the observation consists simply in reading the temperature of the air t, and the temperature t' of a thermometer whose bulb is covered with muslin, which is kept constantly moist by means of a wick leading from a supply of water. The wick and muslin must have been previously boiled in a dilute solution of an alkali and well washed before being mounted, as otherwise they rapidly lose the power of keeping up a supply of moisture from the vessel.

The pressure e'' of aqueous vapour can be deduced from the observations of t and t' by Regnault's formula[1] (available when t' is higher than the freezing point)

$$e''=e'-\cdot0009739t'(t-t')-\cdot5941(t-t')$$
$$-\cdot0008(t-t')(b-755)$$

where e' is the saturation pressure of aqueous vapour at the temperature t', and b is the barometric height in millimetres.

Experiments.—Determine the dew-point and the pressure of aqueous vapour by Dines's Hygrometer, and also by the wet and dry bulb thermometer.

[1] The reduction of observations with the wet and dry bulb thermometers is generally effected by means of tables, a set of which is issued by the Meteorological Office. The formula here quoted is Regnault's formula (Ann. de Chimie, 1845) as modified by Jelinek. See Lupton, table 35.

Enter the results thus :—

Appearance of dew	47°·1 F.
Disappearance of dew	47°·75
Dew-point	47°·42
Pressure of aqueous vapour deduced .	8·33 mm.
Pressure of aqueous vapour from wet and dry bulb	8·9 mm

44. Regnault's Hygrometer.

Regnault's hygrometer consists of a brightly polished thimble of very thin silver, forming the continuation of a short glass tube to which the silver thimble is attached by plaster of paris or some other cement not acted upon by ether. Through a cork fitting tightly into the top of the glass tube pass two narrow tubes of glass, one (A) going to the bottom of the thimble, the other (B) opening at the top of the vessel just below the cork; also a sensitive thermometer so placed that when the cork is in position, the bulb (which should be a small one) is close to the bottom of the thimble.

If, then, ether be poured into the thimble until it more than covers the thermometer bulb, air can be made to bubble through the liquid either by blowing into the tube (A) or sucking air through (B) by means of an aspirating pump of any sort. The passage of the air through the ether causes it to evaporate and the temperature of the liquid to fall in consequence, while the bubbling ensures the mixing of the different layers of liquid, and therefore very approximately, at any rate, a uniform temperature of silver, ether, and thermometer. The passage of air is continued until a deposit of dew is seen on the silver, which shews that the temperature of the silver is below the dew-point. The thermometer is then read, and the temperature of the apparatus allowed to rise until the deposit of moisture has completely disappeared, when the thermometer is again read. The temperature is now above that of the dew-point, and the

mean of the two readings so obtained may be taken as the temperature of the dew-point, provided that there is no more difference than two or three tenths of a degree centigrade between them.

In case the difference between the temperatures of appearance and disappearance is a large one, the method of proceeding suggested by Regnault should be adopted. The first observation will probably have given the temperature of dew appearance within a degree; say the observation was 5°; pass air again through the ether and watch the thermometer, and stop when a temperature of 6° is shewn. Then aspirate slowly, watching the thermometer all the time. Stop as each fifth of a degree is passed to ascertain if there be a deposit of dew. As soon as such a deposit is formed, stop aspirating, and the deposit will probably disappear before the temperature has risen 0°·2, and we thus obtain the dew-point correct to 0°·1.

The thermometer should be read by means of a telescope some 6 feet away from the instrument, and every care should be taken to prevent the presence of the observer producing a direct effect upon the apparatus.

It is sometimes very difficult, and never very easy, to be certain whether or not there is a deposit of dew on the silver, the difficulty varying with different states of the light. It is generally best to have a uniform light-grey background of paper or cloth, but no very definite rule can be given, practice being the only satisfactory guide in the matter.

A modification of Regnault's apparatus by M. Alluard, in which the silver thimble is replaced by a rectangular brass box, one face of which is surrounded by a brass plate, is a more convenient instrument; the contrast between the two polished surfaces, one of which may be covered with the dew while the other does not vary, enables the appearance of the deposit to be judged with greater facility. The method of using the instrument is the same as for Regnault's.

The dew-point being ascertained as described, the

pressure of aqueous vapour corresponding to the temperature of the dew-point is given in the table of pressures based on Regnault's experiments,[1] since at the dew point the air is saturated with vapour. We have already seen (p. 301) that we may take the saturation pressure of vapour at the dew-point as representing the actual pressure of aqueous vapour at the time of the experiment.

Experiment.—Determine the dew-point by Regnault's Hygrometer, and deduce the pressure of aqueous vapour. Calculate also the density of air in the laboratory at the time of observation.

Enter results thus :—

Appearance of dew	47°·1 F.
Disappearance	47 ·75
Dew-point	47 ·42
Pressure of aqueous vapour . . .	8 ·33 mm.

CHAPTER XII.

PHOTOMETRY.

THE first experiments to be performed in optics will be on the comparison of the intensities of two sources of light. We shall describe two simple methods for this, Bunsen's and Rumford's, both founded on the law that the intensity of the illumination from a given point varies directly as the cosine of the angle of incidence upon the illuminated surface and inversely as the square of the distance of the surface from the luminous point. So that if I, I′ be the illuminating powers of two sources distant r, r' respectively from a given surface, on which the light from each falls at the same angle, the illumination from the two will be respectively I/r^2 and I'/r'^2, and if these are equal we have

$$I : I' = r^2 : r'^2,$$

so that by measuring the distances r and r' we can find the ratio of I to I′.

[1] Lupton's *Tables*, No. 34.

Now this supposes that it is possible to make the illumination from each source of light the same by varying the distances of the two sources from the screen. As a matter of fact, this is not necessarily the case; in performing the experiment we compare the two illuminations by the effect produced on the eye, and that effect depends partly on the quantity of energy in the beam of light reaching the eye, partly on the nature of the rays of which that beam is composed. To define the intensity of a beam, we require to know, not merely the quantity of light in it, but also how that light is distributed among the differently coloured rays of which the beam is composed. Any given source emits rays, probably of an infinite number of different colours. The effect produced on the eye depends on the proportion in which these different colours are mixed. If they are mixed in different proportions in the two beams we are considering, it will be impossible for the effect of each of the two, in illuminating a given surface, ever to appear the same to the eye.

This constitutes the great difficulty of all simple photometric measurements. Two different sources of light, a gas flame and a candle for example, emit differently coloured rays in different proportions; the gas light contains more blue than the candle for the same total quantity of light, and so of the two spaces on which the illumination is to be the same, the one will appear bluish, the other reddish.

Strictly, then, two different sources of light can only be compared by the use of a spectro-photometer, an instrument which forms the light from each source into a spectrum and then enables the observer to compare the intensity of the two for the different parts of the spectrum. One such instrument will be described in a subsequent section (§ 67).

45. Bunsen's Photometer.

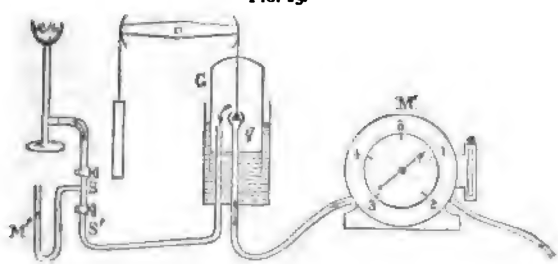
Two standard sperm candles (*see* p. 23) are used as the standard of comparison. These are suspended from the arm

of a balance and counterpoised so that the amount of wax burned can be determined at any moment without moving the candles. This arrangement is also useful in keeping the flames nearly in the same position, for as the candles burn down the arm supporting them rises. The balance is to be placed so that the candle-flames are vertically over the zero of the scale of a photometer bench in a dark room.

As a source to be compared with these, we use a gas-flame, the supply of gas being regulated and measured thus :—

The gas is passed from a gas-holder, where the pressure can be altered by altering the weights on the cover, through a meter, M, fig. 23, which measures the quantity of gas passed

FIG. 23.



through. One complete revolution of the needle corresponds to $\frac{5}{60}$th of a cubic foot of gas, so that the numbers on the dial passed over in one minute give the number of cubic feet of gas which pass through the meter in an hour. The gas enters at the middle of the back of the meter and leaves it at the bottom, passing thence to a governor, G, which consists of an inverted bell, partly sunk in water and counterpoised so that the conical plug attached to its top is very close to the conical opening of the entrance pipe *q*. Any increase of pressure of the gas in the bell raises the bell, narrows the aperture, and diminishes the supply until the pressure falls again. By this means the pressure of the gas at the burner is maintained constant.

The exit pipe from the bell passes to a tube with two stopcocks S, S′. The stopcock S′ is provided with a screw adjustment for regulating the supply of gas with extreme nicety; the stopcock S can then be used, being always either turned on full or quite shut, so as to always reproduce the same flame without the trouble of finely adjusting every time. Between these two stopcocks is a manometer M for measuring the pressure of the gas as it burns.

In stating, therefore, the gas-flame employed, we have to put down (1) the burner employed; (2) the pressure of the gas; (3) the amount of gas passing through the meter per hour.[1]

The gas passes from the stopcocks to the burner, which is fixed on one of the sliding stands of the photometer bar, so that the plane of the flame corresponds to the fiducial mark on the stand. On another sliding stand between the burner and the candles is placed the photometer disc, which consists of a grease spot upon white paper.

The method consists in sliding the photometer disc along the scale until the spot appears of the same brightness as the rest of the paper; the intensities of the lights are then proportional to the squares of their distances from the disc.

The observations should be made by viewing the disc from either side, as it will often be found that when the spot and the rest of the disc appear to be of the same brightness when viewed from one side, they will differ considerably when viewed from the other. This is due, in part, at any rate, to want of uniformity in the two surfaces of the paper of which the disc is made; if the difference be very marked, that disc must be rejected and another used. In all cases, however, observations should be made from each side and the mean taken.

The sources of light should be screened by blackened

[1] In order to test the 'lighting power of gas' with a standard argand burner, the flow through the meter must be adjusted to 5 cubic feet per hour by means of the micrometer tap.

screens, and the position of the disc determined by several independent observations, and the mean taken.

The lights must be very nearly of the same colour, otherwise it will be impossible to obtain the appearance of equality of illumination over the whole disc. (This may be tried by interposing a coloured glass between one of the lights and the disc.) Instead of trying to find a position in which the disc presents a uniform appearance on one side, the position in which it appears the same as viewed from two corresponding points, one on each side, may be sought for. For additional details see the 'Gas Analysts' Manual,' p. 40, §§ 61, 84.

Experiment.—Compare the illuminating power of the gas-flame with that of the standard candle.

Additional experiments.—(*a*) Compare the intensities of the candles and standard argand burner—

(1) Directly.

(2) With a thin plate of glass interposed between one source and the disc. This will give the amount of light lost by reflection and by the absorption of the glass. By rotating the glass plate the variations in the loss at different angles may be tested.

(3) With a thin plate of glass between one source and the disc, and a thick plate on the other side. This will enable you to determine the amount of light lost by the absorption of a thickness of glass equal to the difference of the thicknesses of the two plates.

(*b*) Obtain two burners and arrange them in connection with a three-way tube. Cover one up by a screen, and measure the intensity of the other. Then interchange them, and so obtain the intensity of each separately. Then place them together so that the two flames unite, and measure the intensity of the combined flame and its relation to the sum of the intensities of each.

(*c*) Test the intensity of the light from the same amount of gas used in different burners.

Enter results thus :—

Gas burning at the rate of 5 cubic feet per hour.

Candles " " 16·2 gms. "

Mean distance of gas	Mean distance of candles	Ratio of illuminating powers
75	31	5·85
68	29	5·49
60	25	5·76
52	22	5·59
46	19	5·86

Mean ratio of illuminating powers 5·71.

46. Rumford's Photometer.

The apparatus for making the comparison consists simply of a bar, at the end of which a ground glass or paper screen is fixed, and on which a support is made to slide, carrying the gas jet or other source of light.

On the bar, and in front of the screen, is placed a wooden rod, about 3 inches from the screen. The two lights to be compared are placed one on the sliding support and the other on the table at a fixed distance (taking care that both are the same height), the positions being so adjusted that the two shadows of the rod thrown on the screen are just in contact with each other without overlapping. The screen must be turned so that it makes equal angles with the direction of the light from each source. The distance of the sliding light has to be adjusted so that the two shadows are of the same depth.

Consider a unit of area, e.g. a square centimetre, of each shadow A and B; let the distance of the unit of area of A from the two sources of light be x, X, and let the distance of the unit of area of the shadow B from the same sources be y, Y respectively. Then the unit of area of A is illuminated only by the one source of light, distant X from it, and therefore its illumination is I/X^2, where I is the illumination per unit area at unit distance from the source. The unit of area of B is illuminated only by the source of light at distance y, and the illumination therefore is I'/y^2, when I′ is the illumination per unit area at unit distance from the second source.

Hence, since the illuminations of the shadowed portions of the screen are equal,

$$\frac{I}{x^2}=\frac{I'}{y^2} \therefore \frac{I}{I'}=\frac{x^2}{y^2}.$$

If the two unit areas considered be immediately adjacent to the line of junction of the shadows, then we may measure x and y from the same point. Hence the ratio of the intensities of the two sources is the square of the ratio of the distances of the two sources from the line of contact of the shadows. The method has the advantage that the observations do not need a dark room.

The shadows may be so arranged that the line of contact is on the middle line of the bar on which the one source slides, and accordingly the distance may be measured along the bar. The other distance may be measured by a tape.

The arrangements necessary for determining the rate at which the gas is being burnt or the quantity of wax consumed are described in section 45.

Experiment.—Compare the illuminating power of the gas-flame and standard candle.

Enter results thus :—

Candle burns at the rate of 8·1 gms. per hour.
Gas „ „ 5 cubic feet per hour.

Distance of gas	Distance of candle	Ratio of illuminating powers
128·5	39·5	10·5
98	30·5	10·4
Mean ratio of illuminating powers		10·45

CHAPTER XIII.

REFLEXION AND REFRACTION—MIRRORS AND LENSES.

NEARLY all the methods used in optical measurements are indirect. The quantity required is deduced by calculation from the quantities actually measured, or the law to be demonstrated is inferred from the actual observations by a process of reasoning. This is illustrated by the following experiment on the law of reflexion and by the experiments on focal lengths. The law of refraction may also be verified by the measurements of the refractive index of a transparent medium.

47. Verification of the Law of Reflexion of Light.

In order to prove the law, that the angle which a reflected ray makes with the normal to a plane surface is equal to the angle made by the incident ray with the normal, and that the two rays are in the same plane with the normal, two methods may be adopted :—

(1) The direct method, in which the angles of incidence and reflexion are measured and compared, and the positions of the rays determined.

(2) An indirect method, in which some result is verified which may be theoretically deduced on the assumption that the law holds.

The following experiment is an example of the second method.

It may be proved, by assuming the law of reflexion, that an image of a luminous point is formed by a plane mirror at a point on the normal to the plane surface drawn through the luminous point, and at a distance behind the mirror equal to the distance of the luminous point from the front of the mirror. This we can verify experimentally.

Take as the luminous point the intersection of cross-wires mounted on a ring, which can be placed in any position in a clip.

We can place another similar cross in the exact position occupied by the image in the mirror of the first, in the following manner.

Scrape a horizontal strip of the silvering off the back of the mirror and place the one cross in front, so that on setting the eye on a level with the cross, half of the image is seen coming just to the edge of the silvering.

Then place the other cross behind, so that it can be seen through that part of the glass from which the silvering has been scraped. Place this second cross so that the upper half of it can be seen through the gap, and so that the intersection of the second appears to coincide with the image of the intersection of the first. In order to determine whether or not this is really the case, move your eye from side to side across the first cross-wire, then if the second cross and the image are coincident, the two will appear to move together as the eye moves, and will remain coincident wherever the eye is placed. If, however, the actual cross is nearer to the mirror than the image, then on moving the eye to the right the two will appear to separate, the further, viz. the image, going to the right hand, the real cross to the left.

Place, then, the second cross so that on moving the eye from side to side no separation between the cross and the image occurs. It is then in exactly the same position as that occupied by the image of the first cross in the mirror.

Let the first cross be placed at a distance of 1 foot (about) from the reflecting surface of the mirror. Measure the distance by means of a pair of compasses and a scale, and measure, also, the distance between the same surface of the mirror and the second cross, which has been accurately placed to coincide with the image of the first in the mirror. Then displace the second cross from coincidence with the image and replace it and read the distance again in order

to ascertain the limit of accuracy to which your observation can be carried. Repeat three times.

The experiment may be very conveniently made with a piece of unsilvered plate glass instead of the mirror. The image of the first cross formed by reflexion at the surface of the glass is generally sufficiently bright to permit of the second cross being accurately placed to coincide with it. If the glass is very thick, allowance must be made for the displacement of the image of the second cross as seen through the glass. A corresponding allowance may, of course, also be necessary in the case of the mirror whose thickness will alter the apparent position of the reflected image of the first cross.

Two vertical pins in stands may be used instead of cross-wires, and the upper part of the second one may be viewed directly *over the top* of the mirror, while the lower part of the image of the first is seen in the mirror.

In order to verify that the image and object are on the same normal to the mirror, place the eye so that the image and object are in the same straight line with it, and notice that the image of the eye is in the same line too, no matter how far from or how near to the mirror the eye be placed; this can only be the case if the line is a normal.

In case the result obtained does not apparently confirm the law of reflexion, the discrepancy may be due to the fact that the mirror is cylindrical or spherical and not truly plane. To distinguish between the cases, repeat the experiment, moving the eye vertically up and down instead of horizontally.

Experiment.—Verify the truth of the law of reflexion of light.

Enter results thus:—

Distance of object	Distance of image
75 cm.	75 cm.
65 "	63 "
80·5 "	78 "
71·5 "	71·5 "
61 "	59 "

The following method of finding the angle of a prism is another illustration of the law of reflexion :—

Place the prism on a sheet of paper attached to a drawing-board. Let B A C (fig. 23*a*) be its trace, A being the angle to be measured.

Stick a pin (P) vertically into the board at some distance from A, in such a position that images by reflexion can be obtained from the faces A B and A C respectively. Determine the positions of these images as for a plane mirror, or proceed as follows :—Look at one image, moving your eye about until it is seen as nearly as possible in a line with A ; and place pins at Q, R, so that the image of P, the edge A, and the pin at Q appear in one straight line, while the image, the edge, and the pin at R are seen in another. Then a ray P A falling on one face very close to A is reflected along A Q, while an almost coincident ray incident on the other face is reflected along A R. Join A P, A Q, A R, and measure with a protractor the angles Q A R and B A C. It will be found that Q A R = 2 B A C ; that is, that the angle between the reflected rays is twice the angle of the prism. This can be proved to be a consequence of the law of reflexion.

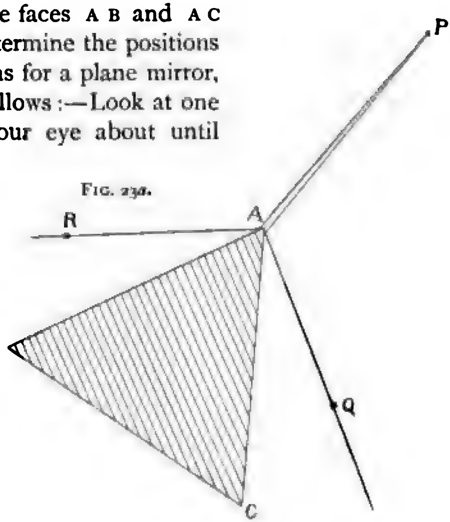


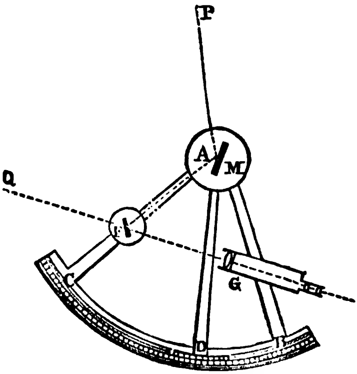
FIG. 23*a*.

Experiment.—Determine the positions of the images of a pin formed by the light reflected from the two surfaces of a prism, and thence measure the angle of the prism.

48. The Sextant.

The sextant consists of a graduated circular arc, B C (fig. 24), of about 60°, connected by two metal arms, A B, A C, with its centre A. A D is a third movable arm, which turns round an axis passing through the centre A, at right angles to the plane of the arc, and is fitted with a clamp and tangent screw. A vernier is attached to this arm at D, and by means of it the position of the arm with reference to the scale can be determined. The vernier is generally constructed to read to 15″.

FIG. 24.



A plane mirror, M, is attached to this arm and moves with it. The plane of the mirror passes through the centre of the circular arc and is at right angles to the plane of the scale.

The mirror is known as the index glass, and is held by adjustable screws in a frame which is rigidly connected to the arm A D. By means of the screws it can be placed so that its plane is accurately perpendicular to that of the arc.

At F on the arm A C is another mirror called the horizon glass, also secured by adjustable screws to the arm. Its plane should be perpendicular to that of the arc and parallel to that of the movable mirror M when the index at D stands at the zero of the scale.

The upper half of the mirror F is left unsilvered.

At G on the arm A B is a small telescope, directed towards the mirror F. The axis of the telescope is parallel to the plane of the arc, and by means of a screw at the

back of the instrument the telescope can be moved at right angles to this plane, so as to direct its axis towards the silvered or unsilvered part of the horizon glass. This is placed in such a position that its normal bisects the angle A F G, and hence a ray of light, parallel to the plane of the sextant, travelling along A F, is reflected by the horizon glass parallel to the axis of the telescope. Let P A be such a ray reflected by the mirror M in direction A F, and suppose P to be some distant object the position of which we wish to observe. Let the telescope be so placed with reference to the plane of the instrument that light from a second distant object Q, also travelling parallel to the plane of the sextant, can enter the telescope through the unsilvered part of the glass F. Then an observer, looking through the telescope, will see the point Q directly, and the point P after reflexion at the two mirrors M and F.

The telescope is fitted with cross-wires, and by altering the position of the arm A D the image of P can be made to coincide with that of Q in the centre of the field of view.

Let us suppose this adjustment made. Then by reflexion at the two mirrors the ray P A has been made to coincide in direction with the ray Q F. Hence, the angle between P A and Q F is twice the angle between the two mirrors. But when the index read zero the two mirrors were parallel, so that twice the angle between the two mirrors is twice the angle through which the arm and vernier have been turned from zero.

In many instruments the graduations are numbered to read as double of their real value; each degree is reckoned as two degrees and so on, so that, if the instrument be in adjustment, the reading of the vernier gives us directly the angle between P A and Q F, that is, the angle which the two distant points P and Q subtend at the observer's eye.

The requisite adjustments are :—

(1) The plane of the index glass M should be at right angles to that of the graduated arc.

(2) The plane of the horizon glass F should also be at right angles to that of the arc.

(3) The axis of the telescope should be parallel to the plane of the arc.

(4) The index and horizon glasses should be parallel when the vernier reads zero.

We proceed to consider how to make these adjustments.

The two glasses are held in their frames by screws, and can be set in any position by altering these screws.

(1) Place the eye close to the index glass and look towards the glass so as to see part of the arc CD and its reflexion, meeting at the surface of the glass. If the two, the arc and its image, appear to be in the same plane, then the glass is perpendicular to that plane. If, however, the image appears to rise out of the plane of the arc, the upper portion of the glass leans forward towards the eye, while if the image appears to drop below the plane of the arc, the glass leans back away from the eye. Adjust the screws till the arc and its image appear to be in the same plane; then the plane of the glass is at right angles to that plane.

(2) To set the horizon glass. Hold the instrument so as to view directly with the telescope some distant point—a star if possible. On turning the index arm round, an image of the point, formed by reflexion at the two glasses, will cross the field. If the two glasses be accurately parallel, this image can be made to coincide exactly with the object seen by the direct rays. If the plane of the horizon glass be not at right angles to that of the arc, so that the two mirrors can never be parallel, the image will appear to pass to one side or the other of the object.

By altering the adjusting screws of the horizon glass, the image seen after two reflexions, and the object seen directly, can be made to coincide in position. When this is the case the two mirrors are strictly parallel, and the horizon glass, therefore, is at right angles to the plane of the arc.

(3) To set the axis of the telescope parallel to the plane of the arc. For this it is necessary that the ring to which the telescope is fixed should be capable of being moved about an axis parallel to the line of intersection of its plane with that of the arc.

The eye-piece of the telescope is usually fitted with two cross-wires, very approximately parallel to the plane of the arc, and one wire at right angles to these, passing through their middle points. The line joining the centre of the object glass to the middle point of this wire is the optical axis of the telescope. Hold the instrument so as to view two distant points, such as two stars, the one directly and the other by reflexion at the two glasses, and incline it to the plane through the eye and the two stars in such a way that the two images seen in the telescope appear to coincide at the point in which the third wire cuts one of the two parallel wires. Then, without moving the index glass, incline the plane of the instrument until the image of the star seen directly falls on the intersection of the third wire and the other of the two parallel wires. If the image of the second star again coincides with that of the first, it follows that the optical axis of the telescope is parallel to the plane of the arc; to make the two parallel the position of the telescope with reference to the arc must be adjusted until it is possible to observe such a coincidence.

(4) To set the two mirrors parallel when the vernier-index reads zero. It will be found that one of the glasses with its frame and adjusting-screws can be moved about an axis at right angles to the plane of the arc. Set the vernier to read zero and clamp it, and direct the telescope to some distant point. If the two glasses are parallel this point, and its image after reflexion at the two mirrors, will appear to coincide. If they do not coincide they can be made to do so—supposing adjustments (1) and (2) have been made—by turning the movable mirror about the axis just

spoken of, and when the coincidence is effected the mirrors will be parallel, while the vernier reads zero.

Instead, however, of making this last adjustment, it is better to proceed as follows to determine the index error of the instrument.

Direct the telescope to a distant point and turn the index glass until the image of the point, after reflexion at the two mirrors, coincides with the point itself as seen directly. Clamp the vernier and read; let the reading be α. If the instrument were in perfect adjustment, the value of α would be zero. Suppose, now, we find that when proceeding to measure the angular distance between two distant points, as already described, the scale and vernier reading is β, then the angular distance required is $\beta-\alpha$. Generally it gives less trouble to determine the index error than to set the mirrors so that there is no such error.

It may, of course, happen that the value of α is negative—in other words, that to bring a point and its image into coincidence we have to push the vernier back beyond the zero of the scale; for this reason the scale graduations are continued beyond the zero.

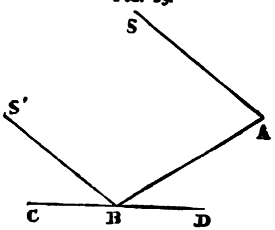
It is important for accurate work that the two images which are brought into coincidence should be about equally bright. Now, the light from one has suffered two reflexions, each of which somewhat diminishes its intensity. If, then, the two distant objects are unequally bright, we should choose the duller one as that to be viewed directly. Again, we have said already that the telescope can be moved in a direction at right angles to the plane of the arc. In its normal position the axis of the telescope will pass through the boundary between the silvered and unsilvered parts of the horizon glass. Half the object-glass will accordingly be filled with direct light, half with reflected. If the direct light is very much stronger than the reflected, we can, by moving the telescope, still keeping its axis parallel to the plane of the circle, place it so that the reflected rays fill

more than half and the direct rays less than half the object glass, and thus reduce the brightness of the direct and increase that of the reflected image. There are also shades of coloured glass attached to the instrument, which can be interposed in the path of either pencil and so decrease its intensity.

The instrument is frequently used to observe the altitude of the sun or of a star; and in this case the horizon, if it is visible, forms one of the distant points, and when the instrument is adjusted, the image of the sun's lower limb should appear to coincide with this.

If the horizon be not visible, an 'artificial horizon' is obtained by reflexion from some horizontal surface—that of pure mercury in a trough is most frequently used. For consider two parallel rays S A, S′ B (fig. 25) coming from a distant object, and let S′ B be reflected at B from a horizontal surface C D. B A appears to come from the image of the distant object formed by reflexion at C D, and if an observer with a sextant at A determine the angle between the distant object and its image, he will measure the angle S A B. But since S A is parallel to S′ B and the angle A B D is equal to S′ B C, the angle S A B is twice the angle S′ B C, that is, twice the altitude of the distant object.

FIG. 25.



If mercury be used for the artificial horizon, it should be covered with a piece of carefully worked plate glass. After one observation the cover should be taken up and turned round and a second taken. The mean of the two will be free from any small error which might arise from the faces of the glass not being parallel. Sometimes a piece of glass, which can be carefully levelled, is used instead of the mercury.

Experiments.

(1) Test the accuracy of the various adjustments of the sextant.

(2) Measure the angular distance between two distant points.

(3) Measure the altitude of a distant point, using an artificial horizon.

Enter results thus :—

	Index error	Angular distance
	2′ 15″	32° 35′ 30″
	2′ 30″	32° 35′ 15″
	2′ 30″	32° 35′ 15″
Mean	2′ 25″	32° 35′ 20″
	True angular distance	32° 32′ 55″

Similarly for observations of altitude.

O. Refraction of Light through a Plate and through a Prism.

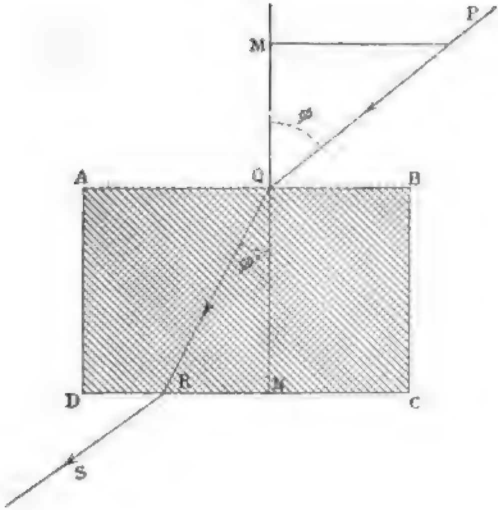
The path of a pencil of light through a plate or a prism may be traced and the law of refraction verified by a graphical construction in the following manner.

Place a rectangular block of glass, which should be of considerable size—say 8 or 10 cm. square by 1 cm. high—on a sheet of paper fastened to a drawing-board, and mark its position A B C D (fig. xxv) on the paper. Draw a line P Q meeting the glass obliquely, and stick two pins vertically into the board at two points some distance apart in P Q.

On looking obliquely through the opposite face (C D) of the glass the two pins will be seen, and it will be usually possible to place the eye in such a position that the one may appear exactly behind the other. Do this, and stick two more pins into the board in front of the glass in such a way that these two are seen in the same straight line as the first two, so that all four appear to be in line one behind the other. Draw with a ruler a line R S through

the feet of the last two, and let it meet the surface of the glass in R. Join Q R. Then a ray of light falling on the glass in the direction P Q is refracted into the glass along Q R, and

FIG. XXV.



on emergence travels along R S. On completing the figure it will be seen that P Q is parallel to R S. Draw M Q N normal to the glass at Q. Then M Q P is the angle of incidence ϕ, and N Q R the angle of refraction ϕ'.

To Verify the Law of Refraction (viz. that sin ϕ/sin ϕ' is constant) and find the Refractive Index.

With Q as centre and Q R as radius describe a circle cutting Q P in P. Draw P M perpendicular to the normal Q M. Measure the distances P M and R N, and take the ratio. Then

$$\frac{\sin\ \phi}{\sin \phi'}=\frac{PM}{QP}\times\frac{QR}{RN}=\frac{PM}{RN},$$

for $QP = QR$.

Take a second incident ray $P_1 Q$, incident at a different angle, and determine the refracted ray $R_1 S_1$ in the same way. Then we shall have

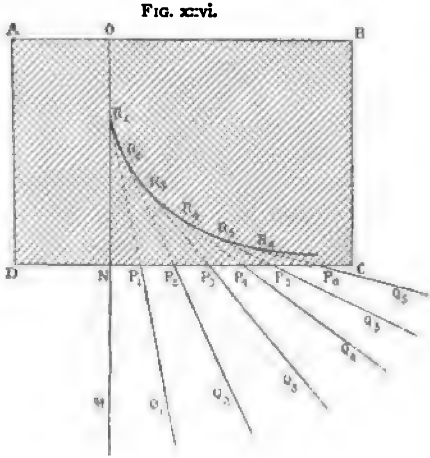
$$\frac{\sin \phi_1}{\sin \phi_1{}'} = \frac{P_1 M_1}{R_1 N_1}.$$

and it will be found that the ratios $\frac{PM}{RN}$ and $\frac{P_1 M_1}{R_1 N_1}$ are equal. Thus this ratio is a constant for all angles of incidence, and the value of this constant is the refractive index. We have thus verified the law of refraction and found μ, the index of refraction.

To Illustrate the formation of a Caustic Curve by Refraction.

Stick a vertical pin into the board in contact with the block at O. Let ON be a normal meeting the opposite

FIG. xxvi.



face in N, and along that face mark off a number of points P_8. . . . such that $NP_1 = P_1 P_2 = P_2 P_3 = . . . = 1$ cm.

At each of the points N, P_1, P_2, . . . place pins in contact with the block. Look at the block from a little distance, and place another series of pins in the board successively in such positions that the pin O, each of the pins N, P_1, P_2, . . in turn, and the corresponding pins, M, Q_1, Q_2, . . . of this next series appear successively in straight lines. Remove the block, join Q_1 P_1, Q_2 P_2, Q_3 P_3, . . . , and produce each of these lines backwards to the point in which it meets the next preceding line. Let these points be R_1, R_2, R_3. Then a ray travelling in the block along O P_2 is refracted so as to emerge along P_2 Q_2, and so for the other rays.

Again, if we can suppose two consecutive emergent rays $P_2 Q_2$, $P_3 Q_3$, to reach the eye, these rays will appear to diverge from R_2, and the position of the image of O which the eye sees when looking along $Q_2 P_2$ will be R_2. In reality, the rays $P_2 Q_2$, $P_3 Q_3$ are too far apart to be treated as consecutive rays ; we should have to suppose incident rays to fall on all the points of the glass between N and P, and draw all the emergent rays. In this way we should obtain a series of points, such as R_1, . . . R_6, all lying on a curve, each point being the intersection of two consecutive emergent rays. This curve is called the caustic curve, and to it all the emergent rays are tangents, while the virtual image of O seen in the direction of any given ray is the point in which that ray touches the caustic curve.

If, then, the figure be constructed as already described, and a curve drawn to touch all the emergent rays, this curve will be the caustic. The same figure can be used to verify by a geometrical construction the law of refraction.

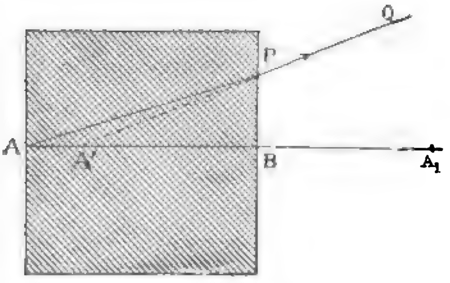
To find the Refractive Index.

The following is another method of finding μ :—

Make a mark at a point A (fig. xxvii) on one face of the block. This may be done by sticking on to it a small piece of sealing-wax. Place the block on the table, and stick a pin upright into the board in such a way that A_1,

the head of the pin, is at the same height as A. On looking through the block the reflected image of the pin and

FIG. xxvii.



the image of A can both be seen. Move the block about until these two, when viewed directly from a point behind the pin, appear to coincide.

In this case A B A_1, cutting the block in B, will be normal to the block, and if A$'$ is the refracted image of A, it is also the reflected image of A_1.

Since the light is nearly directly incident, we know that

$$BA = \mu \, BA' ;$$

and since A$'$ is the reflected image of A_1,

$$BA' = BA_1 ;$$

Thus

$$BA = \mu \, BA_1,$$

$$\therefore \; \mu = BA/BA_1.$$

Hence to find μ, measure the thickness of the block and the distance B A_1 of the pin from the block. The ratio of the two is the refractive index.

To Verify the Law of Reflexion.

Similar experiments can be performed to verify the law of reflexion. In this case let P Q (fig. xxviii) be an incident

ray falling on the block at Q. At two points on this ray stick two pins vertically into the board, and then look at the reflected images of these pins. Move your eye about until these images appear in the same straight line, and in that line stick two other upright pins, R, S. On joining R S it will be found to meet the surface in Q, and to make an angle with the normal at Q equal to that made by P Q. If, moreover, a number of incident rays $P\,Q_1$, $P\,Q_2$ . . . be taken, and the directions of the reflected rays determined, it will be found that these all meet in a point P_1, and if $P\,P_1$ be joined, cutting the face of the block, or this face produced, in N, $P\,P_1$ is at right angles to that face, and is bisected in N.

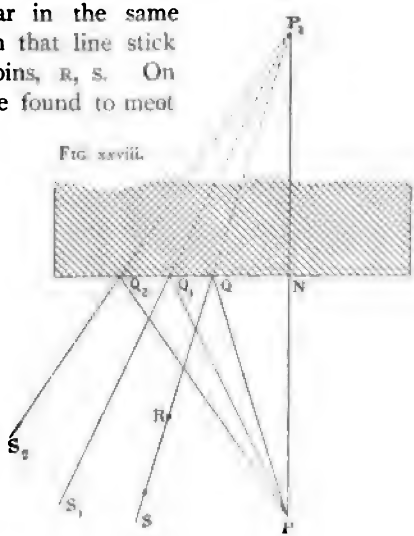


FIG. xxviii.

By replacing the block by a prism, the laws of refraction through the prism may be verified.

Refraction through a Prism.

Draw a ray P Q (fig. xxix) incident obliquely on a prism. The direction of the refracted ray and of the ray in the prism can be found in exactly the same way as in the case of a plate.

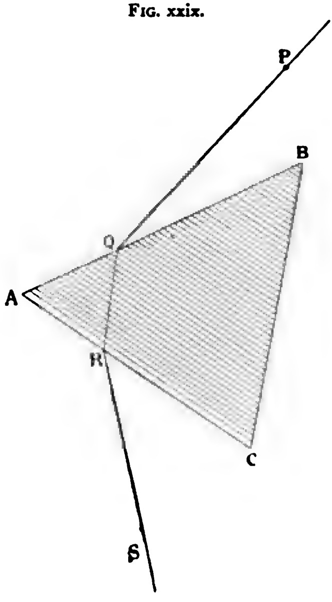
By the aid of a protractor the angles, ϕ and ψ, of incidence and emergence respectively can be found, and the deviation D, which is the angle between the incident ray P Q

and the emergent ray R S. If the angle of the prism, i, be measured, we can verify the formula

$$D = \phi + \psi - i.$$

Moreover, by varying ϕ from zero up to grazing incidence, for which $\phi = 90°$, we can examine the changes in the deviation. We shall find that as the angle of incidence increases the deviation decreases at first, then reaches a minimum value, and afterwards increases again as the angle of incidence is still further increased up to grazing incidence. In the position of minimum deviation we can shew that the incident and emergent rays are equally inclined to the surface of the prism, so that $\phi = \psi$ for this position. Moreover, in general we have

FIG. xxix.



$$\phi' + \psi' = i,$$

ϕ', ψ' being the angles which the ray in the prism makes with the normals to the two faces. Hence, in the position of minimum deviation, for which

$$\phi' = \psi',$$

we have

$$\phi' = \tfrac{1}{2} i,$$

$$\phi = \tfrac{1}{2}(D + i);$$

$$\therefore \mu = \frac{\sin \phi}{\sin \phi'} = \frac{\sin \frac{1}{2}(D + i)}{\sin \frac{1}{2} i}.$$

The images of the pin seen by refraction will usually be slightly coloured, but unless the dispersion of the prism is very great this will not seriously affect the results. The

instrument described in § 62, the spectrometer, enables us to make the measurements above described to a much higher degree of accuracy than is possible with the ruler and pencil.

Experiment.—Trace the path of a ray of light through a plate of glass, and hence verify the law of refraction, and find the refractive index of glass.

Trace the caustic curve formed by rays diverging from a point and emerging from glass.

Trace the path of a ray through a prism, and verify the formulæ

$$\phi' + \psi' = i$$
$$\phi + \psi = D + i.$$

Shew that in the position of minimum deviation $\phi = \psi$, and find the refractive index of the prism.

On Optical Measurements.

Many of the simpler optical experiments described below depend on the determination of the positions of some luminous object and its real image formed after reflexion or refraction. A formula is obtained expressing the quantity sought for, e.g. the focal length of a lens, in terms of distances which can be readily determined. These are measured and their values substituted in the formula; the value of the quantity in question is determined by calculation.

Now, in almost every case, the formula is one giving the relation between the position of a point and its geometrical image, and to obtain this the assumption is made that we are only concerned with a small pencil, the axis of which is incident directly on the reflecting or refracting surfaces.

If this be not the case, there is no such thing as a point image of a point. The rays diverging from a given point of the object do not all converge again exactly to one and the same point. For each point in the object we have—supposing still that the incidence is direct—a least circle of aberration through which all the rays from that point pass, and the nearest approach to an image is the

figure formed by the superposition of all these least circles of aberration, which will be a representation of the object, more or less blurred, and differing in position from the geometrical image.

Now, frequently this happens with the images produced by the optical combinations with which we shall have to do. The pencils which go to form the various images are not small pencils incident directly, and the phenomena are thus complicated by the effects of aberration.

Thus, for example, we may require the radius of a concave mirror, three or four inches across and six or eight inches in radius; or we may be experimenting with a lens of an inch or so in diameter and only one or two inches in focal length. In both these cases we should meet with aberration difficulties. We shall see best how to allow for this in each separate experiment.

There is one measurement common to many optical experiments, the mode of making which may best be described here.

Two objects—the one may be a lens, the other a screen on which an image is focussed—are attached to the supports of an optical bench described below. This is graduated, and the supports possibly are fitted with verniers; at any rate, there is a mark attached to them, the position of which, with reference to the scale of the bench, can be found.

We can thus find easily the distance between the two fixed marks on the supports; but suppose we require the distance between the screen and one face of the lens. To obtain this we must know their positions with reference to the fixed marks. Now, the apparatus is generally constructed so that the central plane of the lens and the plane of the screen respectively are in the same vertical plane as the marks in question, so that, neglecting the thickness of the lens, the distance between the marks is, as a matter of fact, identical with the distance required. But for some purposes this is not sufficiently accurate. We may, for example, wish to consider the thickness of the lens in our measurements

In this case, take a rod with two pointed ends, and measure carefully its length. Let it be a. Put one end against the screen and move up the support carrying the other surface, until this is in contact with the other end of the rod. Let the distance between the marks on the supports, as read at the same time by the scale and vernier, be b. Then, clearly, if in any other position of the supports the distance between the marks on them is c, the distance between the surfaces is $c+a-b$, for a was the distance between them in the first position, and $c-b$ is the distance by which it has been altered.

We may make the same measurement by the following slightly different method which can be used conveniently for determining the distance between two objects measured parallel to any fixed scale. Fix securely to the vernier of the scale a stiff piece of wire, and bend it until its end comes in contact with one of the objects in question, and read the vernier. Now move the vernier with the wire fixed relatively to it, along the scale, until the same end of the wire comes in contact with the second object, then read the vernier again. The difference between the two readings is the distance required.

This will be found a convenient way in making the measurements, described in § 49, if the mirror can be fitted to one of the supports of the optical bench.

Of course, if the distance required be only small, the simplest method of all is to use a pair of compasses and take it off along a finely divided scale.

49. Measurement of the Focal Length of a Concave Mirror.

This may be obtained optically by means of the formula [1]

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f},$$

[1] For the formulæ required in this and the next chapter we may refer to Glazebrook, *Physical Optics*, chap. iv.

f being the focal length, and r the radius of the surface, u and v respectively the distances from the surface of an object and its image; u and v can be measured, and then r or f calculated.

In practice the following modification of the method will be found most convenient.

It depends on the fact that when the image of an object formed by a concave mirror coincides with the object itself, then the object is at the geometrical centre of the spherical surface.

Place a needle in a clip and set it in front of the mirror; place the eye some distance further away from the mirror than the needle. An inverted image of the needle will be seen, unless the needle has been placed too close to the mirror. Adjust the position of the needle relatively to the mirror, so that the point of the image coincides with the point of the needle. When this is the case the image will be of the same size as the object.

The adjustment can be made as finely as necessary, either by moving the eye about and noting whether the relative positions of image and needle vary, or by using a strong magnifying lens, and noticing whether both needle and image are in focus at the same time.

If the aperture of the mirror be very large, and its surface not perfectly spherical, it may be impossible to see the image when using the lens, in consequence of the aberration of the rays from the outer portions of the surface. These defects may, in some cases, be corrected by covering the mirror with black paper, leaving at the centre only a small hole, which may be either oblong or circular.

When the position of the needle has been carefully adjusted, measure its distance from the reflecting surface by means of a pair of compasses and a scale, if the radius be small, or by the method already described if the mirror be fitted to the optical bench.

The result gives the length of the radius of the mirror surface. Half of it is the focal length.

Experiment.—Determine the radius of curvature of the given mirror, and check your result by the use of the spherometer.

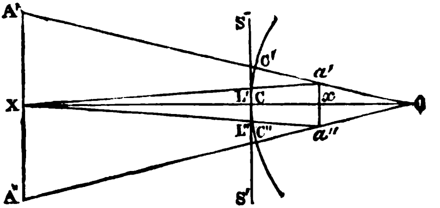
Enter results thus :—

Radius of curvature by optical observations	19·52 cm.
Radius of curvature by spherometer . .	19·8 cm.

50. Measurement of the Radius of Curvature of a Reflecting Surface by Reflexion.

The method of § 49 is applicable only when the reflecting surface is concave, so that the reflected image is real. The following method will do for either a concave or convex surface.

FIG. 26.



Let O, fig. 26, be the centre of the reflecting surface, O C X the axis.

Suppose two objects A′, A″ placed at equal distances on each side of O C X, and at the distance O X from O.

Images of these two points will be formed by reflexion at points *a*′, *a*″ on the axes O A′, O A″, such that (calling the points where the axes O A, O A′ cut the spherical surface C′, C″)

$$\frac{1}{A'C'} - \frac{1}{a'C'} = -\frac{2}{OC'},$$

or

$$\frac{1}{A'C'} = \frac{1}{a'C'} - \frac{2}{OC'},$$

and

$$\frac{1}{A''C''} = \frac{1}{a''C''} - \frac{2}{OC''}$$

Now, the points being very distant, and therefore C′ A′ very nearly equal to C X, we may assume that the straight line $a' a''$ cuts the axis O C X at a point x where

$$\frac{1}{\mathrm{CX}} = \frac{1}{\mathrm{C}x} - \frac{2}{\mathrm{OC}} \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

and for the size of the image, we have

$$\frac{a' a''}{\mathrm{A}' \mathrm{A}''} = \frac{\mathrm{O}x}{\mathrm{OX}} . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

Hence, if $\mathrm{CX} = \mathrm{A}$, $\mathrm{OC} = r$, $\mathrm{A}' \mathrm{A}'' = \mathrm{L}$, $\mathrm{C}x = x$, and $a' a'' = \lambda$ we get from (1)

$$\frac{1}{\mathrm{A}} = \frac{1}{x} - \frac{2}{r} . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Hence

$$\frac{1}{\mathrm{A}} + \frac{1}{r} = \frac{1}{x} - \frac{1}{r};$$

$$\therefore \frac{r + \mathrm{A}}{\mathrm{A}} = \frac{r - x}{x};$$

$$\therefore \frac{x}{\mathrm{A}} = \frac{r - x}{r + \mathrm{A}};$$

and

$$\frac{\lambda}{\mathrm{L}} = \frac{r - x}{r + \mathrm{A}},$$

$$\therefore \frac{\lambda}{\mathrm{L}} = \frac{x}{\mathrm{A}}.$$

From these two equations

$$x = \frac{\mathrm{A} r}{2\mathrm{A} + r},$$

$$\lambda = \frac{\mathrm{L} r}{2\mathrm{A} + r}.$$

Place a small, finely divided scale S S′ immediately in front of the reflecting surface (but not so as to prevent all the light falling upon it) i.e. place it horizontally to cover nearly half the reflecting surface, and observe the images

a', a'' and the scale S S$'$ by means of a telescope placed so that its object-glass shall be as nearly as possible in the middle of the line joining A A$'$; we may with sufficient accuracy suppose the centre of the object-glass to be at the point X. Join Xa', Xa'' and let the lines Xa', Xa'' cut the scale S S$'$ in L$'$ and L$''$, and let l denote the length L$'$ L$''$ of the scale intercepted by them.

Then we get

$$\frac{l}{\lambda}=\frac{\mathrm{XL}'}{\mathrm{X}a'}=\frac{\mathrm{A}}{\mathrm{A}+x}$$

$$\therefore l=\frac{\mathrm{A}}{\mathrm{A}+x}\times\frac{\mathrm{L}r}{2\mathrm{A}+r},$$

or

$$l=\frac{\mathrm{A}}{\mathrm{A}+\dfrac{\mathrm{A}r}{2\mathrm{A}+r}}\times\frac{\mathrm{L}r}{2\mathrm{A}+r}$$

$$=\frac{\mathrm{L}r}{2(\mathrm{A}+r)},$$

or

$$r=\frac{2\mathrm{A}l}{\mathrm{L}-2l}.$$

The formula proved above refers to a convex surface; if the surface be concave we can find similarly the equation

$$r=\frac{2\mathrm{A}\,l}{\mathrm{L}+2\,l}.$$

To make use of this method to find the radius of curvature of a surface, place the surface opposite to, but at some distance from, a window. Then place horizontally a straight bar of wood, about half a metre in length between the surface and the window, fixing it with its ends equidistant from the surface, and at such a height that its reflexion in the surface is visible to an eye placed just below the bar, and appears to cross the middle part of the surface. Fix a telescope under the centre of the bar, with its object-glass

in the same vertical plane as the bar, and focus it so as to see the image reflected in the surface.

It is best that the whole of the bar should be seen reflected in the surface. If this cannot be secured, two well-defined marks, the reflected images of which can be clearly seen, should be made on the bar. These may be obtained by fixing two strong pins into the upper edge, or by laying on it two blocks of wood with clearly defined edges.

In any case the reflected image should appear in the telescope as a well-marked dark object against the bright background of the reflexion of the window. If it be more convenient to work in a dark room, arrangements must be made to illuminate the bar brightly, so that its reflexion may appear light against a dark background.

Now place against the reflecting surface a finely graduated scale—one divided to half-millimetres or fiftieths of an inch will do—arranging it so that one edge of the image of the bar is seen against the divided edge of the scale. If the curvature of the surface be considerable, and the magnifying power of the telescope not too great, the scale will be fairly in focus at the same time as the image of the bar. At any rate, it will be possible to read the graduations of the scale which the image of the bar appears to cover. This gives us the length l of the above formula. Measure the length of the bar or the distance between the two marks—this we call L; and measure with a tape the distance between the reflecting surface and the centre of the object-glass of the telescope—this gives A.

Then the formula gives us r.

In some cases it may be possible to see more than one reflected image of the bar; e.g. if a reflecting surface be one surface of a lens, we may have a reflexion from the back surface as well as from the front. A little consideration enables us to choose the right image. Thus, if the first surface is convex, the reflected image will be erect and will,

therefore, appear inverted if we are using an astronomical telescope.

Experiment.—Determine the radius of the given surface, checking the result by the use of the spherometer.

Enter results thus:—

Surface Convex
A = 175·6 cm.
L = 39·4 cm.
l = 2·06 cm.
r = 20·5 cm.

Value found by spherometer 20·6 cm.

Measurement of Focal Lengths of Lenses.

The apparatus generally employed to determine the focal length of a lens is that known as the optical bench.

It consists simply of a horizontal scale of considerable length, mounted on a substantial wooden beam, along which upright pieces can slide, and to these are severally attached the lens, the luminous object, and a screen on which the image formed by the lens is received. These sliding-pieces carry verniers, by which their position with reference to the scale can be determined. The position of each face of the lens relatively to the zero of the vernier is known or can be found as described on p. 337.

51. Measurement of the Focal Length of a Convex Lens.—First Method.

For this purpose a long bar of wood is employed, carrying at one end a ground-glass screen, fixed at right angles to the length of the bar. A stand, in which the lens can conveniently be fixed with its axis parallel to the length of the bar, slides along it, and the whole apparatus is portable, so that it can be pointed towards the sun or any other distant object.

Place the lens in the stand and withdraw to a dark corner of the laboratory; point the apparatus to a distant

well-defined object—a vane seen through a window against the sky is a good object to choose if the sun be not visible—and slide the lens along the bar until a sharply defined image of the object is formed upon the ground glass. Since the object is very distant, the distance of the lens from the screen is practically equal to the focal length, and can be measured either with a tape or by means of graduations on the bar itself.

The observation should, of course, be made more than once, and the mean of the measurements taken.

52. Measurement of the Focal Length of a Convex Lens.—Second Method.

Mount on one of the stands of the bench a diaphragm with a hole in it across which two fine threads are stretched, or, if more convenient, a piece of fine wire grating, or a pin in a vertical position with its point about the centre of the hole. Place a light behind the hole, taking care that the brightest part of the light is level with the hole and exactly behind it, while the light is as close to the hole as may be.

In the second stand place the lens, fixing it so that its centre is on the same level as that of the hole in the diaphragm, while its axis is parallel to the length of the bench.

In the third stand fix an opaque white screen ; a piece of ground glass or unglazed paper is most suitable. For the present purpose the objects can generally be fixed on their respective stands so as to occupy with sufficient accuracy the same relative positions with regard to the zeros of the verniers, and thus the distances between the different objects in question can be obtained at once, by reading the verniers and subtracting.

If the distance between the first and third stand be more than four times the focal length of the lens, the latter can be placed so that there is formed on the screen a distinct image of the object in the first stand. Move the stand carrying the lens till this is the case. Then measure

by means of the verniers fixed to the stands, or as described on p. 337, the distance, u, between the object and the first surface of the lens and the distance, v, between the image and the second surface.

Then if we neglect the thickness of the lens the focal length f is given by the formula[1]

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

The values of v should be observed for at least three different values of u.

Experiment.—Determine by the methods of this and the preceding sections the focal length of the given lens.

Enter results thus :—

Lens A.

Approximate focal length (§ 51) 58 cm.

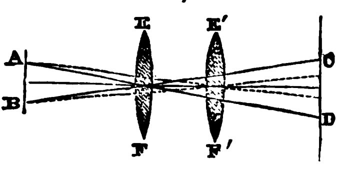
By method of § 52—

u	v	f
105·6	128·8	58·02
99·4	140·1	58·15
85·0	181·9	57·92
Mean value of focal length		58·03

53. Measurement of the Focal Length of a Convex Lens.—Third Method.

The methods already described for finding the focal lengths of lenses involve the measurement of distances from the lens surface, and consequently a certain amount of error is caused by neglecting the thickness of the glass of which the lens is composed. This becomes very important in the case of short-focus lenses and of lens combinations.

FIG. 27



[1] Glazebrook, *Physical Optics*, chap. iv.

The following method avoids the difficulty by rendering the measurement from the lens surfaces unnecessary.

We know that for a convex lens, if u, v are the distances respectively of the image and object from the principal points[1] of the lens E F (fig. 27), and f its focal length; then

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v},$$

u and v being on opposite sides of the lens. Now, if we have two screens A B, C D a distance l apart, and we place the lens E F, so that the two screens are in conjugate positions with regard to it, then $u+v=l$, provided we neglect the distance between the two principal points.

In strictness, $u+v$ is not equal to l, as the distances u and v are not measured from the same point, but from the two principal points respectively, and these are separated by a distance which is a fraction of the thickness of the lens. Thus, if t be the thickness of the lens, it may be shewn that the distance between the principal points is $\frac{\mu-1}{\mu}t$, if we neglect terms involving t^2; the value of this for glass is about $\frac{1}{3}t$.

The image of a cross-wire or a piece of wire-grating at the one screen A B will be formed at the other, C D. Now we can find also another position of the lens, E′ F′, between the screens, such that the image of the cross-wire or grating is again focussed on the second screen. This will evidently be the case when the lens is put so that the values of u and v are interchanged. Let u' and v' be the values which u and v assume for this new position of the lens, and let the distance $u'-u$ or $v-v'$ through which the lens has been moved be a.

Then we have

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$
$$u+v=l$$
$$u'-u=a.$$

[1] See Pendlebury's *Lenses and Systems of Lenses*, p. 39 et seq.

But

$$u' = v \therefore v - u = a.$$

Hence

$$v = \frac{l+a}{2}; \qquad u = \frac{l-a}{2}.$$

Substituting

$$\frac{1}{f} = \frac{2}{l+a} + \frac{2}{l-a} = \frac{4l}{l^2-a^2};$$

$$\therefore f = \frac{l^2-a^2}{4l};$$

so that the focal length may be determined by measuring the distance between the screens (which must be greater than four times the focal length), and the distance through which the lens has to be moved in order to transfer it from one position in which it forms an image of the first screen on the second, to the other similar position. This latter measurement should be made three or four times and the mean taken.

For screens, in this case, we may use small pieces of wire gauze mounted in the circular apertures of two of the stands of the optical bench, or we may fix two pins with their points at the centres of these apertures.

The coincidence of the image of the first object with the second may be determined by the parallax method described in §§ 47 and 49; or the following very convenient arrangement may be adopted:—In the apertures of the two stands of the optical bench mount two pieces of gauze, as suggested above, setting one of them with its wires horizontal and vertical, and the other with its wires inclined at an angle of 45° to these directions. On the stand carrying the gauze on which the image is to be received, mount a magnifying glass of high power—the positive eye-piece of a telescope serves the purpose admirably—and adjust it so that the gauze is accurately in focus. To obtain the coincidence of the image of the first gauze with the second, we have now only to move

whence the expression for the focal length becomes

$$f=\frac{\left(l-\frac{\mu-1}{\mu}t\right)^2-a^2}{4\left(l-\frac{\mu-1}{\mu}t\right)},$$

and this reduces to

$$f=\frac{l^2-a^2}{4l}-\frac{\mu-1}{\mu}\frac{l^2+a^2}{4l^2}t;$$

we have, therefore, to correct our first approximate value by subtracting the quantity

$$\frac{\mu-1}{\mu}\frac{l^2+a^2}{4l^2}t.$$

Experiment.—Determine the focal length of the given lens for red, green, and blue light, and verify your results by the modified method.

Enter results thus :—

Lens A.

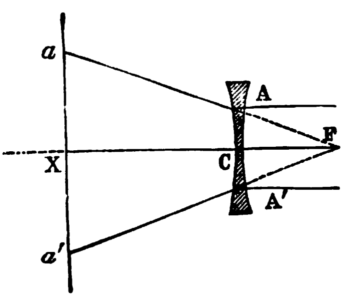
$l = 255.$

	a	f	f (method 2)
Red .	70·5	58·8	58·65
Green .	73·7	58·4	58·27
Blue .	75·8	58·1	57·8

54. Measurement of the Focal Length of a Concave Lens.

Method 1 (requiring a more or less darkened room):—

FIG. 28.



Place in front of the lens a piece of black paper with two narrow slits A, A′ cut parallel to each other at a known distance apart, and let light which is quite or nearly parallel fall on the lens (fig. 28). Two bright patches will be formed on a screen at a, a', by the light passing through the two slits, and the rays forming them will be in the same directions as if they came

from the principal focus F of the lens. If then we measure $a\,a'$ and C X, and if C F $=f$, we have

$$\frac{f}{f+\text{C X}}=\frac{\text{A A}'}{a\,a'},$$

from which f can be found. The distance between the centres of the bright patches can be measured with a pair of compasses and a finely divided scale, or by using a scale as the screen on which the light falls.

In consequence of the indistinctness of the bright patches, this is only a very rough method of determining the focal length.

Method 2:—

The second method consists in placing in contact with the given concave lens a convex lens sufficiently powerful to make a combination equivalent to a convex lens. Let the focal length (numerical) of the concave lens be f, that of the auxiliary convex lens f', and that of the combination F.

Then

$$\frac{1}{f}=\frac{1}{f'}-\frac{1}{\text{F}},$$

$$\therefore f=\frac{\text{F}f'}{\text{F}-f'}.$$

The values of F and f' can be found by one of the methods described for convex lenses.

In selecting a lens with which to form the combination it should be noticed that, if F and f' differ only slightly, say by 1 centimetre, an error of 1 millimetre in the determination of each, unless the errors happen to be in the same direction, will make a difference of one-fifth in the result. The auxiliary lens should therefore be chosen to make the difference $\text{F}-f'$ as large as possible—i.e. the concave lens should with the convex produce a combination nearly equivalent to a lens with parallel faces, so that $\frac{1}{f}$ may be very nearly equal to $\frac{1}{f'}$.

For greater accuracy the light used should be allowed to pass through a plate of coloured glass, so as to render it more nearly homogeneous.

Experiment.—Determine by the two methods the focal length of the given lens.

Lens D.

Enter results thus :—

Method 1.—	Distance between slits .	2·55 cm.
	Distances between images	4·75 „
	Distance from lens to screen .	33·00 „
	Focal length	38·24 „
Method 2.—	Focal length of convex lens .	29·11 cm.
	Focal length of combination .	116·14 „
	Focal length required . .	38·85 „

P. Focal Lengths. Additional Methods of Measurement.

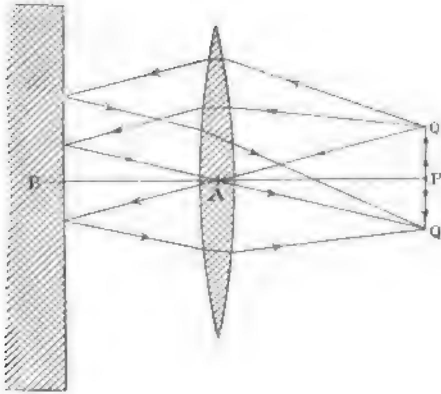
Other methods for measuring focal lengths depending on various properties of lenses and mirrors have been devised.

Thus, consider a source of light on the axis of a convex lens, so placed that a real image of the source is formed on the other side of the lens. If the light fall on a plane mirror, it will be reflected back through the lens, and form an image real or virtual, as the case may be. If the mirror be placed so that the image formed by the lens falls on the mirror, the light will be reflected back, and a real image of the source will be formed coincident with the source itself.

In general this will only happen for one position of the mirror ; but suppose the object is at the principal focus of the lens, then the rays from any point on the object form a parallel pencil on falling on the mirror. They will therefore be reflected as a parallel pencil from the mirror whatever be the distance between it and the lens, and will again be brought to a focus at the same distance from the lens as the object. Thus, if an image be formed at the same distance from the lens as the object, and if this image is not altered by shifting the mirror, keeping its plane normal to the axis of the lens, we know that the object is at the principal focus

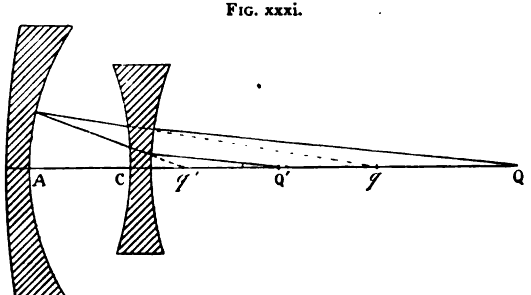
of the lens, and the distance between the object and the lens is the focal length. The image in this case is inverted. Fig. xxx shews the paths of the rays. To perform the experiment, place a pin in a clip, having adjusted the lens and mirror so that the axis of the lens is approximately normal to the mirror, and move the pin about until, looking at it from some little distance, the image of the pin is seen, as in § 49, to coincide with the pin. Then measure the distance PA between the pin and the lens. The convex lens and plane mirror are equivalent to the concave mirror (cf. § 49).

FIG. xxx.



A similar method may be used with a concave lens and concave mirror to find the focal length of the lens.

FIG. xxxi.



Light diverging from an object Q (fig. xxxi) is allowed, after refraction through a concave lens, to fall on a concave mirror. It is reflected from this, and converges after re-

flexion towards a point q', which is the real image in the mirror of q, the image of Q formed by the lens. But before reaching q' the light again falls on the lens and is refracted by it to Q′, at which point a real image of Q is formed. If the distances of Q and Q′ from the lens be observed, and if the focal length of the mirror and the distance between it and the lens be known, then the focal length of the lens can be found. The simplest case is that in which Q and Q′ coincide. When this happens it is clear that the light after reflexion retraces its path; it falls normally on the mirror. Thus q and q' coincide at the centre of the mirror, and if r be the radius of the mirror and $CA = a$, then $Cq = r - a$, and we have

$$\frac{1}{Cq} - \frac{1}{CQ} = \frac{1}{f}, \quad \therefore \frac{1}{f} = \frac{1}{r-a} - \frac{1}{CQ},$$

and by observing CQ we can find f, the focal length of the concave lens.

55. Focal Lines.

When light falls obliquely on a convex lens a refracted pencil does not converge to a point, but to two focal lines in planes at right angles. Let us suppose the lens placed normal to the incident light which is travelling in a horizontal direction, and then turned about a vertical axis till the angle of incidence is ϕ, then the primary focal line is vertical, the secondary is horizontal, and if u be the distance of the source of light from the lens, v_1, v_2, the distances of the focal lines, supposed to be real, and f the focal length of the lens, we have [1]

$$\frac{1}{v_1} + \frac{1}{u} = \frac{\mu \cos \phi' - \cos \phi}{(\mu - 1) \cos^2 \phi} \frac{1}{f},$$

$$\frac{1}{v_2} + \frac{1}{u} = \frac{\mu \cos \phi' - \cos \phi}{\mu - 1} \frac{1}{f};$$

[1] See Parkinson's *Optics*, p. 101. The sign of u has been changed.

$$\therefore \frac{\frac{1}{v_1}+\frac{1}{u}}{\frac{1}{v_2}+\frac{1}{u}} = \frac{1}{\cos^2\phi};$$

$$\therefore \sec^2\phi = \frac{u+v_1}{u+v_2}\cdot\frac{v_2}{v_1}.$$

If, then, we determine v_1 and v_2, this equation will give us the value of ϕ, and if the apparatus can be arranged so that ϕ can readily be measured, the comparison of the value given by the formula with the result of the measurement enables us to check the formula.

To measure ϕ, the stand carrying the lens should be capable of rotation about a vertical axis, and a horizontal circle attached to it so that its centre is in the axis. A pointer fixed to the moving part of the stand turns over the circle. The reading of the pointer is taken when the lens is placed at right angles to the light, and again when it has been placed in the required position. The difference between the two gives the angle of incidence. To find v_1 and v_2, it is best to use as object a grating of fine wire with the wires vertical and horizontal, and to receive the light after traversing the lens on a screen of white paper. For one position of the screen the vertical lines will appear to be distinctly focussed, while the horizontal are hardly visible. The screen then is in the position of the primary focus, and the distance between it and the lens is v_1. For a second position of the screen the horizontal lines are in focus and the vertical are not seen. This gives the secondary focus, and we can thus find v_2.

Each observation will require repeating several times, and in no case will the images formed be perfectly clear and well-defined. A very good result may, however, be obtained by using the homogeneous light of a sodium flame behind the gauze, and receiving the image upon a second gauze provided with a magnifying lens, as described in § 53.

Experiment.—Light falls obliquely on a lens; determine the position of the primary and secondary foci, and hence, find the angle of incidence.

Enter results thus :—

$$u = 102; \quad v_2 = 120; \quad v_1 = 83.$$

Hence
$$\cos^2\phi = \cdot 83,$$
$$\phi = 24^\circ\, 39'.$$

Q. Focal Lines formed by a Prism.

When light diverging from a point falls on a prism, the emergent light diverges from two focal lines. If the edge of the prism be vertical, and if the axis of the incident pencil be at right angles to the edge, the focal lines are horizontal and vertical. The position of the horizontal focal line is independent of the angle of incidence; that of the vertical focal line changes as the incidence is varied. The vertical line is known as the primary focal line, the horizontal line as the secondary. If u be the distance of the object from the prism, which we suppose to be thin, and v_1, v_2 the distances of the primary and secondary focal lines, then it is shewn (Parkinson's 'Optics,' p. 88) that

$$v_1 = \frac{\cos^3\phi' \cos^2\psi}{\cos^2\phi \cos^2\psi'}\, u; \qquad v_2 = u,$$

where ϕ, ψ are angles of incidence and emergence.

If the light after passing through the prism fall on a suitable convex lens, real images of the focal lines are formed by the lens. Thus, in fig. xxxii, O is the source of light; q_1, q_2 the two focal lines formed by the prism A; Q_1, Q_2 the real images of these formed by the lens C.

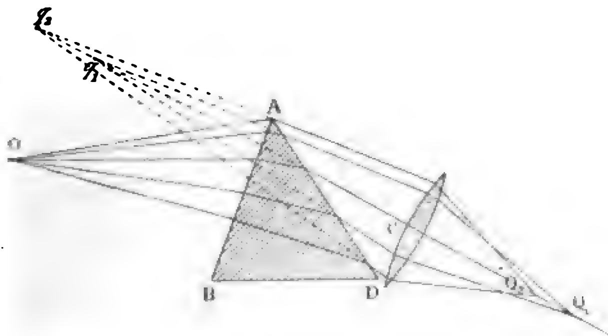
These images Q_1, Q_2 might be received on a screen; it is better to look at them from behind with an eye-piece—an ordinary watchmaker's glass will do, though a Ramsden's eye-piece with cross-wires set at 45° to the horizon is preferable.

If the focal length of the lens C be known, and the distances CQ_1, CQ_2 be measured, the values of v_1 and v_2 can

be calculated, and then by measuring the angles of incidence and emergence the formula can be verified.

In performing the experiment it is best to use for the source O a wire gauze, the wires being set vertically and

FIG xxxii.



horizontally. This is illuminated by a Bunsen burner with a sodium flame. In the position of the primary focal line distinct images of the vertical wires will be formed ; in the position of the secondary line the horizontal wires will be seen clearly. If the position of the prism be that of minimum deviation, so that $\phi = \psi$, then we shall have

$$v_1 = v_2 = u.$$

Thus q_1 and q_2, and therefore Q_1 and Q_2, coincide, and if the eye-piece be focussed on the image both vertical and horizontal wires will be seen. If now the angle of incidence be changed, the vertical wires will become indistinct, while the others remain clear, shewing that the position of the secondary focus is independent of the angle of incidence. On drawing the eye-piece back or pushing it forwards, as the case may be, a badly defined image of both sets of wires, corresponding to the position of the circle of least confusion, comes into view, while on moving the eye-piece still further in the same direction the horizontal wires disappear,

but the vertical wires are seen sharply defined as a set of vertical bars against a uniform field.

Experiment.—Shew that the primary and secondary focal lines formed by a prism coincide when the deviation of the prism is a minimum, and measure the distance between their images formed by a convex lens when the prism is turned 10° from this position.

On the Measurement of Magnifying Powers of Optical Instruments.

The magnifying power of any optical instrument is the ratio of the angle subtended at the eye by the image as seen in the instrument to the angle subtended at the eye by the object when seen directly. If the object to be seen is at a short distance from the eye, and the distance can be altered, the eye must always be placed so that the object is at the distance of most distinct vision (on the average, 25 cm.); and any optical instrument is focussed so that the image seen is at the distance of most distinct vision. Thus the magnifying power of a lens or microscope is the ratio of the angle subtended at the eye by the image in the instrument to the angle subtended at the eye by the object when placed at the distance of most distinct vision.

Telescopes are, however, generally used to observe objects so distant that any alteration which can be made in the distance by moving the eye is very small compared with the whole distance, and hence for a telescope the magnifying power is the ratio of the angle subtended by the image in the telescope to the angle subtended by the object. Then again this image is at the distance of distinct vision for the eye, but the focal length of the eye-piece is generally so short that the angle subtended by the image at the eye is practically the same as if the eye-piece were focussed so that the image was at an infinite distance.

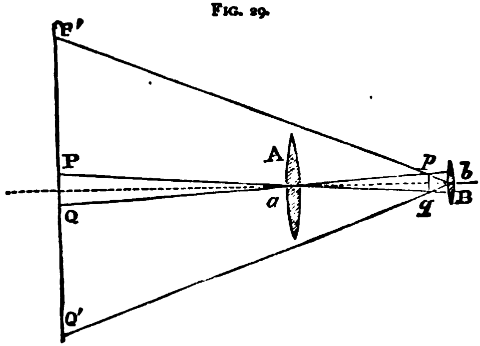
Thus suppose the small image $p\ q$ (fig. 29), formed by the object-glass A, is in such a position with reference to the

eye-piece B that the image of it P′Q′ formed by the eye-piece is at the same distance as the object P Q.

Since the object is very distant the angle subtended by it at the centre *a* of the object-glass, which is equal to the angle *p a q*, is practically the same as that subtended by it at the eye, and the angle subtended by the image at the eye is practically the same as the angle *p b q*.

These angles being very small, they will be proportional to their tangents, and the magnifying power will be equal to either (1) the ratio of the focal length of the object-glass

FIG. 29.



to the focal length of the eye-piece; or (2) the ratio of the absolute magnitude (diameter) of the image P′Q′ to that of the object P Q when the telescope is so focussed that these two are at the same distance from the eye.

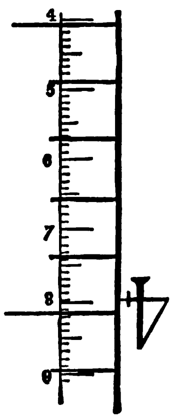
On this second definition of the magnifying power depends the first method, described below, of finding the magnifying power of a telescope.

56. Measurement of the Magnifying Power of a Telescope.—First Method.

Place the telescope at some considerable distance from a large scale, or some other well-defined object divided into a series of equal parts—the slates on a distant roof, for example. Then adjust the eye-piece so that the image

seen in the telescope coincides in position with the scale itself. In doing this, remember that when the telescope is naturally focussed the image is about ten inches off; and as the eye-piece is pulled further out, the image recedes until the small image formed by the object-glass is in the principal focus of the eye-glass, when the image seen is at infinity. The required position lies between these two limits, and is attained when the image seen through the telescope with the one eye is quite distinct, while at the same time the scale, as seen directly, is distinctly seen by the other eye looking along the side of the telescope; and, moreover, the two do not appear to separate as the eyes are moved from side to side.

FIG. 30.



Then the appearance to the two eyes is as sketched in fig. 30, where the magnifying power is about 8.

The number of divisions of the scale, as seen directly, covered by one of the divisions of the image of the scale can be read off, and this gives evidently the ratio of the tangents of the two angles, $p\,b\,q$, $P\,a\,Q$, and hence the magnifying power of the telescope.

If the scale used be in the laboratory, so that its distance from the telescope can be measured, the experiment should be made at different distances. Instead of reading the number of divisions of the scale occupied by one division of the image, it is best to count those occupied by some six or eight divisions of the image and divide one number by the other.

Experiment.—Determine, at two different distances, the magnifying power of the given telescope.

Enter results thus:—

Telescope No. 3.

Distance between scale and telescope . 1000 cm.

Lower edge of image of division 76 is at 0 on scale.
Lower edge of image of division 69 is at 99 on scale.

$$\text{Magnifying power} = \frac{99-0}{76-69} = 14{\cdot}14$$

Distance = 500 cm.

Lower edge of image of division 72 is at 95.
Lower edge of image of division 78 is at 3.

$$\text{Magnifying power} = \frac{95-3}{78-72} = 15{\cdot}3$$

57. Measurement of the Magnifying Power of a Telescope.—Second Method.

The magnifying power of a telescope for an infinitely distant object may be taken as the ratio of the focal length of the object-glass to that of the eye-piece, and may be found by the following method :—

Focus the telescope for parallel rays as follows :—

(1) Focus the eye-lens by sliding in the socket until the cross-wires are seen distinctly.

(2) Direct the telescope to the most distant object visible from an open window—a vane is generally a convenient object—and move the eye-piece and cross-wires together as one piece (there is generally a screw for doing this, but sometimes it has to be done by pulling out the tube by hand) until the distant object is clearly seen as well as the cross-wires, and so that there is no parallax, i.e. so that on moving the eye across the aperture of the eye-piece the cross-wires and image do not move relatively to each other. This will be the case when the image of the distant object formed by the object-glass is in the plane of the cross-wires. The telescope is then said to be focussed for infinity or for parallel rays.

Next, screw off the cover of the eye-piece—without altering the focus—and screw out the object-glass and substitute for it an oblong-shaped diaphragm, the length of which must be accurately measured : let it equal L. The

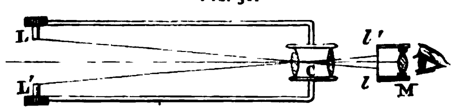
measurement can be easily effected by means of a pair of dividers and a fine scale.

The distance of this from the optical centre of the eye-piece is $F+f$, F and f being the focal lengths of the object-glass and eye-piece respectively. An image of this oblong aperture will be formed by the eye-piece at a distance v on the other side of its optical centre, where

$$\frac{1}{f} = \frac{1}{F+f} + \frac{1}{v}.$$

Now measure the length of this image by bringing up to it a micrometer scale engraved on glass, such as is made for use in a microscope, graduated to tenths of a millimetre and having a lens mounted in front of it to facilitate the reading. Place the micrometer in a clip, and adjust the height and distance until the scale and the image of the aperture are both distinctly seen on looking through the lens attached to the micrometer. In this way the length of the image of the diaphragm can be determined in terms of millimetres and tenths. Let this be l. Of course any other convenient form of micrometer may be used for this purpose.

FIG. 31.



Then the magnifying power $\frac{F}{f} = \frac{L}{l}$

For if L L′ (fig. 31) be the diaphragm aperture, so that L L′ = L, M the micrometer, and $l\,l'$ the image formed, C the optical centre of the eye-piece, then

$$\frac{L}{l} = \frac{LL'}{l\,l'} = \frac{CL}{Cl} = \frac{F+f}{v}.$$

But

$$\frac{1}{f} = \frac{1}{F+f} + \frac{1}{v}; \quad \therefore \frac{1}{v} = \frac{F}{f(F+f)}; \quad \therefore \frac{L}{l} = \frac{F}{f}$$

In measuring the length of the image by the micrometer scale, the aperture should not be too brightly illuminated, or the image may be blurred and indistinct. The telescope should on this account be pointed at a sheet of grey filter-paper or other slightly illuminated uniform surface, giving just light enough for reading the micrometer scale.

Experiment.—Determine the magnifying power of the given telescope.

Enter results thus:—

Telescope No. 2.

Length of aperture	2·18 cm.
Length of image	·16 cm.
Magnifying power	13·6

58. Measurement of the Magnifying Power of a Lens or of a Microscope.

A lens or microscope is used for the purpose of viewing objects whose distance from the eye is adjustable, and in such cases the magnifying power is taken to be the ratio of the angle subtended at the eye by the image as seen in the instrument to the angle subtended at the eye by the object when placed at the distance of most distinct vision (generally 25 cm.). The instrument is supposed to be focussed so that the image appears to be at the distance of most distinct vision.

The method described for a telescope in § 56 is applicable, with slight alteration, to the case of a lens or microscope. The instrument is focussed on a finely divided scale ; one eye looks at the magnified image while the other looks at another scale placed so as to be 25 cm. away from the eye, and to appear to coincide in position with the image of the first scale viewed through the instrument. Suppose the two scales are similarly graduated, and that x divisions of the magnified scale cover X divisions of the scale seen directly, then the magnifying power is X / x. If the two scales be not

similarly divided—and it is often more convenient that they should not be so—a little consideration will shew how the calculation is to be made. Thus, if the magnified scale be divided into m^{ths} of an inch, and the unmagnified one into n^{ths}, and if x divisions of the magnified scale cover x unmagnified divisions, then the magnified image of a length of x/m inches covers an unmagnified length of x/n inches, and the magnifying power is therefore $m\mathrm{x}/nx$.

The following modification of the method gives the two images superposed when only one eye is used :—Mount a camera-lucida prism so that its edge passes over the centre of the eye-lens of the microscope. Then half the pupil of the eye is illuminated by light coming through the microscope, and the other half by light reflected at right angles by the prism. If a scale be placed 25 cm. away from the prism, its image seen in the camera-lucida may be made to coincide in position with the image of the scale seen by the other half of the pupil through the microscope.

To make this experiment successful, attention must be paid to the illumination of the two scales. It must be remembered that magnifying the scale by the microscope reduces proportionately the brightness of the image. Thus the magnified scale should be as brightly illuminated as possible, and the reflected scale should be only feebly illuminated. It should also have a black screen behind it, to cut off the light from any bright object in the background.

A piece of plane unsilvered glass set at 45°, or a mirror with a small piece of the silvering removed, may be used instead of the camera lucida prism.

The magnifying power of a thin lens may be calculated approximately from its focal length. The eye being placed close to the lens, we may take angles subtended at the centre of the lens to be equal to angles subtended at the eye. Now a small object of length l placed at a distance of 25 cm. subtends an angle whose measure may be taken to be $l/25$. When the lens is interposed the *image* is to be at a distance

of 25 cm., and the distance between the object and eye must be altered; the object will therefore be at a distance u where

$$\frac{1}{u} - \frac{1}{25} = \frac{1}{f}.$$

The angle subtended by the image is similarly measured by its length divided by 25, and this is equal to l/u, or

$$l\left(\frac{1}{f} + \frac{1}{25}\right).$$

Thus the magnifying power is

$$\frac{\frac{1}{f} + \frac{1}{25}}{\frac{1}{25}},$$

or

$$\frac{25}{f} + 1.$$

A microscope with a micrometer scale in the eye-piece is sometimes used to measure small distances. We may therefore be required to determine what actual length corresponds, when magnified, to one of the divisions of the micrometer scale in the eye-piece.

For this purpose place below the object-glass a scale divided, say, to tenths of a millimetre, and note the number of divisions of the eye-piece scale which are covered by one division of the object scale seen through the microscope; let it be a. Then each division of the eye-piece scale corresponds clearly to $1/a$ of one-tenth of one millimetre, and an object seen through the microscope which appears to cover b of these eye-piece divisions is in length equal to b/a of one-tenth of a millimetre.

If we happen to know the value of the divisions of the eye-piece scale we can get from this the magnifying power of the object-glass itself, in the case in which the microscope is fitted with a Ramsden's or positive eye-piece, and thence,

on determining the magnifying power of the eye-piece, find that of the whole microscope. For if m_1 be the magnifying power of the object-glass, m_2 that of the eye-piece, then that of the whole microscope is $m_1 \times m_2$.

Thus, if the eye-piece scale is itself divided to tenths of millimetres, since one-tenth of a millimetre of the object scale appears to cover a tenths of a millimetre of the eye-piece scale, the magnifying power of the object-glass is a.

If, on the other hand, the microscope is fitted with a Huyghens or negative eye-piece, then the eye-piece scale is viewed through only the second or eye lens of the eye piece, while the image of the object scale, which appears to coincide with it, is that formed by refraction at the object-glass and the first or field lens of the eye-piece ; the magnifying power determined as above is that of the combination of object-glass and field lens. To determine the magnifying power for the whole microscope, in this case we must find that of the eye-lens and multiply the two together.

It should be noticed that the magnifying power of a microscope depends on the relative position of the object-glass and eye-piece. Accordingly, it the value of the magnifying power is to be used in subsequent experiments, the focussing of the object viewed must be accomplished by moving the whole instrument.

Experiment.—Determine by both methods the magnifying power of the given microscope.

Enter the results thus:—

First method.—Scale viewed through microscope graduated to half-millimetres. Scale viewed directly graduated to millimetres.

Three divisions of scale seen through microscope cover 129 of scale seen directly.

$$\text{Magnifying power} = \frac{129}{\frac{3}{2}} = 86.$$

Three divisions of scale viewed cover 14·57 divisions of eye-piece scale.

Magnifying power of eye-piece 18.

$$\therefore \text{ Magnifying power of microscope} = \frac{14 \cdot 57}{3} \times 18 = 87 \cdot 4.$$

59. The Testing of Plane Surfaces.

The planeness of a reflecting surface can be tested more accurately by optical means than in any other way.

The method depends on the fact that a pencil of parallel rays remains parallel after reflexion at a plane surface.

To make use of this, a telescope is focussed on a very distant object—so distant that the rays coming from it may be regarded as parallel. The surface to be tested is then placed so that some of the parallel rays from the distant object fall on it and are reflected, and the telescope is turned to receive the reflected rays—to view, that is, the reflected image. If the surface be plane, the reflected rays will be parallel and the image will be as far away as the object. When viewed through the telescope, then, it will be seen quite sharp and distinct. If, on the other hand, the surface be not plane, the rays which enter the object-glass will not be parallel, and the image seen in the telescope will be blurred and indistinct.

We can thus easily test the planeness of a surface. If the surface is found to be defective, the defect may arise in two ways :—

(*a*) From the surface being part of a regular reflecting surface—a sphere or paraboloid, for example—and not plane.

In this case a distinct image of the distant object is formed by reflexion at the surface ; but, the surface not being plane, the pencils forming the image will not be parallel, and therefore, in order to see it, we must alter the

focussing of the telescope. We shall shew shortly how, by measuring the alteration in the position of the eye-piece of the telescope, we can calculate the radius of curvature of the surface.

(*b*) In consequence of the general irregularity of the surface. In this case we cannot find a position of the eye-piece, for which we get a distinct image formed—the best image we can get will be ill-defined and blurred. We may sometimes obtain a definite image by using only a small part of the reflecting surface, covering up the rest. This may happen to give regular reflexion, and so form a good image.

To test roughly the planeness of a surface or to measure its curvature, if the latter be considerable, an ordinary observing telescope may be used.

Focus it through the open window on some distant, well-defined object. A vane, if one be visible, will be found convenient. Place the surface to reflect some of the rays from the distant object at an angle of incidence of about 45°, and turn the telescope to view the reflected image.

If the image is in focus, the surface is plane.

If by altering the focus we can again get a well-defined image, the surface reflects regularly, and is a sphere or something not differing much from a sphere; if the image can never be made distinct and clear, the surface is irregular. Let us suppose we find that by a slight alteration in the focus we can get a good image, we shall shew how to measure the radius of curvature of the surface. To do this accurately, we require a rather large telescope with an object-glass of considerable focal length, say about 1 metre.

It will be better, also, to have a collimator. This consists of a tube with a narrow slit at one end of it and a convex lens at the other, the focal length of the lens being the length of the tube; the slit is accordingly in the principal focus of the lens, and rays of light coming from it are rendered parallel by refraction at the lens. Sometimes a tube carrying the slit slides in one carrying the lens, so that the distance between the two can be adjusted.

We shall suppose further that there is a distinct mark on the telescope tube and another on the sliding tube to which the eye-piece is attached. We shall require to measure the distance between these marks; the line joining them should be parallel to the axis of the telescope. The telescope should also be furnished with cross-wires.

Focus the eye-piece on the cross-wires. Turn the telescope to the distant object and adjust the focussing screw, thus moving both eye-piece and cross-wires relatively to the object-glass, until the object is seen distinctly and without any parallax relatively to the cross-wires. To determine when this is the case move the eye about in front of the eye-piece and note that there is no relative displacement of the image and the cross-wires.

Measure with a millimetre scale, or otherwise, the distance a, say, between the two marks on the telescope tubes. Repeat the observation four or five times. Take the mean of the distances observed and set the instrument so that the distance between the marks is this mean.

Now point the telescope to the collimator, place a lamp behind the slit of the latter, and adjust the distance between the slit and the lens until the slit appears to be properly focussed when viewed through the telescope. When this is the case the rays issuing from the collimator lens are accurately parallel.

Place the reflecting surface to reflect at an angle of incidence of about 45° the light from the collimator, and turn the telescope to view it. When the reflecting material is transparent and has a second surface nearly parallel to the first, the light reflected from it will form an image which will be visible and may cause inconvenience; if this be so, cover the second surface with a piece of wet coloured blotting-paper

We require to know the angle of incidence. To find this accurately it would be necessary to use for the collimator the collimator of a spectrometer and to mount the surface

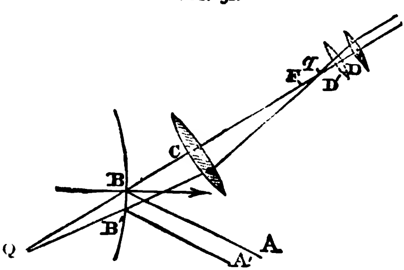
on the table of the spectrometer. The angle then could be found as described in § 62. For most purposes, however, the angle of incidence can be found by some simpler means, e.g. by setting the telescope and collimator so that their axes are at right angles, determining when this is the case by eye or with the help of a square, and then placing the surface so as to bring the reflected image of the slit into the field of view ; the angle required will then not differ much from 45°. Let us call it ϕ. The image seen will not be in focus, but it can be rendered distinct by altering the position of the eye-piece of the telescope. Let this be done four or five times, and measure each time the distance between the two marks on the telescope tubes ; let the mean value be b.

Observe also the distance c between the object-glass and the reflecting surface, this distance being measured parallel to the axis of the telescope. Let F be the focal length of the object-glass, ϕ the angle of incidence, then R the radius of curvature of the reflecting face is, if that face be convex, given by the formula

$$R = 2\,\frac{F^2 + (b-a)(F-c)}{(b-a)\cos\phi}.$$

For let A B (fig. 32) be a ray incident obliquely at B at an angle ϕ, A′ B′ an adjacent parallel ray ; after reflection they will diverge from a point Q behind the surface, and falling on the object-glass C be brought to a focus at q, there forming a real image of the distant object, which is viewed by the eye-piece D. Let F be the principal focus of the object-glass.

FIG. 32.



Then when the distant object was viewed directly, the image fomred by the object-glass was at F, and if D′ be the posi-

tion of the eye-piece adjusted to view it, we have $D'F = Dq$, and hence $Fq = DD'$, but DD' is the distance the eye-piece has been moved; hence we have

$$Fq = b - a, \text{ and } CF = F;$$
$$\therefore \; Cq = F + b - a.$$

Also $CB = c$, and since Q is the primary focal line[1] of a pencil of parallel rays incident at an angle ϕ

$$BQ = \tfrac{1}{2} R \cos\phi;$$
$$\therefore \; CQ = c + \tfrac{1}{2} R \cos\phi.$$

But

$$\frac{1}{CQ} + \frac{1}{Cq} = \frac{1}{F};$$

$$\therefore \; \frac{1}{c + \frac{1}{2} R \cos\phi} = \frac{1}{F} - \frac{1}{F + b - a}$$
$$= \frac{b - a}{F(F + b - a)};$$

$$\therefore \; \tfrac{1}{2} R \cos\phi = \frac{F(F + b - a)}{b - a} - c,$$

and

$$R = 2\,\frac{F^2 + (b - a)(F - c)}{(b - a)\cos\phi}.$$

In the case of a concave surface of sufficiently large radius it will be found that b is less than a; the eye-piece will require pushing in instead of pulling out; and the radius of curvature is given by the formula

$$R = 2\,\frac{F^2 - (a - b)(F - c)}{(a - b)\cos\phi}.$$

We have supposed hitherto that the slit is at right angles to the plane of reflexion, and the primary focus, therefore, the one observed. If the slit be in the plane of reflexion

[1] See Parkinson's *Optics* (edit. 1870), p. 60.

the image seen will be formed at the secondary focal line, and the formula will be

$$R = 2\,\frac{F^2-(a-b)(F-c)}{(a-b)\sec\phi},$$

a, b, c, &c., having the same meaning as before.

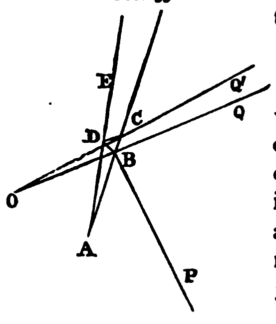
Again let us suppose that the plate of material examined has two faces, each of which has been found to be plane. We can use the method to determine if they are parallel, and if not to find the angle between them.

For make the adjustments as before, removing, however, the wet blotting paper from the back face. If the two faces be strictly parallel only one image of the slit will be seen, for the rays from the front and back surfaces will be parallel after reflexion. If the faces be not parallel, two images of the slit will be seen.

Let us suppose that the angular distance between the two images can be measured either by the circle reading of the spectrometer, if the spectrometer telescope is being used, or by the aid of a micrometer eye-piece if that be more convenient; let this angular distance be D; then the angle between the faces is given by the equation

$$i = \frac{D\cos\phi}{2\mu\cos\phi'},$$

FIG. 33.



where ϕ' is the angle of refraction corresponding to an angle of incidence ϕ, and μ the refractive index of the material; D and i are supposed so small that we may neglect their squares. For (fig. 33) let A B C, A D E be the two faces of the prism, P B Q, P B D C Q′ the paths of two rays; let Q B, Q′ C meet in O, then Q O Q′ = D, B A D = i.

Hence

$$\text{D} = \text{QOQ}' = \text{OBA} - \text{OCA}$$
$$= \tfrac{1}{2}\pi - \phi - \text{OCA},$$
$$\therefore \text{OCA} = \tfrac{1}{2}\pi - \phi - \text{D}.$$

Again

$$\text{DCA} = \text{EDC} - i = \text{ADB} - i$$
$$= \text{DBC} - 2i = \tfrac{1}{2}\pi - \phi' - 2i.$$

Also since DC and CQ′ are the directions of the same ray inside and outside respectively,

$$\cos \text{OCA} = \mu \cos \text{DCA};$$
$$\therefore \sin(\phi + \text{D}) = \mu \sin(\phi' + 2i);$$
$$\therefore \sin\phi + \text{D}\cos\phi = \mu(\sin\phi' + 2i\cos\phi'),$$

neglecting D^2 and i^2.

But

$$\sin\phi = \mu \sin\phi';$$
$$\therefore i = \frac{\text{D}\cos\phi}{2\mu\cos\phi'}.$$

Again, it may happen that one or both faces of the piece of glass are curved; it will then act as a lens, and the following method will give its focal length. The method may be advantageously used for finding the focal length of any long-focussed lens.

Direct the telescope to view the collimator slit, and focus it; interpose the lens in front of the object-glass. The focus of the telescope will require altering to bring the slit distinctly into view again.

Let us suppose that it requires to be pushed in a distance x. Let c be the distance between the lens and the object-glass of the telescope, then the parallel rays from the collimator would be brought to a focus at a distance f behind the lens, i.e. at a distance $f-c$ behind the object-glass; they fall, however, on the object-glass, and are brought by it to a focus at a point distant $\text{F}-x$ from the glass.

$$\therefore \frac{1}{f-c} - \frac{1}{\text{F}-x} = -\frac{1}{\text{F}},$$

and from this we find

$$f = \frac{F^2 - (F - c)x}{x}.$$

If the lens be concave, the eye-piece of the telescope will require pulling out a distance x suppose; and in this case the rays falling on the object-glass will be diverging from a point at a distance $f + c$ in front of it, and will converge to a point at a distance $F + x$ behind it.

$$\therefore \frac{1}{f+c} + \frac{1}{F+x} = \frac{1}{F};$$

$$\therefore f = \frac{F^2 + x(F - c)}{x}.$$

We infer, then, that if the eye-piece requires pushing in the lens is convex, and if it requires pulling out it is concave.

Moreover, we note that all the above formulæ both for reflexion and refraction are simplified if $F = c$; that is to say, if the distance between the object-glass and the reflecting surface or lens, as the case may be, is equal to the focal length of the object-glass.

If this adjustment be made, and if x be the displacement of the eye-piece in either case, we have for the radius of curvature of the surface

$$R = \frac{2F^2}{x \cos \phi},$$

and for the focal length

$$f = \frac{F^2}{x}.$$

Experiments.

(1) Measure the curvature of the faces of the given piece of glass.

(2) If both faces are plane, measure the angle between them.

(3) If either face is curved, measure the focal length of the lens formed by the glass.

Enter results thus:—

(1) Scale used divided to fiftieths of an inch.
Angle of incidence 45°.

First face, concave.

Values of a	.	17·5	17·7	17·5	17·65	17·6	cm.
					Mean	17·59	"
Values of b	.	3·9	3·9	3·8	3·8	3·8	"
					Mean	3·84	"
Value of $a-b$	.	.	.	.	.	13·75	"
Values of c	.	.	.	12·9	13·2	13·0	"
					Mean	13·03	"
Focal length of object-glass			.	.	.	54·3	"
Value of R.	.	.	.	.	.	248·7	"

(2)

$\phi = 45°$
$\mu = 1·496$
$\phi' = 28°12'$
$D = 5'45''$
$i = 1'32''$

(3)

$F = 54$ cm.
$c = 10$ "
$n = 2·35$ "
$f = 11·97$ "

CHAPTER XIV.

SPECTRA, REFRACTIVE INDICES, AND WAVE-LENGTHS.

A BEAM of light generally consists of a combination of differently-coloured sets of rays; the result of the decomposition of a compound beam into its constituents is called a spectrum. If the beam be derived from an illuminated aperture, and the spectrum consist of a series of distinct images of the aperture, one for each constituent set of rays of the compound light, the spectrum is said to be pure.

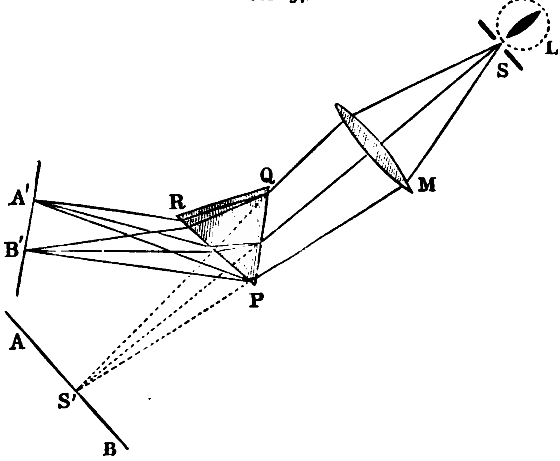
A spectroscope is generally employed to obtain a pure spectrum. The following method of projecting a pure

spectrum upon a screen by means of a slit, lens, and prism, illustrates the optical principles involved.

The apparatus is arranged in the following manner.

The lamp is placed at L, fig. 34, with its flame edgewise to the slit; then the slit S and the lens M are so adjusted as to give a distinct image of the slit at S′ on the screen A B; the length of the slit should be set vertical. The prism P Q R is then placed with its edge vertical to receive the rays after passing through the lens. All the rays from the lens should

FIG. 34.



fall on the front face of the prism, which should be as near to the lens as is consistent with this condition. The rays will be refracted by the prism, and will form a spectrum A′ B′ at about the same distance from the prism as the direct image S′. Move the screen to receive this spectrum, keeping it at the same distance from the prism as before, and turn the prism about until the spectrum formed is as near as possible to the position of S′, the original image of the slit; that is, until the deviation is a minimum. The spectrum thus formed is a pure one, since it contains an image

of the slit for every different kind of light contained in the incident beam.

60. The Spectroscope.

Mapping a Spectrum.

We shall suppose the spectroscope has more than one prism.

Turn the telescope to view some distant object through an open window, and focus it. In doing this adjust first the eye-piece until the cross-wires are seen distinctly, then move the eye-piece and cross-wires by means of the screw until the distant object is clear. The instrument should be focussed so that on moving the eye about in front of the eye-lens no displacement of the image relatively to the cross-wires can be seen.

Remove the prisms, and if possible turn the telescope to look directly into the collimator. Illuminate the slit and focus the collimator until the slit is seen distinctly. Replace one prism and turn the telescope so as to receive the refracted beam. Turn the prism round an axis parallel to its edge until the deviation of some fixed line is a minimum (see § 62, p. 391).

For this adjustment we can use a Bunsen burner with a sodium flame.

If the prism have levelling screws, adjust these until the prism is level.

To test when this is the case fix a hair across the slit, adjusting it so that when viewed directly it may coincide with the horizontal cross-wire of the eye-piece. The hair will be seen in the refracted image cutting the spectrum horizontally. Adjust the levelling screws of the prism until this line of section coincides with the cross-wire.

In some instruments the prisms have no adjusting screws, but their bases are ground by the maker so as to be at right angles to the edge.

Having placed the first prism in position, secure it there

with a clamp, and proceed to adjust the second and other prisms in the same way.

The table of the spectroscope is graduated into degrees and minutes, or in some instruments there is a third tube carrying at one end a scale and at the other a lens whose focal length is the length of the tube. The scale is illuminated from behind by a lamp and is placed so that the rays which issue from the lens fall on the face of the prism nearest the observing telescope, and being there reflected form an image of the scale in the focus of the telescope.

Bring the vertical cross-wire, using the clamp and tangent-screw, over the image of the slit illuminated by the yellow sodium flame and read the scale and vernier, or note the reading of the reflected scale with which it coincides.

Replace the sodium flame by some other source of light the spectrum of which is a line or series of lines, as, for example, a flame coloured by a salt of strontium, lithium, or barium, and take in each case the readings of the reflected scale or of the vernier when the cross-wire coincides with the bright lines.

Now the wave-lengths of these lines are known; we can therefore lay down on a piece of logarithm paper a series of points, the ordinates of which shall represent wave-lengths, while the abscissæ represent the graduations of the circle or scale.

If we make a sufficient number of observations, say from ten to fifteen, we can draw a curve through them, and by the aid of this curve can determine the wave-length of any unknown line; for we have merely to observe the reading of the circle or scale when the cross-wire is over this line and draw the ordinate of the curve corresponding to the reading observed. This ordinate gives the wave-length required.[1]

In using the diagram or 'map' at any future time we must adjust the scale or circle so that its zero occupies the same position with reference to the spectrum. This can be done by arranging that some well-known line—e.g. D—should

[1] See Glazebrook, *Physical Optics*, p. 113.

always coincide with the same scale division or circle reading. The accuracy of readjustment of the spectroscope should also be tested by comparing the reading of some other well-known line with its original reading.

Instead of using the light from a Bunsen burner with metallic salts in the flame, we may employ the electric spark from an induction coil either in a vacuum tube or between metallic points in air.

If the vacuum tube be used, two thin wires from the secondary of the coil are connected to the poles of the tube —pieces of platinum wire sealed into the glass. The primary wire of the coil is connected with a battery of two or three Grove cells, and on making contact with the commutator the spark passes through the tube. This is placed with its narrow portion close up to and parallel to the slit, and the spectroscope observations made as before. If the spark be taken between two metallic poles in air, the two poles placed in the spark-holder are connected with the secondary and placed at a distance of two or three millimetres apart, and the spark passed between them.

The spark-holder is placed in front of the slit, and either the spark is viewed directly or a real image of it is formed on the slit by means of a convex lens of short focus.

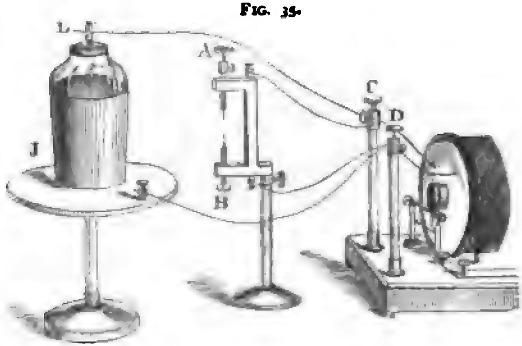
With this arrangement, in addition to the spectrum of the metal formed by the light from the glowing particles of metal, which are carried across between the poles by the spark, we get the spectrum of the air which is rendered incandescent by the passage of the spark. The lines will probably be all somewhat faint, owing to the small quantity of electricity which passes at each discharge.

To remedy this, connect the poles of the secondary coil with the outside and inside coatings of a Leyden jar, as is shewn in fig. 35. Some of the electricity of the secondary coil is used to charge the jar; the difference of potential between the metallic poles rises less rapidly, so that discharges take place less frequently than without the jar; but when the spark does pass, the whole charge of the jar

passes with it, and it is consequently much more brilliant. Even with the jar, the sparks pass so rapidly that the impression on the eye is continuous.[1]

In experiments in which the electric spark is used, it is

FIG. 35.



well to connect the spectroscope to earth by means of a wire from it to the nearest gas-pipe; this helps to prevent shocks being received by the observer.

Sometimes after the spark has been passing for some time it suddenly stops. This is often due to the hammer of the induction coil sticking, and a jerk is sufficient to start it again; or in other cases it is well to turn the commutator of the coil and allow the spark to pass in the other direction.

It may of course happen that the screws regulating the hammer of the coil require adjustment.

Experiments.

Draw a curve of wave-lengths for the given spectroscope, determining the position of ten to fifteen points on it, and by means of it calculate the wave-length of the principal lines of the spectrum of the given metal.

Map the spectrum of the spark passing through the given tubes.

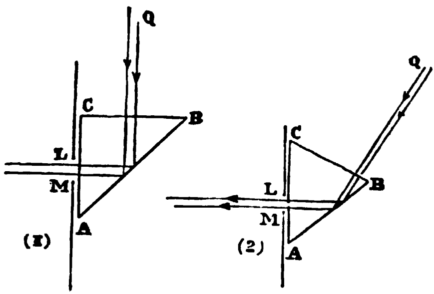
[1] The intensity of the spark may often be sufficiently increased without the use of the jar by having a second small break in the circuit between A and C across which a spark passes.

Comparison of Spectra.

Many spectroscopes are arranged so as to allow the spectra of two distinct sources of light to be examined simultaneously.

To effect this a rectangular prism A B C (fig. 36 [1]) is placed behind the slit of the collimator in such a way as to cover one half, suppose the lower, of the slit.

FIG. 36.



Light coming from one side falls normally on the face B C of this prism, and is totally reflected at the face A B emerging normally from the face C A; it then passes through the slit L M and falls on the object glass of the collimator. In some cases a prism of 60° is used (fig. 36 [2]).

The second source of light is placed directly behind the slit and is viewed over the top of the prism.

One half of the field then, the upper, in the telescope is occupied by the spectrum of the light reflected by the prism, while the other is filled by that of the direct light.

We may use this apparatus to compare the spectra of two bodies.

Suppose we have to determine if a given substance contain strontium.

Take two Bunsen burners and place in one a portion of the given substance on a piece of thin platinum foil, while some strontium chloride moistened with hydrochloric acid is placed in the other on a similar piece of foil. The two spectra are brought into the field. If the strontium lines appear continuous through both spectra, it is clear that the first spectrum is at least in part that of strontium.

As we have seen already, if we pass a spark in air between metallic poles we get the air lines as well as those due to the metal. We may use this comparison method to distinguish between the air lines and those of the metal. For let one set of poles be made of the metal in question, and take for the other set some metal with a simple known spectrum, platinum for example. Arrange the apparatus as described to observe the two spectra. The lines common to both are either air lines or are due to some common impurity of the two metals; the other lines in the one spectrum are those of platinum, in the second they arise from the metal in question.

After practice it is quite easy to recognise the distinctive lines of many substances without actual comparison of their spectra with that of a standard.

Experiment.—Compare the spectra of the sparks passing between platinum poles and poles of the given metal.

Note the wave-lengths of the principal lines in the spark spectrum of the given metal.

On Refractive Indices.

If a ray of homogeneous light fall on a refracting medium at an angle of incidence ϕ, the angle of refraction being ϕ'; then the ratio $\sin\phi/\sin\phi'$ is constant for all values of ϕ, and is the refractive index for light of the given refrangibility going from the first to the second medium.

Let us suppose the first medium is air, then it is not difficult to determine by optical experiments the value of the angle ϕ, but ϕ' cannot be determined with any real approach to accuracy. The determination of μ, the refractive index, is therefore generally effected by indirect means. We proceed to describe some of these.[1]

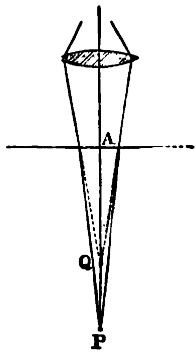
[1] For proofs of the optical formulæ which occur in the succeeding sections, we may refer the reader to Glazebrook's *Physical Optics*, chaps. iv. and viii.

61. Measurement of the Index of Refraction of a Plate by means of a Microscope.

Let P (fig. 37) be a point in a medium of refractive index μ, and let a small pencil of rays diverging from this point fall directly on the plane-bounding surface of the medium and emerge into air.

Let A be the point at which the axis of the pencil emerges, and Q a point on PA, such that $AP = \mu AQ$; then the emergent pencil will appear to diverge from Q, and if we can measure the distances AP and AQ we can find μ. To do this, suppose we have a portion of a transparent medium in the form of a plate, and a microscope, the sliding tube of which is fitted with a scale and vernier or at least a pointer, so that any alteration in the position of the object-glass when the microscope is adjusted to view objects at different distances may be measured.

FIG. 37.



Place under the object-glass a polished disc of metal with a fine cross engraved on it, and bringing the cross into the centre of the field, focus the microscope to view and read the scale. Repeat the observation several times, taking the mean. Now bring between the metal plate and the object-glass the transparent plate, which, of course, must not be of more than a certain thickness. One surface of the plate is in contact with the scratch on the metal, which thus corresponds to the point P; the emergent rays therefore diverge from the point Q, and in order that the scratch may be seen distinctly through the plate, the microscope will require to be raised until its object-glass is the same distance from Q as it was originally from P. Hence, if we again focus the microscope to see the cross, this time through the plate, and read the scale, the difference between the two readings

will give us the distance PQ. Let us call this distance a, and let t be the thickness of the plate, which we can measure by some of the ordinary measuring apparatus, or, if more convenient, by screwing the microscope out until a mark, made for the purpose, on the upper surface of the plate comes into focus, and reading the scale on the tube.

We thus can find $\text{PA} = t$, $\text{PQ} = a$

But we have

$$\text{AP} = \mu\,\text{AQ} = \mu(\text{AP} - \text{PQ});$$

$$\therefore\ t = \mu(t - a),$$

and

$$\mu = \frac{t}{t - a}.$$

A modification of this method is useful for finding the index of refraction of a liquid.

Suppose the liquid to be contained in a vessel with a fine mark on the bottom.

Focus on the mark through the liquid, and then on a grain of lycopodium dust floating on the surface. If the depth be d_1, the difference between the readings gives us d_1/μ; let us call this difference a. Then

$$a = \frac{d_1}{\mu}.$$

Now add some more liquid until the depth is $d_1 + d_2$. Focus on the mark again, and then a second time on the floating lycopodium which has risen with the surface; let the difference between these two be b; then

$$b = \frac{d_1 + d_2}{\mu}.$$

But the difference between the second and fourth reading, that is to say, of the two readings for the lycopodium grains is clearly the depth of liquid added, so that from these two readings d_2 is obtained, and we have

$$b - a = \frac{d_2}{\mu},$$

$$\mu = \frac{d_2}{b - a}.$$

given plate and (2) of the given liquid.

Enter results thus:—

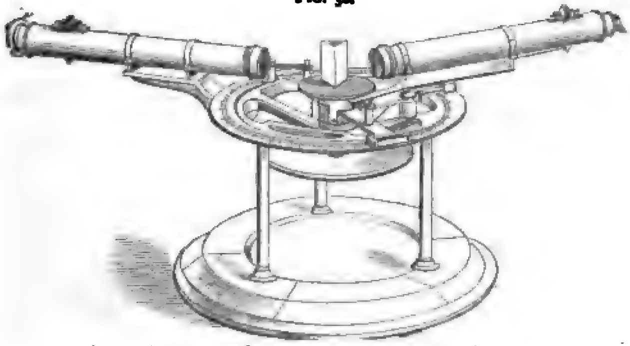
Index of refraction of water.

(2)	a	b	d_a	μ
	1·12	4·54	4·56	1·333
	·95	4·41	4·65	1·344
	·68	4·07	4·56	1·345
	·38	3·76	4·53	1·340
	·43	4·16	4·92	1·319
		Mean value of μ	. .	1·336

62. The Spectrometer.

This instrument (fig. 38) consists of a graduated circle, generally fixed in a horizontal position. A collimator is rigidly attached to the circle. The axis of the collimator is in a plane parallel to that of the circle, and is directed to a point vertically above the centre of the circle. A movable arm,

FIG. 38.



fitted with a clamp and tangent screw, carries an astronomical telescope which is generally provided with a Ramsden's eye-piece and cross-wires. The position of the telescope with reference to the circle is read by means of a vernier.[1] Above the centre of the circle there is a horizontal table, which is generally capable of rotation about the vertical axis of the

[1] See Frontispiece, figs. 5 and 6.

C C

circle; and this table has a vernier attached to it, so that its position can be determined. The whole instrument rests on levelling screws, and the telescope and collimator are held in their positions by movable screws, so that their axes can be adjusted till they are parallel to the circle.

On the Adjustment of a Spectrometer.

The line of collimation, or axes of the telescope and collimator should lie in one plane, and be always perpendicular to the axis about which the telescope rotates. To secure absolute accuracy in this is a complicated problem.

In practice it is usually sufficient to assume that the axes of the telescope and collimator are parallel to the cylindrical tubes which carry the lenses. Level the table of the instrument by means of a spirit-level and the levelling screws, afterwards level separately the telescope and collimator by means of a level and the set screws attached to each. The axis of each is now parallel to the plane of the circle.

See that the clamp and tangent screw work properly, and that the instrument is so placed that the vernier can be read in all positions in which it is likely to be required.

Focus the eye-piece of the telescope on the cross-wires or needle-point. Turn the telescope to some very distant object, and focus the object-glass by the parallel method described on p. 369. Turn the telescope to look into the collimator; illuminate the slit, and then focus it by altering its position with reference to the lens of the collimator. When the slit is in focus, the light issuing from the collimator forms a series of pencils of parallel rays.[1]

[1] This is a very important adjustment; if it be properly carried out the direction of the rays forming an image after reflexion or refraction at the surface of a prism, and hence the circle readings, will be the same, no matter to what extent the prism may be moved parallel to itself about the spectrometer table. In the absence of such an adjustment the measurements would require a prism with indefinitely small width of face and its edge *coincident* with the axis of rotation. It will be seen that the faces of a prism for accurate optical work must be *plane*. A prism which shews by the alteration of focus which it produces that its faces are not plane must be discarded except for roughly approximate measurements.

In experiments in which a prism is used it is generally necessary that the edge of the prism should be parallel to the axis of rotation of the telescope. Turn the telescope to view the slit directly. Fix by means of soft wax a hair or silk fibre across the slit, so that it may appear to coincide with the horizontal cross-wire or point of the needle when seen through the instrument; or, as is often more convenient, cover up part of the slit, making the junction of the covered and uncovered portions coincide with the horizontal wire. Fix the prism with wax or cement on to the levelling table in the centre of the instrument, so that the light from the collimator is reflected from two of its faces, and adjust it by hand, so that the two reflected images of the slit can be brought in turn into the field of view of the telescope. Alter the set screws of the levelling table until the image of the hair across the slit when reflected from either of the two faces, and seen through the telescope, coincides with the intersection of the cross-wires. When this is the case the prism is in the required position.

The edge of the prism may also be adjusted to be parallel to the axis of rotation by setting the two faces successively at right angles to the line of collimation of the telescope. This may be done with great accuracy by the following optical method. Illuminate the cross-wires of the telescope, and adjust the face of the prism so that a reflected image of the cross-wires is seen in the field of view of the telescope coincident with the wires themselves. This can only be the case when the pencil of light from the intersection of the wires is rendered parallel by refraction at the object-glass of the telescope, and reflected normally by the face of the prism, so that each ray returns along its own path (*see* § P). An aperture is provided in the eye-piece tubes of some instruments for the purpose of illuminating the wires; in the absence of any such provision, a piece of plane glass, placed at a suitable angle in front of the eye-piece, may be used. It is sometimes difficult to catch sight

of the reflected image in the first instance, and it is generally advisable, in consequence, to make a rough adjustment with the eye-piece removed, using a lens of low magnifying power instead.

When fixing the prism on to the table, it is best to take care that one face of the prism is perpendicular to the line joining two of the set screws of the levelling table. Level this face first. The second face can then be adjusted by simply altering the third screw, which will not disturb the first face. It is well to place the prism so that the light used passes as nearly as possible through the central portion of the object-glasses of the collimator and telescope.

Measurements with the Spectrometer.

(1) *To verify the Law of Reflexion.*

This requires the table on which the prism is fixed to be capable of motion round the same axis as the telescope, and to have a vernier attached.

Adjust the apparatus so that the reflected image of the slit coincides with the cross-wire, and read the position of the telescope and prism. The slit should be made as narrow as possible.

If the instrument has two verniers for the telescope opposite to each other, read both and take the mean of the readings. Errors of centering are thus eliminated.

Move the prism to another position, adjust the telescope as before, and take readings of the position of the prism and telescope. Subtract these results from the former respectively. It will be found that the angle moved through by the telescope is always twice that moved through by the prism.

(2) *To Measure the Angle of a Prism.*

(*a*) *Keeping the prism fixed*.—Adjust the prism so that an image of the slit can be seen distinctly by reflexion from

each of two of its faces, and its edge is parallel to the axis of rotation of the telescope.

Adjust the telescope so that the image of the slit reflected from one face coincides with the vertical cross-wire, and read the verniers. Move the telescope until the same coincidence is observed for the image reflected from the second face, and read again.

The difference of the two readings is twice the angle required, provided the incident light is parallel.

(*b*) *Keeping the telescope fixed.*—Move the prism until the image of the slit reflected from one face coincides with the vertical cross-wire, and read the verniers for the prism.

Turn the prism until the same coincidence is observed for the other face, and read again.

Then the defect of the difference of the two readings from 180° is the angle required.

Verify by repeating the measurements.

Experiments.

(1) Verify the law of reflexion.

(2) Measure by methods (*a*) and (*b*) the angle of the given prism.

Enter results thus:—

(1) Displacement of telescope .	5° 42′	24° 0′ 15″
" " prism .	2° 51′	12° 0′ 0″

(2) Angle of prism—

By method (*a*)	60° 7′ 30″	60° 7′ 50″	mean 60° 7′ 40″
By method (*b*)	60° 8′ 15″	60° 7′ 45″	mean 60° 8′ 0″

(3) *To Measure the Refractive Index of a Prism.*

First Method.—The spectrometer requires adjusting and the prism levelling on its stand, as before. The angle of the prism must be measured, as described. To obtain an accurate result, it is necessary that the light which falls on the face of the

prism should be a parallel pencil. One method of obtaining this has already been given. The following, due to Professor Schuster, may often be more convenient, and is, moreover, more accurate. Let us suppose that the slit is illuminated with homogeneous light, a sodium flame, for example, and the prism so placed that the light passes through it, being deflected, of course, towards the thick part. Place the telescope so as to view the refracted image. Then it will be found that, on turning the prism round continuously in one direction, the image seen appears to move towards the direction of the incident light, and after turning through some distance the image begins to move back in the opposite direction and again comes into the centre of the field. There are thus, in general, for a given position of the telescope, two positions of the prism, for which the image can be brought into the centre of the field of the telescope. In one of these the angle of incidence is greater than that for minimum deviation, in the other less. Turn the prism into the first of these positions; in general the image will appear blurred and indistinct. Focus the telescope until it is clear. Then turn the prism into the second position. The image now seen will not be clear and in focus unless the collimator happens to be in adjustment. Focus the collimator. Turn the prism back again into the first position and focus the telescope, then again to the second and focus the collimator. After this has been done two or three times, the slit will be in focus without alteration in both positions of the prism, and when this is the case the rays which fall on the telescope are parallel; for since the slit remains in focus, its virtual image formed by the prism is at the same distance from the telescope in the two positions of the prism; that is to say, the distance between the prism and the virtual image of the slit is not altered by altering the angle of incidence, but, since the image corresponds to the primary focal line (*see* § Q), this can only be the case when that distance is infinite—that is, when the rays are parallel on leaving the

prism ; and since the faces of the prism are plane, the rays emerging from the collimator are parallel also. Thus both telescope and collimator may be brought into adjustment.

The simplest method of measuring the refractive index is to observe the angle of the prism and the minimum deviation. We have seen how to measure the former. For the latter, turn the telescope to view the light coming directly from the collimator. When the prism is in position, it of course intercepts the light, but it can generally be turned round so as to allow sufficient light for the purpose to pass on one side of it. Clamp the telescope and adjust with the tangent screw until the intersection of the cross-wires or the end of the needle comes exactly into the centre of the slit; then read the scale and vernier. Repeat the observation several times and take the mean of the readings. If it be impossible to turn the prism without removing it from its place, so as to view the direct image, a method to be described later on may be used.

Turn the prism to receive on one face the light emerging from the collimator, and move the telescope to view the refracted image.

Place the prism so that the deviation of the refracted light is a minimum. To determine this position accurately, turn the prism round the axis of the circle so that the refracted image appears to move towards the direction of the incident light, and continue the motion until the image appears to stop. This position can easily be found roughly. Bring the cross-wire of the telescope to cover the image of the slit, and again turn the prism slightly first one way and then the other. If for motion in both directions the image appears to move away from the direction of the incident light, the prism is in the required position. In general however, for the one direction of rotation the motion of the image will be towards the direct light, and the prism must be turned until the image ceases to move in that direction. The first setting gave us an approximate position for the

prism. By bringing the cross-wires over the image, and then moving the prism, we are able to detect with great ease any small motion which we should not have noticed had there been no mark to which to refer it. Having set the prism, place the telescope, using the clamp and tangent screw so that the cross-wire bisects the image of the slit, and read the vernier.

Displace the prism and telescope, set it again, and take a second reading. Repeat several times. The mean of the readings obtained will be the minimum deviation reading, and the difference between it and the mean of the direct readings the minimum deviation. With a good instrument and reasonable care the readings should not differ among themselves by more than 1′.

Having obtained the minimum deviation D, and the angle of the prism i, the refractive index μ is given by

$$\mu = \frac{\sin \frac{1}{2}(D+i)}{\sin \frac{1}{2} i}.$$

To check the result, the prism should be turned so that the other face becomes the face of incidence, and the deviation measured in the opposite direction.

If we cannot observe the direct light, we may note the deviation reading on each side of it—that is, when first one face and then the other is made the face of incidence—the difference between the two readings is twice the minimum deviation required, while half their sum gives the direct reading.

To determine the refractive index of a liquid we must enclose it in a hollow prism, the faces of which are pieces of accurately worked plane parallel glass, and measure its refractive index in the same way as for a solid.

Experiment.—Determine the refractive index of the given prism.

Enter results thus:—

	Direct reading	Deviation reading (1)	Deviation reading (2)
	183° 15′ 40″	143° 29′	223° 2′
	183° 15′ 50″	143° 28′ 50	223° 1′ 30″
	183° 15′ 30″	143° 29′ 10″	223° 1′ 30″
Mean	183° 15′ 40″	143° 29′	223° 1′ 40″

Deviation (1)	39° 46′ 40″
Deviation (2)	39° 46′ 0″
Mean	39° 46′ 20″
Angle of the prism . . .	60° 0′ 0″

Hence $\mu = 1{\cdot}5295$.

Second Method.—The following is another method of measuring the refractive index, which is useful if the angle of the prism be sufficiently small. Let the light from the collimator fall perpendicularly on the face of incidence. Then if i be the angle of the prism and D the deviation, since, using the ordinary notation,

$$\phi = \phi' = 0;$$

$$\therefore \psi' = i \qquad \psi = \mathrm{D} + i,$$

and $\mu = \sin\psi/\sin\psi' = \sin(\mathrm{D}+i)/\sin i.$

We require to place the prism so that the face of incidence is at right angles to the incident light.

Turn the telescope to view the direct light and read the vernier.

Place the prism in position and level it, as already described. Turn the telescope so that the vernier reading differs by 90° from the direct reading. Thus, if the direct reading be 183° 15′ 30″, turn the telescope till the vernier reads 273° 15′ 30″. This can easily be done by the help of the clamp and tangent screw. Clamp the telescope in this position; the axes of the collimator and telescope are now at right angles.

Turn the prism until the image of the slit reflected from one face comes into the field, and adjust it until there is

coincidence between this image and the cross-wire. The light falling on the prism is turned through a right angle by the reflexion. The angle of incidence is therefore 45° exactly. Read the vernier attached to the table on which the prism rests, and then turn the prism through 45° exactly, so as to decrease the angle of incidence; then the face of incidence will evidently be at right angles to the incident light. Now turn the telescope to view the refracted image, and read the vernier; the difference between the reading and the direct reading is the deviation. The angle of the prism can be measured by either of the methods already described; it must be less than $\sin^{-1}(1/\mu)$, which for glass is about 42°, otherwise the light will not emerge from the second face, but be totally reflected there. The refractive index can now be calculated from the formula.

A similar observation will give us the angle of incidence at which the light falls on any reflecting surface; thus turn the telescope to view the direct light, and let the vernier reading be α, then turn it to view the reflected image, and let the reading be β. Then $\alpha-\beta$ measures the deflection of the light, and if ϕ be the angle of incidence, we can shew that the deviation is $180°-2\phi$.

$$\therefore\ 180°-2\phi=\alpha-\beta;$$
$$\therefore\ \phi=90°-\tfrac{1}{2}(\alpha-\beta).$$

Experiment.—Determine the refractive index of the given prism for sodium light.

Enter the results thus:—

Angle of prism		15° 35′ 20″
	Direct reading	Deviation reading
	183° 15′ 10″	191° 53′ 30″
	183° 15′ 50″	191° 54′ 20″
	183° 15′ 30″	191° 53′ 40″
Mean	183° 15′ 30″	191° 53′ 50″
Deviation . . .		8° 38′ 20″
Value of μ . . .		1·5271.

(4) *To Measure the Wave-Length of Light by means of a Diffraction Grating.*

A diffraction grating consists of a number of fine lines ruled at equal distances apart on a plate of glass—a transmission grating; or of speculum metal—a reflexion grating. We will consider the former. If a parallel pencil of homogeneous light fall normally on such a grating, the origin of light being a slit parallel to the lines of the grating, a series of diffracted images of the slit will be seen, and if θ_n be the deviation of the light which forms the nth image, reckoning from the direction of the incident light, d the distance between the centres of two consecutive lines of the grating, and λ the wave-length, we have

$$\lambda = \frac{1}{n} d \sin \theta_n.$$

The quantity d is generally taken as known, being determined at the time of ruling the grating. The spectrometer is used to determine θ_n.

The telescope and collimator are adjusted for parallel rays, and the grating placed on the table of the instrument with its lines approximately parallel to the slit. For convenience of adjustment it is best to place it so that its plane is at right angles to the line joining two of the levelling screws. The grating must now be levelled, i.e. adjusted so that its plane is at right angles to the table of the spectrometer. This is done by the method described above for the prism. Then place it with its plane approximately at right angles to the incident light, and examine the diffracted images of the slit. The plane of the grating is at right angles to the line joining two of the levelling screws; the third screw then can be adjusted without altering the angle between the plane of the grating and the table of the spectrometer. Adjust the third screw until the slit appears as distinct as possible; the lines of the grating will then be parallel to the slit.

Turn the table carrying the grating so as to allow the direct light to pass it; adjust the telescope so that the vertical cross-wire bisects the image of the slit seen directly, and read the vernier. This gives us the direct reading. Place the grating with its plane accurately perpendicular to the incident rays, as described above (p. 393), and turn the telescope to view the diffracted images in turn, taking the corresponding readings of the vernier. The difference between these and the direct reading gives us the deviations θ_1, θ_2, &c. A series of diffracted images will be formed on each side of the direct rays. Turn the telescope to view the second series, and we get another set of values of the deviation θ'_1, θ'_2, &c. If we had made all our adjustments and observations with absolute accuracy, the corresponding values θ_1, θ'_1, &c., would have been the same; as it is their mean will be more accurate than either.

Take the mean and substitute in the formula

$$\lambda = \frac{1}{n} d \sin \theta_n.$$

We thus obtain a set of values of λ.

If the light be not homogeneous, we get, instead of the separate images of the slit, more or less continuous spectra, crossed it may be, as in the case of the solar spectrum, by dark lines, or consisting, if the incandescent body be gas at a low pressure, of a series of bright lines.

In some cases it is most convenient to place the grating so that the light falls on it at a known angle, ϕ say. Let ψ be the angle which the diffracted beam makes with the normal to the grating, and θ the deviation for the nth image, ϕ and ψ being measured on the same side of the normal, then it may be shewn that

$$\theta = \phi + \psi$$

and

$$\begin{aligned} n\lambda &= d(\sin\phi + \sin\psi) \\ &= d\{\sin\phi + \sin(\theta - \phi)\}. \end{aligned}$$

The case of greatest practical importance is when the deviation is a minimum, and then $\phi = \psi = \frac{1}{2}\theta$, so that if θ_n denote the minimum deviation for the nth diffracted image, we have

$$\lambda = \frac{2}{n} d \sin \tfrac{1}{2}\theta_n.$$

In the case of a reflexion grating, if ϕ and ψ denote the angles between the normal and the incident and reflected rays respectively, ϕ and ψ now being measured on opposite sides of the normal, the formula becomes

$$n\lambda = d(\sin\psi - \sin\phi);$$

and if θ be the deviation

$$\theta = \pi - (\phi + \psi).$$

If the value of d be unknown, it may be possible to find it with a microscope of high power and a micrometer eye-piece. A better method is to use the grating to measure θ_n for light of a known wave-length. Then in the formula, $n\lambda = d \sin \theta_n$, we know λ, n, and θ_n, and can therefore determine d.

Experiment.—Determine by means of the given grating the wave-length of the given homogeneous light.

Value $d = \frac{1}{3000}$ Paris inch

$= \cdot 0009023$ cm.

Values of deviations, each the mean of three observations—

			Mean
1	3° 44′ 30″	3° 44′ 45″	3° 44′ 37″·5
2.	7° 29′ 0″	7° 29′ 45″	7° 29′ 22″·5
3	11° 16′ 45″	11° 17′ 30″	11° 17′ 7″·5

	Tenth metres[1]
Values of λ	5895
	5893
	5915
Mean	5901

[1] A 'tenth metre' is 1 metre divided by 10^{10}.

63. The Optical Bench.

The optical bench (fig. 39) consists essentially of a graduated bar carrying three upright pieces, which can slide along the bar; the second upright from the right in the

FIG. 39.

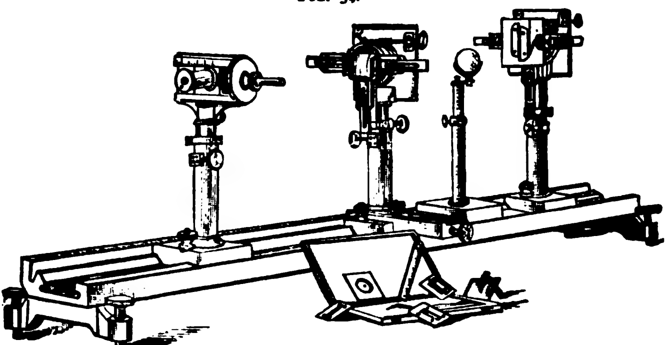


figure is an addition to be described later. The uprights are provided with verniers, so that their positions relatively to the bar can be read. To these uprights are attached metal jaws capable of various adjustments; those on the first and second uprights can rotate about a vertical axis through its centre and also about a horizontal axis at right angles to the upright; they can also be raised and lowered.

The second upright is also capable of a transverse motion at right angles to the length of the bar, and the amount of this motion can be read by means of a scale and vernier. The jaws of the first upright generally carry a slit, those of the second are used to hold a bi-prism or apparatus required to form the diffraction bands.

To the third upright is attached a Ramsden's eye-piece in front of which is a vertical cross-wire; and the eye-piece and cross-wire can be moved together across the field by means of a micrometer screw. There is a scale attached to the frame above the eye-piece, by which the amount of displacement can be measured. The whole turns of the screw are read on the scale by means of a pointer attached

to the eye-piece. The fractions of a turn are given by the graduations of the micrometer head.

The divisions of the scale are half-millimetres and the micrometer head is divided into 100 parts.

(1) *To Measure the Wave-Length of Light by means of Fresnel's Bi-prism.*

The following adjustments are required :—

(1) The centre of the slit, the centre of the bi-prism, and the centre of the eye-piece should be in one straight line.

(2) This line should be parallel to the graduated scale of the bench.

(3) The plane face of the bi-prism should be at right angles to this line.

(4) The plane of motion of the eye-piece should also be at right angles to the same line.

(5) The cross-wire in the eye-piece, the edge of the prism, and the slit should be parallel to each other, and vertical, that is to say, at right angles to the direction of motion of the eye-piece.

To describe the adjustments, we shall begin with (5).

Focus the eye-piece on the cross-wire, and by means of the flat disc to which it is attached, turn the latter round the axis of the eye-piece until it appears to be vertical; in practice the eye is a sufficiently accurate judge of when this is the case.

Draw the third upright some way back, and insert between it and the slit a convex lens.[1] Illuminate the slit by means of a lamp, and move the lens until a real image of the slit is formed in the plane of the cross-wire. Turn the slit round by means of the tangent screw until this image is parallel to the cross-wire. The slit must be held securely and without shake in the jaws.

Move the eye-piece up to the slit and adjust the vertical and micrometer screws until the axis of the eye-piece appears to pass nearly through the centre of the slit, turning at the same time the eye-piece round the vertical axis until its axis appears parallel to the scale. This secures (4) approximately.

[1] This is shewn in the figure.

Draw the eye-piece away from the slit, say 20 or 30 cm. off, and place the bi-prism in position, turning it about until its plane face appears to be at right angles to the scale of the bench. This secures (3) approximately.

Look through the eye-piece. A blurred image of Fresnel's bands may probably be visible. By means of the traversing screw move the second upright at right angles to the scale until this image occupies the centre of the field. If the bands be not visible, continue to move the screw until they come into the field.

It may be necessary to alter the height of the bi-prism by means of the vertical adjustment so that its centre may be at about the same level as those of the slit and eye-piece.

By means of the tangent screw turn the bi-prism round the horizontal axis at right angles to its own plane until the lines appear bright and sharp.

Adjustment (5) is then complete.

Now draw the eye-piece back along the scale; if the lines still remain in the centre of the field of view, it follows that the slit, the centre of the bi-prism, and the centre of the eye-piece are in one straight line parallel to the scale.

If this be not the case, alter the position of the eye-piece by means of the micrometer screw and that of the bi-prism by means of the traversing screw with which the second stand is furnished, until the lines are seen in the centre of the field for all positions of the eye-piece along the scale bar of the instrument.

Adjustments (1) and (2) have thus been effected.

For (3) and (4) it is generally sufficient to adjust by eye, as already described. If greater accuracy be required, the following method will secure it.

Move the lamp to one side of the slit and arrange a small mirror so as to reflect the light through the slit and along the axis of the instrument. The mirror must only cover one-half of the slit, which will have to be opened somewhat widely. Place your eye so as to look through the other half of the slit in the same direction as the light. Images

of the slit reflected from the faces of the bi-prism and probably from other parts of the apparatus will be seen.

Suppose the flat face of the bi-prism is towards the slit. Turn the prism round a vertical axis until the image reflected at the flat face appears directly behind the centre-line of the bi-prism, then clearly the plane of the bi-prism is at right angles to the incident light, and that is parallel to the scale.

In making the adjustment, the stand holding the prism should be placed as far as may be from the slit.

If the bevelled face be towards the slit, two images will be seen, and these must be adjusted symmetrically one on each side of the centre.

To adjust the eye-piece employ the same method, using the image reflected from the front lens or from one of the brass plates which are parallel to it. To do this it may be necessary to remove the bi-prism—if this be the case, the eye-piece adjustment must be made first.

As soon as the adjustments are made the various moving pieces must be clamped securely.

It is necessary for many purposes to know the distance between the slit and the cross-wire or focal plane of the eye-piece. The graduations along the bar of the instrument will not give us this directly; for we require, in addition, the horizontal distance between the zero of the vernier and the slit or cross-wire respectively.

To allow for these, take a rod of known length, a centimetres suppose; place one end in contact with the slit, and bring up the eye-piece stand until the other end is in the focal plane. Read the distance as given by the scale between the slit and eye-piece uprights; let it be b centimetres.

Then clearly the correction $a-b$ centimetres must be added to any scale reading to give the distance between the slit and the eye-piece. This correction should be determined before the bi-prism is finally placed in position.

To use the bi-prism to measure λ, the wave-length of

Draw the eye-piece away from the slit, say 20 or 30 cm. off, and place the bi-prism in position, turning it about until its plane face appears to be at right angles to the scale of the bench. This secures (3) approximately.

Look through the eye-piece. A blurred image of Fresnel's bands may probably be visible. By means of the traversing screw move the second upright at right angles to the scale until this image occupies the centre of the field. If the bands be not visible, continue to move the screw until they come into the field.

It may be necessary to alter the height of the bi-prism by means of the vertical adjustment so that its centre may be at about the same level as those of the slit and eye-piece.

By means of the tangent screw turn the bi-prism round the horizontal axis at right angles to its own plane until the lines appear bright and sharp.

Adjustment (5) is then complete.

Now draw the eye-piece back along the scale; if the lines still remain in the centre of the field of view, it follows that the slit, the centre of the bi-prism, and the centre of the eye-piece are in one straight line parallel to the scale.

If this be not the case, alter the position of the eye-piece by means of the micrometer screw and that of the bi-prism by means of the traversing screw with which the second stand is furnished, until the lines are seen in the centre of the field for all positions of the eye-piece along the scale bar of the instrument.

Adjustments (1) and (2) have thus been effected.

For (3) and (4) it is generally sufficient to adjust by eye, as already described. If greater accuracy be required, the following method will secure it.

Move the lamp to one side of the slit and arrange a small mirror so as to reflect the light through the slit and along the axis of the instrument. The mirror must only cover one-half of the slit, which will have to be opened somewhat widely. Place your eye so as to look through the other half of the slit in the same direction as the light. Images

of the slit reflected from the faces of the bi-prism and probably from other parts of the apparatus will be seen.

Suppose the flat face of the bi-prism is towards the slit. Turn the prism round a vertical axis until the image reflected at the flat face appears directly behind the centre-line of the bi-prism, then clearly the plane of the bi-prism is at right angles to the incident light, and that is parallel to the scale.

In making the adjustment, the stand holding the prism should be placed as far as may be from the slit.

If the bevelled face be towards the slit, two images will be seen, and these must be adjusted symmetrically one on each side of the centre.

To adjust the eye-piece employ the same method, using the image reflected from the front lens or from one of the brass plates which are parallel to it. To do this it may be necessary to remove the bi-prism—if this be the case, the eye-piece adjustment must be made first.

As soon as the adjustments are made the various moving pieces must be clamped securely.

It is necessary for many purposes to know the distance between the slit and the cross-wire or focal plane of the eye-piece. The graduations along the bar of the instrument will not give us this directly; for we require, in addition, the horizontal distance between the zero of the vernier and the slit or cross-wire respectively.

To allow for these, take a rod of known length, a centimetres suppose; place one end in contact with the slit, and bring up the eye-piece stand until the other end is in the focal plane. Read the distance as given by the scale between the slit and eye-piece uprights; let it be b centimetres.

Then clearly the correction $a-b$ centimetres must be added to any scale reading to give the distance between the slit and the eye-piece. This correction should be determined before the bi-prism is finally placed in position.

To use the bi-prism to measure λ, the wave-length of

image by reflexion at a large angle of incidence from a plane glass surface (Lloyd's Experiment).

Diffraction Experiments.

The apparatus may be used to examine the effects of diffraction by various forms of aperture.

The plate with the aperture is placed in the second upright in the place of the bi-prism.

If we have a single edge at a distance a from the slit, and if b be the distance between the edge and the eye-piece, x the distance between two bright lines

Then[1]

$$x = \sqrt{\left\{\frac{\lambda(a+b)b}{a}\right\}}$$

If the obstacle be a fibre of breadth c, then $x = \frac{b\lambda}{c}$, where b is distance between the fibre and the screen or eye-piece.

This formula, with a knowledge of the wave-length of the light, may be used to measure the breadth of the fibre. (Young's Eriometer.)

In order to obtain satisfactory results from diffraction experiments a very bright beam of light is required. It is best to use sunlight if possible, keeping the beam directed upon the slit of the optical bench by means of a heliostat.

Experiments.—Measure the wave-length of light by means of the bi-prism.

Enter results thus:—

$a = 56$ cm.
$x = {\cdot}0359$ cm., (mean of 5)
$c = {\cdot}092$ cm., („ 3)
$\lambda = {\cdot}0000589$ cm.

[1] Glazebrook's *Physical Optics*, p. 172.

CHAPTER XV.

POLARISED LIGHT.

On the Determination of the Position of the Plane of Polarisation.[1]

THE most important experiments to be made with polarised light consist in determining the position of the plane of polarisation, or in measuring the angle through which that plane has been turned by the passage of the light through a column of active substance, such as a solution of sugar, turpentine, or various essential oils, or a piece of quartz.

The simplest method of making this measurement is by the use of a Nicol's or other polarising prism. This is mounted in a cylindrical tube which is capable of rotation about its own axis. A graduated circle is fixed with its centre in the axis of the tube, and its plane at right angles to the axis, and a vernier is attached to the tube and rotates with it, so that the position, with reference to the circle, of a fiducial mark on the tube can be found. In some cases the vernier is fixed and the circle turns with the Nicol. If we require to find the position of the plane of polarisation of the incident light, we must, of course, know the position of the principal plane of the Nicol relatively to the circle. If we only wish to measure a rotation a knowledge of the position of this plane is unnecessary, for the angle turned through by the Nicol is, if our adjustments be right, the angle turned through by the plane of polarisation.

For accurate work two adjustments are necessary :—

(1) All the rays which pass through the Nicol should be parallel.

(2) The axis of rotation of the Nicol should be parallel to the incident light.

To secure the first, the source of light should be small;

[1] See Glazebrook, *Physical Optics*, chap. xiv.

in many cases a brightly illuminated slit is the best. It should be placed at the principal focus of a convex lens; the beam emerging from the lens will then consist of parallel rays.

To make the second adjustment we may generally consider the plane ends of the tube which holds the Nicol as perpendicular to the axis of rotation. Place a plate of glass against one of these ends and secure it in this position with soft wax or cement. The incident beam falling on this plate is reflected by it. Place the plate so that this beam after reflexion retraces its path. This is not a difficult matter; if, however, special accuracy is required, cover the lens from which the rays emerge with a piece of paper with a small hole in it, placing the hole as nearly as may be over the centre of the lens. The light coming through the hole is reflected by the plate, and a spot of light is seen on the paper. Turn the Nicol about until this spot coincides with the hole; then the incident light is evidently normal to the plate—that is, it is parallel to the axis of rotation of the Nicol.

If still greater accuracy be required, the plate of glass may be dispensed with, and a reflexion obtained from the front face of the Nicol. This, of course, is not usually normal to the axis, and hence the reflected spot will never coincide with the hole, but as the Nicol is turned, it will describe a curve on the screen through which the hole is pierced. If the axis of rotation have its proper position and be parallel to the direction of the incident light, this curve will be a circle with the hole as centre. The Nicol then must be adjusted until the locus of the spot is a circle with the hole as centre.

When these adjustments are completed, if the incident light be plane-polarised, and the Nicol turned until there is no emergent beam, the plane of polarisation is parallel to the principal plane of the Nicol; and if the plane of polarisation be rotated and the Nicol turned again till the emergent beam is quenched, the angle turned through by

the Nicol measures the angle through which the plane of polarisation has been rotated.

But it is difficult to determine with accuracy the position of the Nicol for which the emergent beam is quenched. Even when the sun is used as a source of light, if the Nicol be placed in what appears to be the position of total extinction, it may be turned through a considerable angle without causing the light to reappear. The best results are obtained by using a very bright narrow line of light as the source—the filament of an incandescence lamp has been successfully employed by Mr. McConnel—as the Nicol is turned, a shadow will be seen to move across this line from one end to the other, and the darkest portion of the shadow can be brought with considerable accuracy across the centre of the bright line. Still, for many purposes, white light cannot be used, and it is not easy to secure a homogeneous light of sufficient brightness. Two principal methods have been devised to overcome the difficulty; the one depends on the rotational properties of a plate of quartz cut normally to its axis; the other, on the fact that it is comparatively easy to determine when two objects placed side by side are equally illuminated if the illumination be only faint. We proceed to describe the two methods.

64. The Bi-quartz.

If a plane-polarised beam of white light fall on a plate of quartz cut at right angles to its axis, it has, as we have said, its plane of polarisation rotated by the quartz. But, in addition to this, it is found that the rays of different wave-lengths have their planes of polarisation rotated through different angles. The rotation varies approximately inversely as the square of the wave-length; and hence, if the quartz be viewed through another Nicol's prism, the proportion of light which can traverse this second Nicol in any position will be different for different colours, and the quartz will appear coloured. Moreover, the colour will vary as the

analysing Nicol, through which the quartz is viewed, is turned round. If the quartz be about 3·3 mm. in thickness, for one position of the Nicol it will appear of a peculiar neutral grey tint, known as the tint of passage. A slight rotation in one direction will make it red, in the other blue. After a little practice it is easier to determine, even by eye, when this tint appears, than to feel certain when the light is completely quenched by a Nicol. It can be readily shewn moreover that when the quartz gives the tint of passage, the most luminous rays, those near the Fraunhofer line E, are wanting from the emergent beam; and if the quartz have the thickness already mentioned, the plane of polarisation of these rays has been turned through 90°.

A still more accurate method of making the observation is afforded by the use of a bi-quartz. Some specimens of quartz produce a right-handed, others a left-handed rotation of the plane of polarisation of light traversing them. A bi-quartz consists of two semicircular plates of quartz placed so as to have a common diameter. The one is right-handed, the other left. The two plates are of the same thickness, and therefore produce the same rotation, though in opposite directions, in any given ray. If, then, plane-polarised white light pass normally through the bi-quartz, the rays of different refrangibilities are differently rotated, and that too in opposite directions by the two halves, and if the emergent light be analysed by a Nicol, the two halves will appear differently coloured. If, however, we place the analysing Nicol so as to quench in each half of the bi-quartz the ray whose plane of polarisation is turned through 90°—that is to say, with its principal plane parallel to that of the polariser—light of the same wave-length will be absent from both halves of the field, and the other rays will be present in the same proportions in the two; and if the thickness of the bi-quartz be about 3·3 mm. this common tint will be the tint of passage. A very slight rotation of the analyser in one direction renders one half red, the other blue, while if

the direction of rotation be reversed, the first half becomes blue, the second red. Hence the position of the plane of polarisation of the ray which is rotated by the bi-quartz through a certain definite angle can be very accurately determined.

A still better plan is to form the light after passing the analyser into a spectrum. If this be done in such a way as to keep the rays coming from the two halves of the bi-quartz distinct—e.g. by placing a lens between the bi-quartz and the slit and adjusting it to form a real image of the bi-quartz on the slit, while at the same time the slit is perpendicular to the line of separation of the two halves—two spectra will be seen, each crossed by a dark absorption band. As the analysing Nicol is rotated the bands move in opposite directions across the spectrum, and can be brought into coincidence one above the other. This can be done with great accuracy and forms a very delicate method. Or we may adopt another plan with the spectroscope: we may use a single piece of quartz and form the light which has passed through it into a spectrum, which will then be crossed by a dark band; this can be set to coincide with any part of the spectrum. This is best done by placing the telescope so that the cross-wire or needle-point may coincide with the part in question, and then moving the band, by turning the analyser, until its centre is under the cross-wire.

FIG. 40.

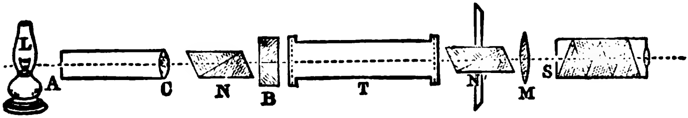


Fig. 40 gives the arrangement of the apparatus: L is the lamp, A the slit, and C the collimating lens. The parallel rays fall on the polarising Nicol N and the bi-quartz B. They then traverse the tube T containing the active rotatory substance and the analysing Nicol N′, falling

on the lens M which forms an image of the bi-quartz on the slit S of the small direct-vision spectroscope. If we wish to do without the spectroscope, we can remove both it and the lens M and view the bi-quartz either with the naked eye or with a lens or small telescope adjusted to see it distinctly. If we use the single quartz, we can substitute it for the bi-quartz, and focus the eye-piece of the telescope to see the first slit A distinctly, and thus observe the tint of passage.

The quartz plate may be put in both cases at either end of the tube T. If it be placed as in the figure, and the apparatus is to be used to measure the rotation produced by some active substance, the tube should in the first instance be filled with water, for this will prevent the necessity of any great alteration in the adjustment of the lens M or in the focussing of the telescope, if the lens be not used, between the two parts of the experiment.

The mode of adjusting the Nicols has been already described.

The light should traverse the quartz parallel to its axis, and this should be at right angles to its faces. This last adjustment can be made by the same method as was used for placing the axis of the Nicol in the right position, provided the maker has cut the quartz correctly. In practice it is most convenient to adjust the quartz by hand, until the bands formed are as sharp and clear as may be.

Care must be taken that each separate piece of the apparatus is securely fastened down to the table to prevent any shake or accidental disturbance.

If a lens is used at M, it is best to have it secured to the tube which carries the analysing Nicol, its centre being on the axis of this tube; by this means it is fixed relatively to the Nicol, and the light always comes through the same part of the lens. This is important, for almost all lenses exert a slight depolarising effect on light, which differs appreciably in different parts of the lens. For most purposes

this is not very material, so long as we can be sure that the effect remains the same throughout our observations. This assurance is given us, provided that the properties of the lens are not altered by variations of temperature, if the lens be fixed with reference to the principal plane of the analyser, so that both lens and analyser rotate together about a common axis.

One other point remains to be noticed. If equality of tint be established in any position, and the analyser be then turned through 180°, then, if the adjustments be perfect, there will still be equality of tint. To ensure accuracy we should take the readings of the analysing Nicol in both these positions. The difference between the two will probably not be exactly 180°; this arises mainly from the fact that the axis of rotation is not accurately parallel to the light. The mean of the two mean readings will give a result nearly free from the error, supposing it to be small, which would otherwise arise from this cause.

To attain accuracy in experiments of this kind needs considerable practice.

Experiments.

(1) Set up the apparatus and measure the rotation produced by the given plate of quartz.

(2) Make solutions of sugar of various strengths, and verify the law that the rotation for light of given wave-length varies as the quantity of sugar in a unit of volume of the solution.

Enter results thus:—

Thickness of quartz :—

1·01 cm. 1·012 cm. 1·011 cm. Mean 1·011 cm.

Analyser readings without quartz plate.

	Position A		Position B
	6° 7′		186° 10′
	6° 9′		186° 12′
	6° 8′		186° 9′
	6° 6′		186° 11′
Mean	6° 7′ 30″	Mean	186° 10′ 30″

Mean of the two . . . . 96° 9′

Analyser readings with quartz plate.

	Position A		Position B
	280° 47′		360 + 100° 48′
	280° 45′		+ 100° 47′
	280° 46′		100° 49′
	280° 48′		100° 50′
Mean	280° 46′ 30″	Mean	360 + 100° 48′ 30″

Mean of the two	370° 47′ 30
Mean rotation	274° 38 30
Rotation deduced from position A .	274° 39′ 0″
" " " B .	274° 38′ 0″

65. Shadow Polarimeters.

The theory of these, as has been stated, all turns on the fact that it is comparatively easy to determine when two objects placed side by side are equally illuminated, the illumination being faint.

Suppose, then, we view through a small telescope or eye-piece placed behind the analyser a circular hole divided into two parts across a diameter, and arranged in such a way that the planes of polarisation of the light emerging from the two halves are inclined to each other at a small angle. For one position of the analyser one half of the field will be black, for another, not very different, the other half will be black, and for an intermediate position the two halves will have the same intensity. The analyser can be placed with the greatest nicety in the position to produce this. If now the planes of polarisation of the light from the two halves of the field be each rotated through any the same angle and the analyser turned until equality of shade is re-established, the angle through which the analyser turns measures the angle through which the plane of polarisation has been rotated.

Whatever method of producing the half-shadow field be adopted, the arrangement of apparatus will be similar to that shewn in fig. 40, only B will be the half-shadow plate,

and instead of the lens M and the spectroscope S we shall have a small telescope adjusted to view the plate B.

In nearly all cases homogeneous light must be used for accurate work. Excellent results can be obtained by placing a bead of sodium on a small spoon of platinum gauze just inside the cone of a Bunsen burner, and then allowing a jet of oxygen to play on the gauze.

Lord Rayleigh has found that a good yellow light is given by passing the gas supplied to a Bunsen burner through a small cylinder containing a finely divided salt of sodium, keeping the cylinder at the same time in a state of agitation, while Dr. Perkin passes the gas over metallic sodium in an iron tube which is kept heated. The brilliancy of the light is much increased by mixing oxygen with the coal gas as in the oxyhydrogen light.

Whenever a sodium flame is used, it is necessary that the light should pass through a thin plate of bichromate of potassium, or through a small glass cell containing a dilute solution of the same salt, to get rid of the blue rays from the gas.

In almost all cases the half-shadow arrangement may be attached to either the polariser or the analyser. If the latter plan be adopted, it must, of course, turn with the analyser, and this is often inconvenient; the other arrangement, as shewn in fig. 40, labours under the disadvantage that the telescope requires readjusting when the tube with the rotating liquid is introduced.

We will mention briefly the various arrangements which have been suggested[1] for producing a half-shadow field, premising, however, that as the sensitiveness depends both on the brightness of the light and the angle between the planes of polarisation in the two halves of the field, it is convenient to have some means of adjusting the latter. With a bright light this angle may conveniently be about 2°.

It is also important that the line of separation between

[1] See also Glazebrook, *Physical Optics*, chap. xiv.

the two halves should be very narrow, and sharp, and distinct.

(1) Jellett's prism :—

The ends of a long rhomb of spar are cut off at right angles to its length, and then the spar cut in two by a plane parallel to its length and inclined at a small angle to the longer diagonal of the end-face. One half is turned through 180° about an axis at right angles to this plane, and the two are reunited.

If a narrow beam of parallel rays fall normally on one end of such an arrangement, the ordinary rays travel straight through without deviation, but their planes of polarisation in the two halves are inclined to each other at a small angle. The extraordinary rays are thrown off to either side of the apparatus, and if the prism be long enough and the beam not too wide, they can be separated entirely from the ordinary rays and stopped by a diaphragm with a small circular hole in it through which the ordinary rays pass.

(2) Cornu's prism :—

A Nicol or other polarising prism is taken and cut in two by a plane parallel to its length. A wedge-shaped piece is cut off one half, the edge of the wedge being parallel to the length of the prism, and the angle of the wedge some 3°. The two are then reunited, thus forming two half-Nicols, with their principal planes inclined at a small angle. The light emerging from each half is plane-polarised, the planes being inclined at a small angle.

Both of these suffer from the defects that the angle between the planes of polarisation is fixed and that the surface of separation of the two halves being considerable, unless the incident light is very strictly parallel, some is reflected from this surface, and hence the line of separation is indistinct and ill-defined.

(3) Lippich's arrangement :—

The polariser is a Glan's prism. Lippich finds this more

convenient than a Nicol, because of the lateral displacement of the light produced by the latter.

A second Glan's prism is cut in two by a plane parallel to its length, and placed so that half the light from the first prism passes through it, while the other half passes at one side. The first prism is capable of rotation about an axis parallel to its length, and is placed so that its principal plane is inclined at a small angle, which can be varied at will, to that of the half-prism. The plane of polarisation of the rays which emerge from this half-prism is therefore slightly inclined to that of the rays which pass to one side of it, and this small angle can be adjusted as may be required.

This arrangement also has the disadvantage that the surface of separation is large, and therefore the line of division is apt to become indistinct.

(4) Lippich has used another arrangement, which requires a divided lens for either the telescope or collimator, and is, in consequence, somewhat complicated, though in his hands it has given most admirable results.

All these four arrangements can be used with white light, and are therefore convenient in all cases in which the rotatory dispersion produced by the active substance, due to variation of wave-length in the light used, is too small to be taken into account.

(5) Laurent's apparatus:—

The polariser is a Nicol followed by a half-wave plate for sodium light, made of quartz or some other crystal.

If quartz cut parallel to the axis be used, the thickness of the plate will be an odd multiple of ·0032 cm. One of the axes of this plate is inclined at a small angle to the principal plane of the Nicol. The plate is semicircular in form and covers half the field—half the light passes through it, the other half to one side. The light on emerging from the plate is plane-polarised, and its plane of polarisation is inclined to the axis of the quartz at the same angle as that of

the incident light, but on the opposite side of that axis. We have thus plane-polarised light in the two halves of the field—the angle between the two planes of polarisation being small.

And, again, by varying the angle between the axis of the quartz and the plane of polarisation of the incident light, we can make the angle between the planes of polarisation in the two halves of the field anything we please; but, on the other hand, since the method requires a half-wave plate, light of definite refrangibility must be used.

(6) Poynting's method:—

Poynting suggested that the desired result might be obtained by allowing the light from one half the field, after traversing a Nicol's prism, to pass through such a thickness of some rotatory medium as would suffice to produce in its plane of polarisation a rotation of 2° or 3°. If quartz cut perpendicular to the axis be used, this will be about ·01 cm. for sodium light. A plate of quartz so thin as this being somewhat difficult to work, Poynting suggested the use of a thicker plate which had been cut in two; one half of this thicker plate is reduced in thickness by about ·01 cm., and the two pieces put together again as before; the light from one half the field traverses ·01 cm. of quartz more than the other, and hence the required effect is produced. This works well, but it is important that the light should pass through both plates of quartz parallel to the axis, otherwise elliptic polarisation is produced. Moreover, the difficulty of obtaining a plate of quartz ·01 cm. thick is not really very great.

Another suggestion of Poynting's was to use a glass cell with a solution of sugar or other active substance in it. A piece of plate glass of 3 or 4 mm. in thickness is placed in the cell, the edge of the plate being flat and smooth. The polarised light from half the field passes through the glass plate, that from the other half traverses an extra thickness of some 3 or 4 mm. of sugar solution, which rotates it through

the required angle. This method has an advantage over the quartz that we are able to adjust the angle between the planes of polarisation in the two halves of the field by varying the strength of the solution. Its simplicity is a strong point in its favour. It has the disadvantage that it is rather difficult to get a clear sharp edge, but care overcomes this.

Of course the adjustments necessary in the position of the Nicols, the method of taking the readings, &c., are the same as those in the last section.

Experiment.—Set up a half-shadow polarimeter and measure the rotation produced in active solutions of various strengths, determining the relation between the strength of the solution and the rotation.

Enter results as in preceding section.

CHAPTER XVI.

COLOUR VISION.[1]

66. The Colour Top.

THIS apparatus consists of a spindle, which can be rapidly rotated by means of a pulley fixed to it, and from this a string or band passes to the driving wheel of some motor.[2] A disc whose edge is graduated in one hundred parts turns with the spindle, and by means of a nut and washer on the end of the spindle, coloured discs can be fixed against this divided circle. From six coloured papers—black, white, red, green, yellow, and blue—discs of two sizes are prepared and are then slit along a radius from circumference to centre so as to admit of being slipped one over the other. Each has a hole at the centre through which the spindle can pass.

The apparatus is arranged to shew that, if any five out

[1] See Deschanel, *Natural Philosophy*, chap. lxiii.

[2] The water motor referred to in § 28 is very convenient for this experiment.

of these six discs be taken, a match or colour equation between them is possible. For instance, if yellow be excluded, the other five may be arranged so that a mixture of red, green, and blue is matched against one of black and white. Take, then, the three large discs of these colours and, slipping them one on the other, fix them against the graduated circle. Start the motor and let it rotate rapidly, looking at the discs against a uniform background of some neutral tint. The three colours will then appear blended into one.

Now place the small discs on these; then on rotating the whole, it will be found that the white and black blend into a grey tint. By continual adjustments an arrangement may be found, after repeated trials, such that the colour of the inner circle is exactly the same both in tint and luminosity as that of the outer ring. The quantities of colour exposed may then be read off on the graduated circle, and it will be found that the proportions are somewhat like the following: 79 parts black and 21 white match 29·2 blue, 29·2 green, and 41·6 red.

With the six discs six equations of this kind can be formed leaving out each colour in turn.

But, according to Maxwell's theory of colour, a match can be found between any four colours, either combining them two and two in proper proportions, or one against three. The colour top is not suited to shew this, for with it we have another condition to fulfil. The whole circumference of the circles has in each case to be filled up with the discs. The vacant spaces must therefore be filled up with black, which alters the intensity of the resultant tints; but the intensity may be adjusted by altering the sizes of all the coloured sectors proportionately, and hence with any four colours and black a match can be made.

And thus from the theory the six final equations are not independent; for between any four of the variables, the colours, there exists a fixed definite relation. If, then, we take two of the equations, we can by a simple algebraical calculation find the others. A comparison between the

equations thus formed and those given directly as the result of the experiments forms a test of the theory; but in practice it is better, in order to insure greater accuracy, to combine all the equations into two, which may then be made the basis of calculation, and from which we may form a second set of six equations necessarily consistent among themselves and agreeing as nearly as is possible with the observations.

A comparison between these two sets gives evidence as to the truth of the theory, or, if we consider this beyond doubt, tests the accuracy of the observations. The six equations referred to are formed from the six found experimentally by the method of least squares. Thus let us denote the colours by the symbols x, y, z, u, v, w, and the quantities of each used by a_1, b_1, c_1, d_1, e_1, f_1 in the first equation, and by the same letters with 2, 3, &c., subscript in the others, and let $\Sigma\{x\}$ denote the sum formed by adding together a series of quantities such as x. Our six equations are

$$a_1 x + b_1 y + c_1 z + d_1 u + e_1 v + f_1 w = 0,$$
&c. &c.

And we have to make

$$\Sigma (a x + b y + c z + d u + e v + f w)^2$$

a minimum, treating x, y, z, u, v, w as variables.

The resulting equations will be the following :—

$$x \Sigma a^2 + y \Sigma a b + z \Sigma a c + u \Sigma a d$$
$$+ v \Sigma a e + w \Sigma a f = 0,$$
$$x \Sigma b a + y \Sigma b^2 + z \Sigma b c + u \Sigma b d$$
$$+ v \Sigma b e + w \Sigma b f = 0.$$
&c. &c.

The calculation of the six equations in this manner is a somewhat long and troublesome process, while the numbers actually arrived at will depend greatly on the exact colours of the discs. In a paper on the subject ('Nature,' Jan. 19, 1871), from which the above account is taken, Lord Rayleigh calls attention to the importance of having the discs accurately cut and centred, otherwise on rotation a

coloured ring appears between the two uniform tints and gives rise to difficulty.

The results also depend to a very considerable extent upon the kind of light with which the discs are illuminated. The difference between light from a cloudless blue sky and light from the clouds is distinctly shewn in the numbers recorded in the paper referred to above.

The numbers obtained may also be different for different observers; the experiment, indeed, forms a test of the colour-perception of the observer.

At the Cavendish Laboratory the colour top is driven by a small water turbine by Baily & Co., of Manchester.

The following table is taken from Lord Rayleigh's paper, being the record of his experiments on July 20, 1870. The circle actually used by him had 192 divisions; his numbers have been reduced to a circle with 100 divisions by multiplying them by 100 and dividing by 192. The second line in each set gives the results of the calculations, while in the first the observed numbers are recorded.

TABLE.

Black	White	Red	Green	Yellow	Blue
0 0	+15·6 +16·1	+60·8 +60·4	+23·6 +23·5	−41·1 −41·5	−58·9 −58·5
+46·8 +44·7	0 0	−66·6 −66·8	−33·4 −33·2	+29·1 +29·6	+24·1 +25·7
−70·7 −71·2	−29·3 −28·8	0 0	+11·4 +11·6	+27 +27	+61·6 +61·4
+52 +51·6	+26 +26·5	+22 +21·9	0 0	−33·3 −33·8	−66·7 −66·2
−79 −79·3	−21 −20·7	+41·6 +42·1	+29·2 +29·2	0 0	+29·2 +28·7
+70·2 +70·6	+10·9 +11·3	−64 −63·8	−36 −36·2	+18·9 +18·1	0 0

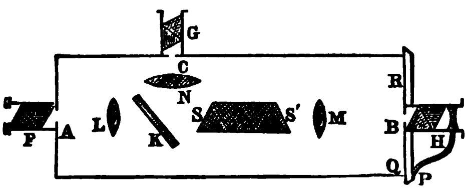
Experiment.—Form a series of colour matches with the six given discs, taken five at a time, and compare your results with those given by calculation.

Enter the results as in the above table.

67. The Spectro-photometer.

This instrument consists of a long, flat rectangular box (fig. 41). At one end of this there is a slit, A, the width of which can be adjusted. The white light from a source

FIG. 41.



behind the slit passes through a collimating lens, L, placed at the distance of its own focal length from A, and falls as a parallel pencil on the set of direct-vision prisms SS′. The emergent beam is brought to a focus by the second lens M, and a pure spectrum thus formed at the end of the box.

A sliding-piece fitted to this end carries a narrow slit B, through which any desired part of the spectrum may be viewed. C is a second slit, illuminated also by white light, the rays from which after passing through the lens N fall on a plane mirror K, and being there reflected traverse the prisms and form a second spectrum directly below the first. By adjusting the positions of the lenses and the mirror K the lines in the two spectra can be made to coincide. The light from A passes over the top of the mirror and the two spectra are seen one above the other. A concave lens enables the observer to focus distinctly the line of separation at K.

In front of the three slits respectively are three Nicol's

prisms, F, G, H. F is fixed with its principal plane vertical, parallel, therefore, to the slits and edges of the prisms ; G has its principal plane horizontal, while H is capable of rotation round a horizontal axis parallel to the length of the box ; P is a pointer fixed to the prism H and moving over a graduated circle Q R, which is divided into 360 parts. The zero of the graduations is at the top of the circle, and when the pointer reads zero the principal plane of H is vertical.

The Nicols F and G polarise the light coming through the slits, the first in the horizontal plane, the second in the vertical. The emergent beam is analysed by the Nicol H. When the pointer reads zero or 180° all the light in the upper spectrum from the slit A passes through H, but none of that from C is transmitted. As the Nicol is rotated through 90° the quantity of light from A which is transmitted decreases, while the amount coming from C increases, and when the Nicol has been turned through 90° all the light from C is transmitted and none from A.

For some position then between 0 and 90° the brightness of the small portions of the two spectra viewed will be the same. Let the reading of the pointer when this is the case be θ. Let the amplitude of the disturbance from A be a, that of the disturbance from C be c, then clearly

$$a \cos \theta = c \sin \theta,$$

and if I_a, I_c be the respective luminous intensities,

$$\frac{I_a}{I_c} = \frac{a^2}{c^2} = \tan^2 \theta.$$

Now place anywhere between L and K a small rectangular cell containing an absorbing solution. The upper spectrum will become darker and the Nicol will require to be moved to establish equality again in the brightness. Let θ' be the new reading, and I'_a the intensity of the light which now reaches the eye from A. Then [1]

$$\frac{I'_a}{I_c} = \tan^2 \theta'.$$

[1] See Glazebrook, *Physical Optics*, pp. 10–27.

Thus

$$\frac{I'_a}{I_a} = \frac{\tan^2 \theta'}{\tan^2 \theta}.$$

But if k represent the fraction of the light lost by absorption and reflexion at the faces of the vessel, we have

$$I'_a = I_a (1 - k).$$

Hence

$$k = 1 - \frac{\tan^2 \theta'}{\tan^2 \theta}.$$

To eliminate the effects of the vessel the experiment should be repeated with the vessel filled with water or some other fluid for which the absorption is small; the difference between the two results will give the absorption due to the thickness used of the absorbing medium.

Of course in all cases four positions of the Nicol can be found in which the two spectra will appear to have the same intensity. At least two of these positions—which are not at opposite ends of the same diameter—should be observed and the mean taken. In this manner the index error of the pointer or circle will be eliminated.

For success in the experiments it is necessary that the sources of light should be steady throughout. In the experiments recorded below two argand gas-burners with ground-glass globes were used. The apparatus and burners must remain fixed, relatively to each other, during the observations.[1]

Dr. Lea has recently suggested another method of using the instrument to compare the concentration of solutions of the same substance of different strengths.

A cell is employed with parallel faces, the distance between which can be varied at pleasure. A standard solution of known strength is taken and placed in a cell of known thickness; let c_1 be the concentration, that is, the

[1] See *Proc. Cam. Phil. Soc.*, vol. iv. Part VI. (Glazebrook on a Spectro-photometer).

quantity of absorbing matter in a unit of volume, m_1 the thickness of this solution. The apparatus is adjusted until the intensity in the two images examined is the same. The other solution of the same medium is put in the adjustable cell, which is then placed in the instrument, the standard being removed, and the thickness is adjusted, without altering the Nicols, until the two images are again of the same intensity, whence, if c be the concentration, m the thickness, we can shew that

$$cm = c_1 m_1 ;$$

$$\therefore c = c_1 m_1 / m \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and from this c can be found, for all the other quantities are known.

We may arrive at equation (1) from the following simple considerations. If c be the concentration, cm will be proportional to the quantity of absorbing material through which the light passes. If, then, we suppose that with the same absorbent the loss of light depends only on the quantity of absorbing matter through which the light passes, since in the two cases the loss of light is the same, we must have

$$cm = c_1 m_1,$$

or

$$c = c_1 m_1 / m.$$

Experiments.

(1) Determine by observations in the red, green, and blue parts of the spectrum the proportion of light lost by passing through the given solution.

(2) Determine by observations in the red, green, and blue the ratio of the concentration of the two solutions.

Enter results thus :—

(1.) Solution of sulphate of copper 1 cm. in thickness.

Colour	θ	θ'	k
Red, near C . . .	60° 50′	49° 50′	·56
Green, near F . . .	61° 30′	56° 30′	·33
Blue-green . . .	64° 30′	58° 30′	·39

(2.) Two solutions of sulphate of copper examined. Standard solution, 10 per cent., 1 cm. in thickness.

Thickness of experimental solution giving the same absorption observed, each mean of five observations.

Colour of Light	Thickness	Ratio of Concentrations
Blue	·74	1·35
Green	·73	1·37
Red	·75	1·33

68. The Colour Box.

The colour box is an arrangement for mixing in known proportions the colours from different parts of the spectrum and comparing the compound colour thus produced with some standard colour or with a mixture of colours from some other parts of the spectrum.

Maxwell's colour box is the most complete form of the apparatus, but it is somewhat too complicated for an elementary course of experiments.

We proceed to describe a modification of it, devised by Lord Rayleigh, to mix two spectrum colours together and compare them with a third. This colour box is essentially the spectro-photometer, described in the last section, with the two Nicols F and G removed. Between the lens L and the mirror K is placed a double-image prism of small angle, rendered nearly achromatic for the ordinary rays by means of a glass prism cemented to it. This prism, as well as the mirror K, is capable of adjustment about an axis normal to the bottom of the box. The prism thus forms two images of the slit, the apparent distance between which depends on the angle at which the light falls on the prism; this distance can therefore be varied by turning the prism round its axis.

The light coming from these two images falls on the direct-vision spectroscope SS′, and two spectra are thus formed in the focal plane QR. These two spectra overlap, so that at any point, such as B, we have two colours mixed, one from each spectrum. The amount of overlapping

and therefore the particular colours which are mixed at each point, depend on the position of the double-image prism, and, by adjusting this, can be varied within certain limits.

Moreover, on passing through the double image prism the light from each slit is polarised, and the planes of polarisation in the two beams are at right angles. We will suppose that the one is horizontal, the other vertical. Thus, in the two overlapping spectra the light in one spectrum is polarised horizontally, in the other vertically. For one position of the analysing prism the whole of one spectrum is quenched, for another position at right angles to this the whole of the second spectrum is quenched. The proportion of light, then, which reaches the eye when the two spectra are viewed, depends on the position of the analyser, and can be varied by turning this round. Thus, by rotating the analyser we can obtain the colour formed by the mixture of two spectrum colours in any desired proportions, and at the same time the proportions can be calculated by noting the position of the pointer attached to the analyser. For if we call A and B the two colours, and suppose that when the pointer reads 0° the whole of the light from A and none of that from B passes through, and when it reads 90° all the light from B and none from A is transmitted, while α, β denote the maximum brightnesses of the two as they would reach the eye if the Nicol H were removed, then when the pointer reads θ° we shall have

$$\frac{\text{Intensity of B}}{\text{Intensity of A}} = \frac{\alpha}{\beta} \tan^2 \theta.$$

The standard light will be that in the lower part of the field, which comes from the slit C, after reflexion at the mirror K. This light being almost unpolarised—the reflexions and refractions it undergoes slightly polarise it—is only slightly affected in intensity by the motion of the analyser. By adjusting the tap of the gas-burner we can alter its intensity, and by turning the mirror K we can bring any desired portion of the spectrum to the point B.

The instrument was designed to shew that a pure yellow, such as that near the D line, could be matched by a mixture of red and green in proper proportions, and to measure those proportions. It is arranged, therefore, in such a way that the red of one spectrum and the green of the other overlap in the upper half of the field at B, while the yellow of the light from C is visible at the same time in the lower half.

Experiment.—Determine the proportions of red and green light required to match the given yellow.

Enter results thus :—

Values of θ	59°
	61°
	60° 15′
	59° 45′
Mean	60°

Ratio of intensities $\frac{3a}{\beta}$.

R. Colour Photometry.

Captain Abney has recently shewn how, by a modification of Rumford's photometer, the luminous intensity at each point of the spectrum may be compared with that from a given source.

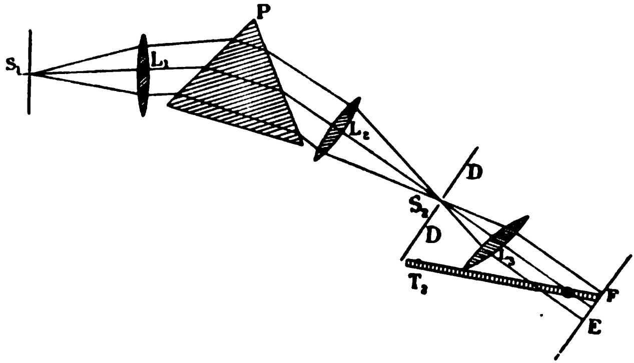
For this purpose a pure spectrum of the given source is produced on a screen. This may be done as in chap. xiv., fig. 34.

It is preferable, however, to use two lenses in such a way that the light from the slit S_1 (fig. xxxiii), which is placed at the principal focus of the first lens, falls as a parallel beam on the prism P. After refraction through it, parallel rays of each different colour fall on the lens L_2, and are brought by it to a focus on the screen D D. In this screen there is a second slit (S_2), through which rays of only one refrangibility pass. These rays fall on a third lens (L_3) arranged so as to produce on a white screen at F E an image of the nearer face of the

prism. This image is illuminated only by light which has passed through S_2—that is, by light of a definite colour, and by moving the slit S_2 a patch of light of any required colour can be thrown on to the screen at F E.

The lenses used will not, in general, be achromatic, and thus the images of S_1 formed by the different colours will not be at the same distance from L_2, but by tilting the screen D D they can all be brought into focus. Again, since the face of the prism P_2 is not at right angles to the

FIG. xxxiii.



direction in which the light travels from it to reach the slit S_2, the lens L_3 is also slightly tilted in order to form on F E a sharp image of the whole of this face.

To apply this to colour photometry, a vertical stick is placed in the path of this coloured beam, casting a shadow on the screen, while a second (standard) light (T_2), mounted on a scale, casts a second shadow close by. This second shadow is coloured, being illuminated by the coloured beam from S_2, while the first shadow receives the light from the standard; still, by moving the comparison light along the scale a point can be found at which the luminosities over the two appear equal. The determination of this point is,

however, attended with some difficulty, much of which is overcome by the adoption of the following oscillation method, the account of which is taken from the Bakerian Lecture for 1886 by Captain Abney and Major-General Festing.

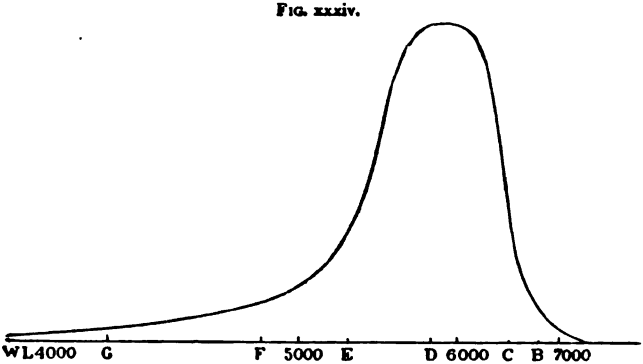
The illuminating value of the spectrum varies greatly in its different parts, the maximum usually being in the yellow, and there is a gradation from this towards either end.

Now suppose that with the standard light at, say, 50 cm. from the screen it is approximately of the same intensity as the yellow light of the spectrum, then if the standard be moved to, say, 60 cm. distance there will be two parts of the spectrum, one towards the red the other towards the blue, which will have the same luminosity as the standard at a distance of 60 cm.; this is, of course, 25/36 of its value when at 50 cm. To find these points, the card to which the slit s_2 is attached is movable, and the slit can be made to slide along the spectrum, its position being determined by means of a scale. When the standard is at 60 cm. distance and the slit in the yellow, the shadow of the rod illuminated by the white light will be palpably darker than the other; when the slit has passed into the green-blue, it will be palpably lighter. Captain Abney finds 'that the best way of determining the intermediate point where the shadows balance is by oscillating the slide gently between two points where first one shadow and then the other is palpably too dark; the oscillations become shorter and shorter until the point of balance is determined.' The slide is then moved through the yellow to the red, and the same process is repeated. Two points in the spectrum whose illumination corresponds to that of the standard at the distance of 60 cm. are thus found. This distance is then varied, and another pair of points determined. In this manner a curve is drawn in which the abscissæ represent the position of the slit, while

the ordinates give the intensity of the light in terms of that of the standard.

By means of an independent series of observations the wave-length of the light which falls on the slit in any given position can be found, as in § 62, and thus a curve giving intensity in terms of wave-length can be determined. This curve is called a luminosity curve. The form of the curve, as found by Captain Abney, is given in fig. xxxiv. The

FIG. xxxiv.



measurements are to some extent affected by the colour of the receiving screen; a card painted with two coats of zinc oxide gives the best results. A portion of this screen about 5 cm. square, limited by a sheet of black paper with a hole cut in it, should be used.

Instead of moving the standard light, the method of varying its intensity adopted by Captain Abney in some later experiments may be employed ('Proc. R. S.' vol. xliii. p. 249).

A circular disc is placed between the standard light and the screen. The disc is divided into four quadrantal sectors, and the alternate sectors are removed. If such a disc is rotated between the light and the screen, it is clear that half the light is cut off. To the disc a pair of movable sectors are fitted, and these can be adjusted so as to close

to a greater or less extent, as may be required, the open sectors of the main disc. If, for example, the open sectors be half closed by the adjustable sectors, the transmitted light has only half the intensity of that previously transmitted.

By means of suitable mechanism the position of these movable sectors can be adjusted relatively to the others while the apparatus is in motion, and thus the amount of light from the standard can be varied until the luminosity of the shadows is the same. In this method of making the observations the slit is fixed in position and the sectors adjusted. When the adjustment has been made the motor is stopped, and the position of the sectors determined; from this the intensity of the standard can be found.

The apparatus can be used to examine the effect of colour mixtures by placing two or more slits in the screen D D. A coloured image of the face of the prism will be formed by light passing through each slit, and these images are superposed. By opening each slit in turn and finding the luminosity, and then making measurements with the two or three slits open simultaneously, we can verify the law that the impression due to a mixed light is the sum of the impressions due to each light separately.

The apparatus has been employed by Captain Abney to study colour-blindness, by comparing the luminosity curves found by various observers, and also for experiments on the scattering of light by small particles. For this purpose a glass trough filled with pure water was placed between the source and the slit S_1, and the luminosity curve found. Then a solution of mastic in alcohol was dropped in various quantities into the water, and the curve again determined. It was found that the intensity of the transmitted light was very closely in accordance with the formula found by Lord Rayleigh, in accordance with which

$$I = I_0 e^{-kx\lambda^{-4}};$$

I_0 being the intensity of the incident light, x the thickness of the absorbing medium, k a constant, and λ the wavelength.

Experiments.

(1) Determine the luminosity curve for the various components of the light from the given source, and compare the result with the normal curve.

(2) Shew that the intensity of a mixture of colours is the sum of the intensities of the components.

(3) Determine the absorption in different parts of the spectrum produced by the given solution of mastic, and compare your result with Lord Rayleigh's formula.

Enter results thus:—

(1)

Scale Reading	Intensity	Scale Reading	Intensity
60	·35	48·9	80
56·4	1·2	47·8	97
53·6	4·2	47·1	100
52	9·6	46·9	100
51	19·4	46·2	96
50	43·5	45·4	82
49·2	73	44·9	59
		44	16
		42·4	0

The curve can be drawn from these.

(2) Slits were placed in the red, green, and violet, and the

Slit Open	Observed	Calculated
R	203	204·25
(R+G)	242	241·75
G	38·5	37·5
(G + V)	45·0	46·0
V	8·5	8·5
(R + V)	214·0	212·5
(R + G + V)	250·0	250·25

luminosities observed for each slit separately, and for the slits

in pairs, and also all three together. The corresponding values were calculated from the curve on the assumption that the resulting impression is the sum of the individual ones.

(3) The intensity for various wave-lengths before and after absorption was determined. The table gives the observed and calculated ratio:—

Wave Length	Observed Ratio	Calculated Ratio
6448	13·1	12·7
6374	12·1	12·3
6210	11·85	11·6
5900	10	9·9
5589	8·25	8·1
5459	7·4	7·3
5180	5·6	5·6
4602	4·8	4·8

CHAPTER XVII.

MAGNETISM.

Properties of Magnets.

CERTAIN bodies, as, for instance, the iron ore called lodestone, and pieces of steel which have been subjected to certain treatment, are found to possess the following properties, among others, and are called magnets.

If a magnet be suspended at any part of the earth's surface, except certain so-called magnetic poles, so as to be free to turn about a vertical axis, it will in general tend to set itself in a certain azimuth—*i.e.* with any given vertical plane, fixed in the body, inclined at a certain definite angle to the geographical meridian—and if disturbed from this position will oscillate about it.

If a piece of iron or steel, or another magnet, be brought

near to a magnet so suspended, the latter will be deflected from its position of equilibrium.

If a magnet be brought near to a piece of soft iron or unmagnetised steel, the iron or steel will be attracted by the magnet. This is illustrated by the experiment of § S, p. 467.

If a long thin magnetised bar of steel be suspended so as to be capable of turning about a vertical axis through its centre of gravity, it will be found to point nearly north and south. We shall call *the end which points north the north end of the magnet*, the other the south end.

Definition of Uniform Magnetisation.—If a magnet be broken up into any number of pieces, each of these is found to be a magnet. Let us suppose that the magnet can be divided into a very large number of very small, equal, similar, and similarly situated parts, and that each of the parts is found to have exactly the same magnetic properties. The magnet is then said to be *uniformly magnetised*.

Definition of Magnetic Axis of a Magnet.—If any magnet be supported so as to be free to turn in any direction about its centre of gravity, it is found that there is a certain straight line in the magnet which always takes up a certain definite direction with reference to the earth. This line is called the *magnetic axis of the magnet*.

Definition of Magnetic Meridian.—The vertical plane through this fixed direction is called the plane of the *magnetic meridian*.

Definition of Magnetic Poles.—If the magnet be a long thin cylindrical bar, uniformly magnetised in such a way that the magnetic axis is parallel to the length of the bar, the points in which the axis cuts the ends of the bar are *the magnetic poles*. The end of the bar which tends to point north, when the magnet is freely suspended, is the *north, or positive pole*; the other is the *south, or negative pole*. Such a magnet is called solenoidal, and behaves to other magnets as if the poles were centres of force, the rest of the magnet being devoid of magnetic action. In all actual

magnets the magnetisation differs from uniformity. No two single points can strictly be taken as centres of force completely representing the action of the magnet. For many practical purposes, however, a well-made bar magnet may be treated as solenoidal with sufficient accuracy; that is to say, its action may be regarded as due to two poles or centres of force, one near each end of the magnet.

The following are the laws of force between two magnetic poles:—

(1) *There is a repulsive force between any two like magnetic poles, and an attractive force between any two unlike poles.*

(2) *The magnitude of the force is in each case numerically equal to the product of the strength of the poles divided by the square of the distance between them.*

This second law is virtually a definition of the strength of a magnetic pole.

In any magnet the strength of the positive pole is equal in magnitude, opposite in sign, to that of the negative pole. If the strength of the positive pole be m, that of the negative pole is $-m$. Instead of the term 'strength of pole,' the term 'quantity of magnetism' is sometimes used. We may say, therefore, that the uniformly and longitudinally magnetised thin cylindrical bar behaves as if it had quantities m and $-m$ of magnetism at its two ends, north and south respectively; we must, however, attach no properties to magnetism but those observed in the poles of magnets. If, then, we have two magnetic poles of strengths m and m', or two quantities of magnetism m and m', at a distance of r centimetres apart, there is a force of repulsion between them which, if m and m' are measured in terms of a proper unit, is

$$mm'/r^2 \text{ dynes.}$$

If one of the two m or m' be negative, the repulsion becomes an attraction.

The C.G.S. unit strength of pole is that of a pole which

repels an equal pole placed a centimetre away with a force of one dyne.

In practice it is impossible to obtain a single isolated pole; the total quantity of magnetism in any actual magnet, reckoned algebraically, is always zero.

DEFINITION OF MAGNETIC FIELD.—A portion of space throughout which magnetic effects are exerted by any distribution of magnetism is called *the magnetic field* due to that distribution.

Let us consider the magnetic field due to a given distribution of magnetism. At each point of the field a pole of strength m is acted on by a definite force. The *Resultant Magnetic Force* at each point of the field is the force which is exerted at that point on a positive pole of unit strength placed there.

This is also called the *Strength of the Magnetic Field* at the point in question.

If H be the strength of the field, or the resultant magnetic force at any point, the force actually exerted at that point on a pole of strength m is m H.

The magnetic force at each point of the field will be definite in direction as well as in magnitude.

DEFINITION OF LINE OF MAGNETIC FORCE.—If at any point of the field a straight line be drawn in the direction of the magnetic force at that point, that straight line will be a tangent to the *Line of Magnetic Force* which passes through the point. *A Line of Magnetic Force* is a line drawn in such a manner that the tangent to it at each point of its length is in the direction of the resultant magnetic force at that point.

A north magnetic pole placed at any point of a line of force would be urged by the magnetic force in the direction of the line of force.

As we shall see shortly, a small magnet, free to turn about its centre of gravity, will place itself so that its axis is in the direction of a line of force.

A surface which at each point is at right angles to the

line of force passing through that point is called a level surface or surface of equilibrium, for since the lines of force are normal to the surface, a north magnetic pole placed anywhere on the surface will be urged by the magnetic forces perpendicularly to the surface, either inwards or outwards, and might therefore be regarded as kept in equilibrium by the magnetic forces and the pressure of the surface. Moreover, if the pole be made to move in any way over the surface, since at each point of its path the direction of its displacement is at right angles to the direction of the resultant force, no work is done during the motion.

DEFINITION OF MAGNETIC POTENTIAL.—The magnetic potential at any point is the work done against the magnetic forces in bringing up a unit magnetic pole from the boundary of the magnetic field to the point in question.

The work done in transferring a unit magnetic pole from one point to another against magnetic forces is *the difference between the values of the magnetic potential at those points.* Hence it follows that the magnetic potential is the same at all points of a level surface. It is therefore called an equipotential surface.

Let us suppose that we can draw an equipotential surface belonging to a certain configuration of magnets, and that we know the strength of the magnetic field at each point of the surface. Take a small element of area, a square centimetres in extent, round any point, and through it draw lines of force in such a manner that if H be the strength of the magnetic field at the point, the number of lines of force which pass through the area a is H a.

Draw these lines so that they are uniformly distributed over this small area.

Do this for all points of the surface.

Take any other point of the field which is not on this equipotential surface; draw a small element of a second equipotential surface round the second point and let its area be a' square centimetres. This area will, of course, be per-

pendicular to the lines of force which pass through it. Suppose that the number of lines of force which pass through this area is n', then it can be proved, as a consequence of the law of force between two quantities of magnetism, that ***the strength of the field at any point of this second small area a' is numerically equal to the ratio n'/a'.***

The field of force can thus be mapped out by means ot the lines of force, and the intensity of the field at each point determined by their aid.

The intensity is numerically equal to the number of lines of force passing through any small area of an equipotential surface divided by the number of square centimetres in that area, provided that the lines of force have originally been drawn in the manner described above.[1]

[1] For an explanation of the method of mapping a field of force by means of lines of force, see Maxwell's *Elementary Electricity*, chaps. v. and vi. and Cumming's *Electricity*, chaps. ii. and iii. The necessary propositions may be summarised thus (leaving out the proofs) :—

(1) Consider any closed surface in the field of force, and imagine the surface divided up into very small elements, the area of one of which is σ; let F be the resultant force at any point of σ, resolved normally to the surface inwards; let $\Sigma F \sigma$ denote the result of adding together the products $F\sigma$ for every small elementary area of the closed surface. Then, if the field of force be due to matter, real or imaginary, for which the law of attraction or repulsion is that of the inverse square of the distance,

$$\Sigma F \sigma = 4 \pi M,$$

where M is the quantity of the real or imaginary matter in question contained inside the closed surface.

(2) Apply proposition (1) to the case of the closed surface formed by the section of a tube of force cut off between two equipotential surfaces. [A tube of force is the tube formed by drawing lines of force through every point of a closed curve.]

Suppose σ and σ' are the areas of the two ends of the tube, F and F' the forces there; then $F\sigma = F'\sigma'$.

(3) Imagine an equipotential surface divided into a large number of very small areas, in such a manner that the force at any point is inversely proportional to the area in which the point falls. Then σ being the measure of an area and F the force there, $F\sigma$ is constant for every element of the surface.

(4) Imagine the field of force filled with tubes of force, with the elementary areas of the equipotential surface of proposition (3) as bases. These tubes will cut a second equipotential surface in a series of elementary areas σ'. Let F' be force at σ', then by propositions (2) and

On the magnetic potential due to a single pole.—The force between two magnetic poles of strengths m and m', at a distance r_1 centimetres apart is, we have seen, a repulsion of mm'/r_1^2 dynes. Let us suppose the pole m' moved towards m through a small distance. Let A (fig. 42) be the position of m, P_1, P_2 the two positions of m'. Then AP_2P_1 is a straight line, and $AP_1 = r_1$. Let $AP_2 = r_2$, $P_1P_2 = r_1 - r_2$.

FIG. 42.

Then, if, during the motion, from P_1 to P_2, the force remained constant and of the same value as at P_1, the work done would be

$$\frac{mm'}{r_1^2}(r_1 - r_2);$$

while if, during the motion, the force had retained the value which it has at P_2, the work would have been

$$\frac{mm'}{r_2^2}(r_1 - r_2).$$

Thus the work actually done lies between these two values. But since these fractions are both very small, we may neglect the difference between r_1 and r_2 in the denominators. Thus the denominator of each may be

(3) $F'\sigma'$ is constant for every small area of the second equipotential surface, and equal to $F\sigma$, and hence $F\sigma$ is constant for every section of every one of the tubes of force; thus $F\sigma = \kappa$.

(5) By properly choosing the scale of the drawing, κ may be made equal to unity. Hence $F = \frac{1}{\sigma}$, or the force at any point is equal to the number of tubes of force passing through the unit of area of the equipotential surface which contains the point.

(6) Each tube of force may be indicated by the line of force which forms, so to speak, its axis. With this extended meaning of the term 'line of force' the proposition in the text follows. The student will notice that, in the chapter referred to, Maxwell very elegantly avoids the analysis here indicated by accepting the method of mapping the electrical field as experimentally verified, and deducing from it the law of the inverse square.

written $r_1 r_2$ instead of r_1^2 and r_2^2 respectively. The two expressions become the same, and hence the work done is

$$mm' \frac{r_1 - r_2}{r_1 r_2},$$

or

$$mm' \left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

Similarly the work done in going from P_2 to a third point, P_3, is

$$mm' \left(\frac{1}{r_3} - \frac{1}{r_2}\right).$$

And hence we see, by adding the respective elements together, that the work done in going from a distance r' to a distance r is

$$mm' \left(\frac{1}{r} - \frac{1}{r'}\right).$$

Hence the work done in bringing the pole m from infinity to a distance r from the pole m is mm'/r. But the potential due to m at a distance r, being the work done in bringing up a unit pole from beyond the influence of the pole m, will be found by dividing this by m'; it is therefore equal to m/r.

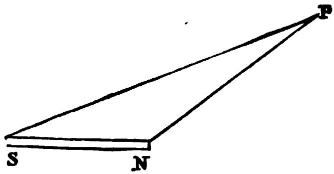
Again, it follows from the principle of conservation of energy that the work done in moving a unit pole from any one point to any other is independent of the path, and hence the work done in moving the unit pole from any point whatever at a distance r' to any point at a distance r from the pole m is

$$m \left(\frac{1}{r} - \frac{1}{r'}\right).$$

For a single pole of strength m, the equipotential surfaces are clearly a series of concentric spheres, with m as centre; the lines of force are radii of these spheres.

If we have a solenoidal magnet of strength m, and r_1, r_2 be the distances of any point, P (fig. 43), from the positive and negative poles N and S of the magnet, then the potential at P due to the north pole is m/r_1, and that due to the south pole is $-m/r_2$; hence the potential at P due to the magnet is

FIG. 43.



$$m\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$

The equipotential surfaces are given by the equation

$$m\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=c,$$

where c is a constant quantity, and the lines of force are at right angles to these surfaces. To find the resultant magnetic force at P we have to compound a repulsion of m/r_1^2 along N P with an attraction of m/r_2^2 along P S, using the ordinary laws for the composition of forces.

Let us now consider the case in which the lines of force in the space in question are a series of parallel straight lines uniformly distributed throughout the space.

The intensity of the field will be the same throughout; *such a distribution constitutes a uniform magnetic field.*

The earth is magnetic, and the field of force which it produces is practically uniform in the neighbourhood of any point provided that there be no large masses of iron near, and the lines of force are inclined to the horizon in these latitudes at an angle of about 67°.

On the Forces on a Magnet in a Uniform Field.

We proceed to investigate the forces on a solenoidal magnet in a uniform field.

Let us suppose the magnet held with its axis at right angles to the lines of force, and let l be the distance between its poles, m the strength of each pole, and H the intensity

of the field. The north pole is acted on by a force m H at right angles to the axis of the magnet, the south pole by an equal, parallel, but opposite force m H. These two forces constitute a couple; the distance between the lines of action, or arm of the couple, is l, so that the moment of the couple is $m\,l$ H. If the axis of the magnet be inclined at an angle θ to the lines of force, the arm of the couple will be $m\,l \sin\theta$, and its moment $m\,l$ H $\sin\theta$. In all cases the couple will depend on the product $m\,l$.

DEFINITION OF MAGNETIC MOMENT OF A MAGNET.—The product of the strength of either pole into the distance between the poles, is called *the magnetic moment of a solenoidal magnet.* Let us denote it by M; then we see that if the axis of the magnet be inclined at an angle θ to the lines of force, the couple tending to turn the magnet so that its axis shall be parallel to the lines of force is M H $\sin\theta$. Thus the couple only vanishes when θ is zero; that is, when the axis of the magnet is parallel to the lines of force.

But, as we have said, the actual bar magnets which we shall use in the experiments described below are not strictly solenoidal, and we must therefore consider the behaviour, in a uniform field, of magnets only approximately solenoidal.

If we were to divide a solenoidal magnet into an infinitely large number of very small, equal, similar, and similarly situated portions, each of these would have identical magnetic properties; each would be a small magnet with a north pole of strength m and a south pole of strength $-m$.

If we bring two of these elementary magnets together so as to begin to build up, as it were, the original magnet, the north pole of the one becomes adjacent to the south pole of the next; we have thus superposed, a north pole of strength m and a south pole of strength $-m$; the effects of the two at any distant point being thus equal and opposite, no external action can be observed. We have therefore a magnet equal in length to the sum of the lengths of the other two, with two poles of the same strength as those of either.

If, however, we were to divide up an actual magnet in this manner, the resulting elementary magnets would not all have the same properties.

We may conceive of the magnet, then, as built up of a number of elementary magnets of equal volume but of different strengths.

Consider two consecutive elements, the north pole of the one of strength m is in contact with the south pole of the other of strength $-m'$ say; we have at the point of junction a north pole of strength $m-m'$, we cannot replace the magnet by centres of repulsive and attractive force at its two ends respectively, and the calculation of its action becomes difficult.

If, however, the magnet be a long bar of well-tempered steel carefully magnetised, it is found that there is very little magnetic action anywhere except near the ends. The elementary magnets of which we may suppose it to consist would have equal strengths until we get near the ends of the magnet, when they would be found to fall off somewhat. The action of such a magnet may be fairly represented by that of two equal poles placed close to, but not coincident with, the ends; and we might state, following the analogy of a solenoid, that the magnetic moment of such a magnet was measured by the product of the strength of either pole into the distance between its poles.

We can, however, give another definition of this quantity which will apply with strictness to any magnet. The moment of the couple on a solenoidal magnet, with its axis at an angle θ to the lines of magnetic force in a field of uniform intensity H, is, we have seen, $MH \sin\theta$, M being the magnetic moment. Thus the maximum couple which this magnet can experience is MH, and the maximum couple which the magnet can be subjected to in a field of uniform force of intensity unity is M.

Now any magnet placed in a uniform field of magnetic force is acted on by a couple, and we may say that for

any magnet whatever, *the magnetic moment of a magnet is measured by the maximum couple to which the magnet can be subject when placed in a uniform magnetic field of intensity unity.*

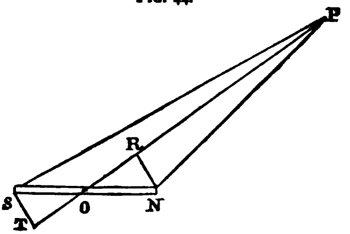
The couple will be a maximum when the magnetic axis of the magnet is at right angles to the lines of force.

If the angle between the axis of the magnet and the lines of force be θ, the magnetic moment M, and the strength of the field H, the couple will be M H $\sin\theta$, just as for a solenoidal magnet.

On the Potential due to a Solenoidal Magnet.

We have seen that if P be a point at distances r_1, r_2 from the north and south poles, N, S, respectively, of a solenoidal magnet N O S (fig. 44) of strength m, the magnetic potential at P is

FIG. 44.



$$m\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$

We will now put this expression into another and more useful form, to which it is for our purposes practically equivalent. Let O, the middle point of the line N S, be the centre of the magnet; let O P $=r$, O N $=$ O S $=l$, so that $2\,l$ is the length of the magnet, and let the angle between the magnetic axis and the radius vector O P be θ, this angle being measured from the north pole to the south, so that in the figure N O P $=\theta$.

Draw N R, S T perpendicular to P O or P O produced, and suppose that O P is so great compared with O N that we may neglect the square and higher powers of the ratio of O N/O P. Then P R N is a right angle, and P N R differs very little from a right angle, for O N is small compared with O P, so that P N $=$ P R very approximately, and similarly P S $=$ P T.

Also $\qquad$ O R $=$ O T $=$ O N cos P O N $= l\cos\theta$.

Thus

$$r_1 = \text{PN} = \text{PO} - \text{OR} = r - l\cos\theta = r\left(1 - \frac{l}{r}\cos\theta\right),$$

and

$$r_2 = r + l\cos\theta = r\left(1 + \frac{l}{r}\cos\theta\right);$$

and, if V denote the magnetic potential at P, we have

$$\begin{aligned} V &= m\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \\ &= \frac{m}{r}\left\{\frac{1}{1 - \frac{l}{r}\cos\theta} - \frac{1}{1 + \frac{l}{r}\cos\theta}\right\} \\ &= \frac{m}{r}\,\frac{2\frac{l}{r}\cos\theta}{1 - \frac{l^2}{r^2}\cos^2\theta}. \end{aligned}$$

But we are to neglect terms involving l^2/r^2, etc.; thus we may put

$$V = \frac{2\,m\,l}{r^2}\cos\theta = \frac{M\cos\theta}{r^2},$$

if M be the moment of the magnet.

We shall see next how to obtain from this expression the magnetic force at P.

On the Force due to a Solenoidal Magnet.

To obtain this we must remember that the work done on a unit pole by the forces of any system in going from a point P_1 to a second point P_2, V_1, V_2 being the potentials at P_1 and P_2, is $V_1 - V_2$. Let a be the distance between these two points, and let $\bar{F}$ be the average value of the magnetic force acting from P_1 to P_2 resolved along the line P_1P_2. Then the work done by the force $\bar{F}$ in moving the pole is $\bar{F}a$.

Hence $$\bar{F}a = V_1 - V_2,$$

and if the distance a be sufficiently small, $\bar{F}$, the average

value of the force between P_1 and P_2 may be taken as the force in the direction $P_1 P_2$ at either P_1 or P_2.

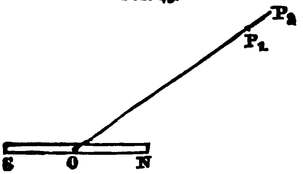
Denoting it by F we have

$$F = \text{limiting value of } \frac{V_1 - V_2}{a}$$

when a is very small.

Let us suppose that P_1, P_2 are two points on the same radius from O, that $OP_1 = r$ and $OP_2 = r + \delta$.

FIG. 45.



Then θ is the same for the two points, and we have

$$V_1 = \frac{M \cos\theta}{r^2},$$

$$V_2 = \frac{M \cos\theta}{(r+\delta)^2}$$

$$= \frac{M \cos\theta}{r^2\left(1+\frac{\delta}{r}\right)^2} = \frac{M \cos\theta}{r^2}\left(1 - \frac{2\delta}{r}\right)$$

neglecting $\left(\frac{\delta}{r}\right)^2$ and higher powers (see p. 42).

Also, in this case, $a = \delta$. Thus

$$F = \text{limiting value of } \frac{V_1 - V_2}{a}$$

$$= \frac{M \cos\theta}{r^2 \delta}\left(\frac{2\delta}{r}\right) = \frac{2M \cos\theta}{r^3}.$$

We shall denote this by R, so that R is the force outwards, in the direction of the radius-vector, on a unit pole at a distance r from the centre of a small solenoidal magnet of moment M. If the radius-vector make an angle θ with the axis of the magnet, we have

$$R = \frac{2M \cos\theta}{r^3}.$$

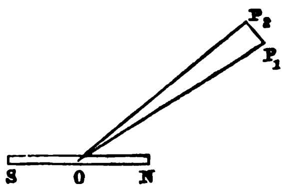
Again, let us suppose that P_1P_2 (fig. 46) is a small arc of a circle with O as centre, so that

$$OP_1 = OP_2 = r$$

let $$P_1ON = \theta,$$

and $$P_2ON = \theta + \phi.$$

Fig. 46.



Thus

$$a = P_1P_2 = OP_1 \times P_1OP_2 = r\phi.$$

The force, in this case, will be that at right angles to the radius vector, tending to increase θ; if we call it T we have

$$T = -\text{limiting value of } \frac{V_2 - V_1}{r\phi}$$

$$= -\frac{M}{r^3\phi}\left\{\cos(\theta+\phi) - \cos\theta\right\}$$

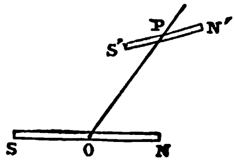
$$= \frac{M}{r^3}\sin\theta \qquad \text{(see p. 45).}$$

These two expressions are approximately true if the magnet N S be very small and solenoidal. We may dispense with the latter condition if the magnet be sufficiently small; for, as we have said, any carefully and regularly magnetised bar behaves approximately like a solenoid with its poles not quite coincident with its ends. In such a case $2\,l$ will be the distance between the poles, not the real length of the magnet, and $2\,m\,l$ will still be the magnetic moment.

On the Effect on a Second Magnet.

In practice we require to find the effect on two magnetic poles of equal but opposite strengths, not on a single pole, for every magnet has two poles.

Fig. 47.



Let us suppose that P (fig. 47) is the centre of a second magnet N′ P S′ so small that we may, when considering the action of the distant magnet N O S, treat it

as if either pole were coincident with P, that m' is the strength, and $2l'$ the length of this magnet, and θ' the angle between P N′ and O P produced.

Then we have, acting outwards parallel to the radius vector O P on the pole N′, a force

$$\frac{2m' M \cos\theta}{r^3},$$

and an equal and parallel force acting inwards towards O on the pole S′; these two constitute a couple, the arm of which will be $2l' \sin\theta'$. Thus, if M′ be the magnetic moment of the second magnet, so that $M' = 2m'l'$, we have acting on this magnet a couple, tending to decrease θ', whose moment will be

$$\frac{2MM' \cos\theta \sin\theta'}{r^3}.$$

This arises from the action of the radial force R.

The tangential force on N′ will be

$$\frac{Mm' \sin\theta}{r^3},$$

tending to decrease θ' and on S′ an equal force also tending to decrease it. These constitute another couple tending to decrease θ'; the arm of this couple will be $2l' \cos\theta'$, and its moment will be

$$\frac{MM' \sin\theta \cos\theta'}{r^3}.$$

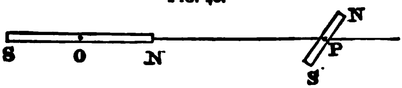
Thus, combining the two, we shall have a couple, the moment of which, tending to increase θ', will be

$$-\frac{MM'}{r^3}(\sin\theta\cos\theta' + 2\sin\theta'\cos\theta).$$

It must of course be remembered that these expressions are only approximate; we have neglected terms which, if the magnets are of considerable size, may become important.

Two cases are of considerable interest and importance. In the first the axis of the first magnet passes through the centre of the second.

FIG. 48.



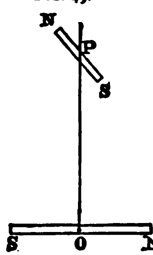
The magnet N S is said to be 'end on.' In this case (fig. 48) we have $\theta = 0$, and the action is a couple tending to decrease θ', the moment of which is

$$\frac{2\,\text{M}\,\text{M}'}{r^3}\sin\theta'.$$

If no other forces act on the second magnet, it will set itself with its axis in the prolongation of that of the first magnet.

In the second case (fig. 49) the line joining the centres of the two is at right angles to the axis of the first magnet, which is said to be 'broadside on'; then $\theta = 90°$, and we have a couple tending to increase θ', the moment of which will be

FIG. 49.



$$\frac{\text{M}\,\text{M}'}{r^3}\cos\theta'.$$

We may notice that for a given value of r, the maximum value of the couple in this second case is only half of its maximum value in the former case.

The position of equilibrium will be that in which $\cos\theta' = 0$, or when the two axes are parallel. Let us suppose that the second magnet is capable of rotating about a vertical axis through its centre, in the same way as a compass needle; it will, if undisturbed, point north and south under the horizontal component of the magnetic force due to the earth; let us call this H. Place the first magnet with its north pole pointing towards the second, and its centre exactly to the west of that of the second. The second will be deflected, its north pole turning to the east. Let ϕ be

the angle through which it turns, then clearly $\theta' = 90 - \phi$. The moment of the earth's force on the magnet is $M'H \sin\phi$, that of the couple due to the other magnet is

$$2\,M\,M' \sin\theta'/r^3, \text{ or } 2\,M\,M' \cos\phi/r^3,$$

in the opposite direction. But the magnet is in equilibrium under these two couples, and hence we have

$$M'\,H \sin\phi = \frac{2\,M\,M'}{r^3} \cos\phi.$$

Thus

$$M = \tfrac{1}{2}\,H\,r^3 \tan\phi.$$

Next place the first magnet with its north pole west and its centre exactly to the south of the second; the north pole of the second will move to the east through an angle ψ, say, and in this case we shall have $\theta' = \psi$.

The moment of the couple due to the earth will be as before $M'H \sin\psi$; that due to the first magnet is

$$\frac{M\,M'}{r^3} \cos\psi$$

and hence $$M = H\,r^3 \tan\psi.$$

We shall see shortly how these formulæ may be used to measure M and H.

On the Measurement of Magnetic Force.

The theoretical magnets we have been considering are all supposed to be, in strictness, simply solenoidal rods without thickness, mere mathematical lines in fact.

The formulæ may be applied as a first approximation, however, to actual magnets, and we shall use them in the experiments to be described.

There remains, however, for consideration the theory of an experiment which will enable us to compare the magnetic moments of a magnet of any form under different

conditions of magnetisation, or of two magnets of known form, or to compare the strengths of two approximately uniform magnetic fields, or, finally, in conjunction with the formulæ already obtained, to measure the moment of the magnet and the strength of the field in which it is.

We have seen (p. 166) that, if a body, whose moment of inertia about a given axis is K, be capable of vibrating about that axis, and if the force which acts on the body after it has been turned through an angle θ from its position of equilibrium, tending to bring it back to that position, be $\mu\,\theta$, then the body will oscillate isochronously about this position; also if the time of a complete oscillation be T, then T is given by the formula

$$T = 2\pi\sqrt{\frac{K}{\mu}}$$

We shall apply this formula to the case of a magnet. We have seen already that, if a magnet be free to oscillate about a vertical axis through its centre of gravity, it will take up a position of equilibrium with its magnetic axis in the magnetic meridian. The force which keeps it in the meridian arises from the horizontal component of the earth's magnetic force; and if the magnet be disturbed from this position through an angle θ, the moment of the couple tending to bring it back is $M\,H \sin\theta$, M being the magnetic moment. Moreover, if θ be the circular measure of a small angle, we know that the difference between θ and $\sin\theta$ depends on θ^3 and may safely be neglected; we may put, therefore, with very high accuracy, if the magnet be made to oscillate only through a small angle, the value θ for $\sin\theta$ in the above expression for the moment of the couple acting on the magnet, which thus becomes $M{\cdot}H\,\theta$; so that, if K be the moment of inertia of the magnet about the vertical axis, the time of a small oscillation T is given by the equation

$$T = 2\pi\sqrt{\left(\frac{K}{M\,H}\right)}.$$

T can be observed experimentally, and hence we get an equation to find M H, viz.

$$\text{M H} = \frac{4\pi^2 \text{K}}{\text{T}^2}.$$

If we have in addition a relation which gives the ratio of M/H from the two we can find M and H. Such a relation has been obtained above (p. 450), and with the notation there employed we have

$$\frac{\text{M}}{\text{H}} = \tfrac{1}{2} r^3 \tan \phi.$$

We shall discuss the experimental details shortly.

Magnetic Induction.

There are some substances in which the action of magnetic forces produces a magnetic state which lasts only as long as the magnetic forces are acting. Such substances, of which iron is the most marked example, become themselves temporary magnets when placed in a magnetic field. They are said to be magnetised by induction. They lose nearly all their magnetic property when the magnetising forces cease to act. In most specimens of iron a certain amount of this remains as permanent magnetism after the cessation of the magnetising forces. In very soft iron the amount is very small; in steel, on the other hand, the greater portion remains permanently. We shall call such substances magnetic.

The attraction between a magnet and a magnetic substance is due to this induction.

Wherever a line of force from a magnet enters a magnetic substance it produces by its action a south pole. Where it leaves the substance it produces a north pole. Thus, if a magnetic body be brought near a north pole, those portions of the surface of the body which are turned towards the pole become endued generally with south

polar properties; those parts of the surface which are away from the north pole acquire north polar properties. An attraction is set up between the north pole of the magnet and the south polar side of the induced magnet, a repulsion of weaker amount between the north pole and the north polar side, so that on the whole the magnetic body is attracted to the north pole. This may even be the case sometimes when the magnetic body is itself a somewhat weak magnet, with its north pole turned to the given north pole. These two north poles would naturally repel each other; but, under the circumstances, the given pole will induce south polar properties in the north end of the weak magnet, and this south polarity may be greater than the original north polarity of the magnet, so that the two, the given north pole and the north end of the given magnet, may actually attract each other.

69. Experiments with Magnets.

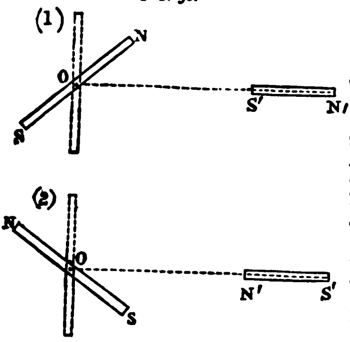
(*a*) *To magnetise a Steel Bar.*

We shall suppose the magnet to be a piece of steel bar about 10 cm. in length and 0·5 cm. in diameter, which has been tempered to a straw colour. The section of the bar should be either circular or rectangular.

We proceed first to shew how to determine if the bar be already a magnet. We may employ either of two methods. Take another delicately-suspended magnet—a well-made compass needle will do—but if great delicacy be required, a very small light magnet suspended by a silk fibre. A small mirror is attached to the magnet, and a beam of light, which is allowed to fall on it, is reflected on to a screen; the motions of the magnet are indicated by those of the spot of light on the screen, as in the Thomson reflecting galvanometer. Bring the bar into the neighbourhood of the suspended magnet, placing it with its axis east and west and its length directed towards the centre of the magnet, at a distance of about 25 cm. away. Then, if N S be the suspended

magnet, N′ S′ the bar, and if N′ be a north end, S′ a south end, N S will be deflected as in fig. 50 (1). On reversing N′ S′ so as to bring it into position (2), N S will be deflected in the opposite direction. If the action between the two be too small to produce a visible permanent deflexion of the magnet N S, yet, by continually reversing the bar at intervals equal to the time of oscillation of the needle, the effects may be magnified, and a swing of considerable amplitude given to the latter. The swing can be gradually destroyed by presenting the reverse poles in a similar way.

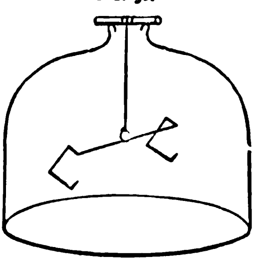
FIG. 50.



This is a most delicate method of detecting the magnetism of a bar, and there are few pieces of steel which will not shew some traces of magnetic action when treated thus.

The following is the second method. Twist a piece of copper wire to form a stirrup (fig. 51) in which the magnet can be hung, and suspend it under a bell-jar by a silk fibre, which may either pass through a hole at the top of the jar and be secured above, or be fixed to the jar with wax or cement. If the magnet to be used be rectangular in section, the stirrup should be made so that one pair of faces may be horizontal, the other vertical when swinging. For very delicate experiments this fibre must be freed from torsion. To do this take a bar of brass, or other non-magnetic material, of the same weight as the magnet, and hang it in

FIG. 51.



the stirrup. The fibre will untwist or twist, as the case may be, and the bar turn round, first in one direction then in the other. After a time it will come to rest. The fibre is then hanging without torsion. Now remove the torsion-bar and replace it by the magnetic bar which is to be experimented on, without introducing any twist into the fibre.

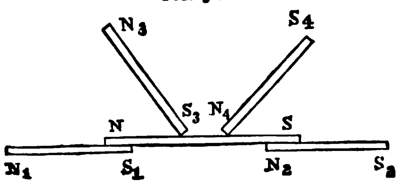
As the stirrup will be frequently used again for suspending the magnet, make a mark on the latter so that it can always be replaced *in the same position* on the stirrup.

If now the bar is at all magnetised, it will, when left to swing freely, take up a position of equilibrium with its north end pointing to the north, and when displaced from that position, will return to it again after a number of vibrations about it. This method would be even more delicate than the last, except that the torsion of the fibre might sometimes make it appear that the bar is magnetised when it is really not so.

Having satisfied yourself that the bar is only feebly magnetised, proceed to magnetise it more strongly.

This can be done by stroking it with another magnet, using the method of divided touch, or by the use of an electric current. In the method of divided touch the bar is placed on two magnets N_1S_1, N_2S_2, Fig. 52; two other magnets are held as in the figure N_3S_3 and N_4S_4. They are then drawn outwards from the centre slowly and regularly, from the position shewn in the figure, in which they are nearly in contact, to the ends. The operation is repeated several times, stroking always from the centre to the ends. Then the bar to be magnetised is turned over top to bottom and again stroked.

FIG. 52.



It will be found to be a magnet with its north pole N

over S_1 and its south pole S over N_2. In all cases the two ends of the bar rest on opposite poles, and the poles above, which are used for stroking, are of the same name as those below, on which the bar rests. The two magnets used for stroking should have about the same strength.

If an electric current be used, the bar may be magnetised either by drawing it backwards and forwards across the poles of an electro-magnet, or by placing it inside of a long coil of thick insulated wire, such as is used for the coils of an electro-magnet, and allowing a powerful current to pass through the wire.

It will be much more strongly magnetised if it be put into the coil when hot and allowed to cool rapidly with the current circulating round it.

To deprive a steel bar entirely of its magnetism is a difficult matter. The best plan is to heat it to a red heat and allow it to *cool gradually, with its axis pointing east and west.* If it be placed north and south, it will be found that the magnetic action of the earth is sufficient to re-magnetise the bar.

(*b*) *To compare the Magnetic Moment of the same Magnet after different Methods of Treatment, or of two different Magnets.*

(1) Suspend the magnet in its stirrup under the bell jar, as in fig. 51, and when it is in equilibrium make a mark on the glass opposite to one end. Displace the magnet slightly from this position, and count the number of times the end crosses the mark in a known interval of time,[1] say one minute—a longer interval will be better if the magnet continue swinging. Divide this number by the number of seconds in the interval, 60 in the case supposed, the result is the number of transits in one second. Call this n. There will be two transits to each complete oscillation, for the period of an oscillation is the interval between two consecutive passages of the needle through the resting point *in the same direction*, and all transits, both right to left

[1] The times of crossing the mark must be counted 0, 1, 2, . . . n.

and left to right, have been taken; $\frac{1}{2}n$ is therefore the number of complete oscillations in one second, and the periodic time is found by dividing one second by the number of oscillations in one second. Hence, T being the periodic time,

$$T = 2/n.$$

But we have shewn (p. 452) that

$$MH = 4\pi^2 K/T^2.$$

Hence

$$MH = \pi^2 n^2 K$$

and

$$M = \pi^2 n^2 K/H.$$

Now K depends only on the form and mass of the magnet, which are not altered by magnetisation; H is the strength of the field in which it hangs, which is also constant; so that if M_1, M_2, &c. be the magnetic moments after different treatments, n_1, n_2, &c. the corresponding number of transits per second,

$$M_1 = \pi^2 n_1^2 K/H$$
$$M_2 = \pi^2 n_2^2 K/H, \text{ \&c.}$$
$$M_1 : M_2 = n_1^2 : n_2^2, \text{ \&c.}$$

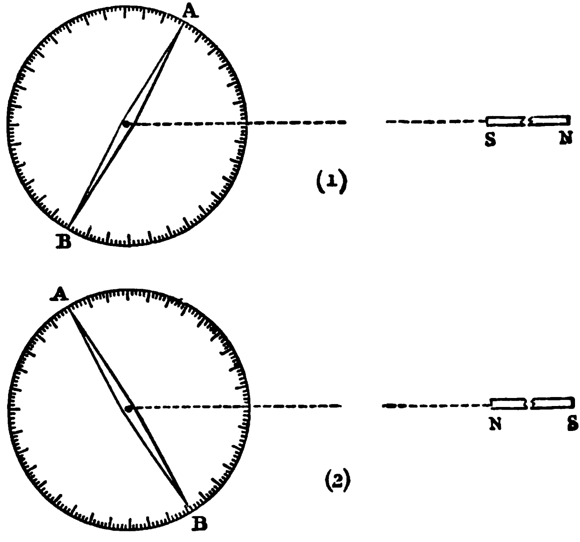
We thus find the ratio of M_1 to M_2.

(2) We can do this in another way as follows :—

Take a compass needle, A B (fig. 53) provided with a divided circle, by means of which its direction can be determined, and note its position of equilibrium. Place the magnet at some distance from the compass needle, with its end pointing towards the centre of the needle and its centre east or west of that of the needle. Instead of a compass needle we may use a small magnet and mirror, with a beam of light reflected on to a scale, as already described (p. 453). The centre of the magnet should be from 40 to 50 cm. from the needle. The needle will be deflected from its position of equilibrium. Let the deflection observed be θ_1; reverse the magnet so that its north pole comes into the position

formerly occupied by the south pole, and *vice versâ*. The needle will be deflected in the opposite direction (fig. 53 [2]). Let the deflection be θ_2. If the magnet had been uniformly magnetised and exactly reversed we should find that θ_1 and θ_2 were the same. Let the mean of the two values be θ; so

FIG. 53.



that θ is the deflection produced on a magnetic needle by a bar magnet of moment M when the line joining the centres of the two is east and west, and is in the same straight line as the axis of the bar magnet. But under these circumstances we have shewn (p. 450) that, if r be the distance between their centres,

$$M = \tfrac{1}{2} Hr^3 \tan \theta.$$

If another magnet of moment M′ be substituted for the first, and a deflection θ' be observed, the distance between the centres being still r, we have

$$M' = \tfrac{1}{2} Hr^3 \tan \theta'.$$

Hence

$$M : M' = \tan\theta : \tan\theta'.$$

We can thus compare the moments of the same magnet under different conditions, or of two different magnets.

(*c*) *To compare the Strengths of different Magnetic Fields of approximately Uniform Intensity.*

Let H_1 be the strength of the first field, let a magnet swing in it, and let the number of transits per second observed as in (*b*) be n_1, then we have, M being the magnetic moment,

$$H_1 = \pi^2 n_1^2 K/M.$$

Now let the magnet swing in the second field, strength H_2, and let n_2 be the number of transits per second. Then

$$H_2 = \pi^2 n_2^2 K/M.$$

Hence

$$H_1 : H_2 = n_1^2 : n_2^2.$$

To realise the conditions of this experiment surround the magnet hanging as in (*a*) with a soft-iron cylinder of considerable radius in comparison with the length of the magnet. The cylinder should be pierced with holes, through which the magnet may be viewed, and the number of transits per second counted in the manner already described (p. 456).

The magnetic field within the iron cylinder is thus compared with that which the earth produces when the cylinder is removed.

(*d*) *To measure the Magnetic Moment of a Magnet and the Strength of the Field in which it hangs.*

For this we have only to combine the results of the observations in (*b*), and determine the moment of inertia of the magnet about the axis of rotation. Thus, weigh the magnet and let its mass be m grammes; measure its length with a rule, the calipers, or the beam compass, as may be

most convenient; let it be l cm. Determine, by means of the screw gauge, its diameter if it be a circular cylinder, let it be c cm.; or if it be rectangular in shape, the length of that side of the rectangle which is horizontal when it is swinging, let this be a cm. Then it can be shewn, by the use of the integral calculus, that in the first case, if the section be circular,

$$\mathrm{K}=m\left(\frac{l^2}{12}+\frac{c^2}{16}\right),$$

and in the second, if it be rectangular,

$$\mathrm{K}=m\left(\frac{l^2+a^2}{12}\right).[1]$$

Thus K can be determined in either case, supposing the stirrup to be so light in comparison with the magnet that its effect may be neglected.

If K cannot be found by direct measurement, we must have recourse to the methods of observation described in § D.

Thus, K being determined, we know all the quantities involved in the two equations of (b), with the exception of M and H.

The two equations are

$$\mathrm{M\,H}=\pi^2 n^2 \mathrm{K},$$

$$\frac{\mathrm{M}}{\mathrm{H}}=\tfrac{1}{2}r^3\tan\theta\,;$$

and from these we obtain by multiplication,

$$\mathrm{M}^2=\tfrac{1}{2}\pi^2 n^2 \mathrm{K} r^3\tan\theta\,;$$

whence

$$\mathrm{M}=\pi n r\sqrt{(\tfrac{1}{2}\mathrm{K} r\tan\theta)},$$

and by division,

$$\mathrm{H}^2=\frac{2\pi^2 n^2 \mathrm{K}}{r^3\tan\theta},$$

[1] Routh's *Rigid Dynamics*, chapter i. See also above, p. 167.

or

$$H = \pi n \sqrt{\left(\frac{2K}{r^3 \tan \theta}\right)}.$$

This is the method actually employed in many unifilar magnetometers, to determine the horizontal intensity of the earth's magnetic force, the only difference consisting in the very delicate arrangements for the accurate determination of the quantities to be measured.

(*e*) *To determine the Magnetic Moment of a Magnet of any shape.*

The method just given involves the measurement of r, the distance between the centre of the magnet and that of the compass needle, and the assumption that this distance is great compared with the dimensions of the magnets, so that they may be treated as solenoidal. In practice these two conditions may not be possible. We might, for example, require to find the magnetic moment about a diameter of a large steel sphere magnetised in any manner.

Now the first equation we have used, viz.,

$$MH = \pi^2 n^2 K,$$

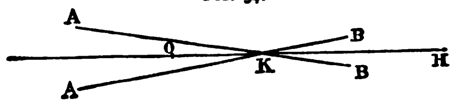
is true for any magnet, provided only that the amplitude of the oscillation is small, and may be applied to the case in point. To find, then, the value of M, determine H as in (*d*), using magnets of a suitable form and size. Suspend the given magnet so that it can oscillate about a suitable axis, and determine K either by calculation from its dimensions, or by observations as in § 23; count also n, the number of transits per second of any fixed point on the magnet across some fixed mark. The formula will then give us M.

(*f*) *To determine the Direction of the Earth's Horizontal Force.*

Consider a magnet, e.g. a magnetised steel disc, free to turn about a vertical axis, which can be inverted on this axis, so that on inversion the side which was the top comes to the

bottom, and *vice versâ*. Then we have seen (p. 434) that a certain straight line in the body will set itself in a certain direction, namely, that of the earth's horizontal force. We wish to determine this direction. It may of course be found approximately by the use of a compass needle. Find it thus and make two marks on the magnet such that the line joining them is approximately in the required direction, and at the same time is horizontal. Let A, B (fig. 54) be the two

FIG. 54.



marks, O the point in the axis round which the magnet turns which is in the same horizontal plane as A B, and O H the required direction. Take the magnet off its support, and turn it over top to bottom through 180°; replacing it, we will suppose, in such a manner on the support that the point O is brought back into its former position. When the magnet again comes to rest, the line in the magnet which originally coincided with O H will clearly do so again; the effect of the change might have been attained by keeping this line fixed and turning the magnet about it through 180°. Hence, clearly if A′ B′ be the new position of A B, A B and A′ B′ meet on O H at K, say, and are equally inclined to it. But A B, A′ B′ being visible marks on the material of the magnet, the directions of these two lines can be identified: the line which bisects them is the direction required, and is thus readily determined.

Moreover, it is not necessary that the point O should, when the magnet is turned round, be brought exactly into its old position. The line O H will in any case after the reversal remain parallel to itself, and A′ B′ will represent not the new position of A B, but its projection on the horizontal plane O A B. The plane of the magnetic meridian will be a vertical plane bisecting the angle between the vertical planes

through the old and new positions of any line A B fixed in the magnet. The experiment then in its simplest form may be performed as follows :—

Fasten a sheet of white paper down on to the table, and suspend over it a magnet of any shape whatever, hanging freely in a stirrup, as already described, by a fibre which has been carefully freed from torsion (p. 454). The magnet should be as close down to the paper as is possible.

Make two marks on the magnet, one at each end, and looking vertically down on it, make two dots on the paper with a fine-pointed pencil, or some other point, exactly under the two marks ; join these two dots by a straight line. Reverse the magnet in its stirrup, turning the top to the bottom, and let it again come to rest. Make two dots as before on the paper vertically below the new positions of the marks, and join these two. The line bisecting the angle between the two lines thus drawn on the paper gives the direction of the horizontal component of the earth's force. In performing the experiment thus, serious error is introduced if the observer's eye be not held vertically over the magnet in each case. This is best ensured by placing a piece of plane mirror on the table below the magnet, leaving the part of the paper which is just below the mark uncovered, and placing the eye at some distance away, and in such a position that the image of the magnet, formed by reflection in the mirror, is exactly covered by the magnet itself ; then if the dot be made on the paper in such a manner as to appear to the observer to be covered by the mark on the magnet, it is vertically below that mark.

If the position of true geographical north at the place of observation be known, we can obtain the angle between the true north and the magnetic north from this experiment. This angle is known as the magnetic declination.

The declinometer, or apparatus used to measure the declination, is constructed on exactly the same principles as those made use of in the foregoing experiment, more

delicate means being adopted to determine the position of the two marks on the magnet with reference to some fixed direction. For an account of these more delicate methods, see Maxwell's 'Electricity and Magnetism,' vol. ii. part iii. chap. vii., and Chrystal, 'Ency. Brit.,' article Magnetism.

(*g*). *Experiments on Two Magnets. Comparison of Magnetic Moments.*

The magnetic moment of a magnet is measured by the maximum couple which the magnet can experience when placed in a field of magnetic force of unit intensity. If we have a series of two or more magnets rigidly connected together, the magnetic moment of the system will be found by combining the moments of the parts according to the law of the composition of couples—i.e. according to the parallelogram law. Thus, if we have two magnets carefully magnetised along the axis of figure, whose moments are m and m', and place them respectively—

1. With their axes parallel and their poles in the same direction;

2. With their axes parallel and their poles in opposite directions;

3. With their axes at right angles;

And if M_1, M_2, M_3 be the magnetic moments of the three combinations, respectively, then we have

$$M_1 = m + m', \qquad M_2 = m - m', \qquad M_3^2 = m^2 + m'^2;$$
$$\therefore\ 2M_3^2 = (m+m')^2 + (m-m')^2 = M_1^2 + M_2^2.$$

Now let the magnets be rigidly connected together in these three positions in turn, so that the centre of one is vertically below that of the other, and let the times T_1, T_2, T_3 of their oscillations about a vertical axis be observed.

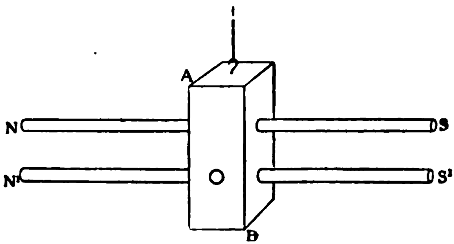
The magnets may most easily be so fixed in the following manner:—

A B (fig. xxxv) is a small rectangular block suspended by a fine silk fibre attached to a hook at the centre of one face.

Two parallel holes are bored through one pair of ver-

tical faces, and by inserting the magnets in these they can be placed in the positions 1 or 2. A third hole is bored at right angles to the former through the other pair of vertical

FIG. XXXV.



faces, and by inserting one of the magnets through this the two can be put in position 3.

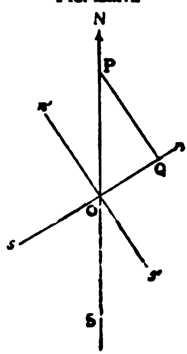
Now the moment of inertia of the system about the vertical fibre is the same for all three positions. Let this moment be K, and let H be the strength of the earth's horizontal field. Then we have

$$M_1 = \frac{4\pi^2}{T_1^2}\frac{K}{H}, \quad M_2 = \frac{4\pi^2}{T_2^2}\frac{K}{H}, \quad M_3 = \frac{4\pi^2}{T_3^2}\frac{K}{H};$$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{M_2}{M_1} = \frac{m-m'}{m+m'};$$

$$\therefore \quad \frac{m}{m'} = \frac{T_1^2+T_2^2}{T_2^2-T_1^2}.$$

FIG. XXXVI.



We thus find the ratio of the magnetic moments without knowing the moments of inertia of the magnets.

Again, when the magnets are in position 3, let $n\ s$, $n'\ s'$ (fig. xxxvi) be traces of the axes of the two, and let N O S be the north and south line marked on paper below the system. From P, any point on O N, draw P Q perpendicular to O n. Then, since the magnets are in equilibrium, and the forces acting

H H

on their poles are parallel to S N, we have by the parallelogram law,

$$\frac{OQ}{QP} = \frac{m}{m'} = \frac{T_1^2 + T_2^2}{T_1^2 - T_1^2}.$$

This can be verified experimentally by construction. Also, since

$$2\,M_3^2 = M_1^2 + M_2^2,$$

we have

$$\frac{2}{T_3^4} = \frac{1}{T_1^4} + \frac{1}{T_2^4}.$$

This formula can be verified by experiment.

Care must be taken in the construction and in measuring the times of swing in order to obtain accuracy in these last two results.

Experiments.

(*a*) Determine if the given bar of steel is magnetised. Magnetise it.

(*b*) Compare the moment of the given magnet after magnetisation (1) by stroking, (2) by the use of an electro-magnet.

(*c*) Compare the strength of the magnetic field within a soft-iron cylindrical screen with the normal strength of the earth's field.

(*d*) Determine the moment of the given bar magnet and the horizontal intensity of the earth's magnetic force.

(*e*) Determine the moment of the given magnetic mass about the given axis, using the known value of the earth's horizontal force.

(*f*) Lay down on the table the direction of the magnetic meridian.

Enter results thus :—

(*a*) Effect on suspended magnet only visible after five or six reversals of position, isochronous with the time of swing.

(*b*)

	Observed values of *n*,		Observed values of *n*,
	·098		·144
	·104		·148
	·101		·140
Mean	·101	Mean	·144

$$M_1/M_2 = (101)^2/(144)^2 = \cdot 492.$$

(*c*) Values of n within the cylinder, using the same magnet after the last magnetisation.

$$\begin{array}{rl} & \cdot 073 \\ & \cdot 070 \\ & \cdot 068 \\ \hline \text{Mean} & \cdot 070 \end{array}$$

Strength of field within : strength without $= (\cdot 070)^2 : (\cdot 144)^2$.

(*d*) Using the last observations in (*b*)

$$n = \cdot 144$$

$$\text{K (calculated from dimensions)} = 379\cdot 9 \text{ gm.(cm.)}^2$$

$$r = 40 \text{ cm.}$$

$$\theta = 4^\circ\ 30'$$

Whence $\text{H} = \cdot 176$ C.G.S. units

$\text{M} = 442\cdot 6$ C.G.S. units.

(*e*) A sphere of radius 2·5 cm. experimented with.

Mass 500 gm.

$\text{K} = 1250 \text{ gm.(cm.)}^2$

$\text{H} = \cdot 176$ C.G.S. units

$n = \cdot 0273$

$\text{M} = 52\cdot 6$ C.G.S. units.

(*f*) Shew on a sheet of paper lines drawn parallel to the edge of the table and to the direction of the horizontal component of the earth's magnetic force respectively.

(*g*) Compare the magnetic moment of the two given magnets, and verify the result that $2\text{M}_3^2 = \text{M}_1^2 + \text{M}_2^2$.

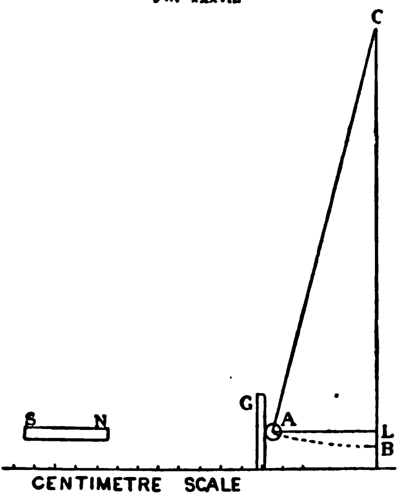
S. Comparison of Gravitational and Magnetic Forces.

The force with which an ordinary bar magnet attracts a piece of soft iron varies very rapidly with change in the distance of the iron from the pole of the magnet. The following experiment illustrates this point.

A small iron sphere, about ·5 cm. in diameter, is suspended from the ceiling by a long fine thread, so as to be a few centimetres above a table. Beneath it is placed a scale of centimetres, on which stands a vertical piece of glass.

The glass is mounted in such a way that its position on the scale can be easily determined. The reading at the point underneath the sphere when the thread hangs vertically is observed. Place the glass at right angles to the scale, and at some few centimetres away from this point. On bringing a magnet near on the side of the glass removed from the sphere, the sphere is attracted and moves up against the glass. Gradually withdraw the magnet, keeping it at the same level as the sphere, until the sphere just leaves the glass, and observe its position on the scale when this occurs. It will be

FIG. XXXVII.



found that this position can be determined with considerable accuracy. Let W be the weight of the sphere, l the length of the string. In fig. xxxvii, let B be the point on the scale vertically under C the point of suspension of the sphere; let A be the centre of the sphere when just leaving the glass. Let A L be horizontal, and equal to y cm. Let S N be the magnet, and A N $= x$ cm. The distances x and y are given by the observations on the scale. Let F be the horizontal component at A of the force due to the magnet in the direction

of the scale. Then the sphere is in equilibrium under its weight, the tension of the string, and the force due to the magnet.

Hence the component of the tension in the direction A N is equal to F. If, moreover, the axis of the magnet N S is at the same level as A, the magnet exerts no vertical force on the sphere, and the vertical component of the tension is equal to the weight of the sphere; but since the string is very long (12 feet), the vertical component of the tension is equal very approximately to the whole tension, and thus we get

$$F = W \frac{AL}{AC} = W \frac{y}{l}.$$

Set the glass plate so that when the sphere is in contact with it its centre may be at distances of 1, 2, 3, . . . cm. respectively from B, and determine the corresponding values of x, x_1, x_2, . . . Then plot a curve, taking the values of x as abscissæ and the corresponding values of y, y_1, y_2 . . . as ordinates.

The curve should be found to take the form given by the equation $y \times x^5 = C$, where C is a constant for reasons which are given in the foot-note.[1]

[1] If H is the strength of the magnetic field at A due to the magnet, and a the radius of the sphere, k the magnetic susceptibility, and if δ H represents the rate of change of H per centimetre increase of x, the distance A N, then it can be shewn that

$$F = \frac{4}{3} \pi \frac{a^3 k}{1 + \frac{4}{3} \pi k} H \delta H.$$

If the force F be due to the action of a long bar magnet, so long that we may without serious error neglect the effect of the pole S compared with that of N, then we have $H = \frac{m}{x^2}$, and from this

$$\delta H = -\frac{2m}{x^3}.$$

Thus the force F acting towards N is

$$\frac{8}{3} \frac{\pi a^3 k m^2}{1 + \frac{4}{3} \pi k} \frac{1}{x^5}.$$

and since $F = W y / l$, we have $y = C / x^5$, where C is constant.

The value of C may be found by taking the values of x and y corresponding to some point P on the curve, and substituting them in the equation; then by drawing the curve $y = Cx^5$, and comparing it with the result of the experiment, or by calculating the values of $y \times x^5$ for the observed points, we may verify the result.

Experiment.—Verify the relation $y \times x^5 = C$, in the circumstances described above, and compare the magnetic force upon the iron sphere when its centre is 2 cm. from the end of the bar magnet with the weight of the sphere.

T. Gauss's Verification of the Law of Magnetic Force.

We have seen already (p. 450) that if the law of force between two magnetic poles be that of the inverse square, and if ϕ be the angle through which a magnet is deflected from the meridian by a second magnet of moment M at a distance r in the 'end-on' position, then

$$H \tan \phi = \frac{2\,M}{r^3}.$$

While if ψ be the deflexion due to the same magnet in the 'broadside-on' position, then

$$H \tan \psi = \frac{M}{r^3}.$$

These results can be verified by the apparatus referred to in § 69 (*b*), fig. 53.

For if we observe the values of ϕ and ψ corresponding to different values of r, we can shew that

$$r^3 \tan \phi = \text{constant} = 2\, r^3 \tan \psi.$$

If we make the more general assumption that the force between two poles m, m' is $m m'/r^n$, then we can find the value of the magnetic potential and the magnetic force

at any point by the same method as we have applied to the simpler case. We shall find

$$V = \frac{M \cos\theta}{r^n},$$

$$R = \frac{n M \cos\theta}{r^{n+1}},$$

$$T = \frac{M \sin\theta}{r^{n+1}};$$

while the equations giving the ratio of M to H become

$$H \tan\phi = \frac{n M}{r^{n+1}}, \text{ and } H \tan\psi = \frac{M}{r^{n+1}}.$$

Hence by observing ϕ and ψ we can find n.[1] There are various ways in which we can carry out the experiment; one has been already described. The following is one which employs a modification of the ordinary method of reading a galvanometer mirror.

The deflected magnet *n s*, which should be very small, is attached to the back of a mirror. This mirror is suspended by a fine silk fibre, the point of suspension being vertically above the point O (fig. xxxviii); P Q is a wooden stand, pivoted so as to turn about a vertical axis through O, and the support carrying the mirror is attached to P Q. By this means the mirror always occupies the same position relative to the support carrying the fibre, and errors due to the torsion of the fibre are eliminated. At P and Q are two vertical pins, equidistant from O, the top of the

[1] In the above we have neglected terms depending on l^2/r^2, $2l$ being the length of the magnet.

If these are included, then it can be shewn that

$$H \tan\phi = \frac{n M}{r^{n+1}}\left\{1 + \frac{(n+1)(n+2)}{2 \cdot 3}\frac{l^2}{r^2} + \ldots\right\},$$

$$H \tan\psi = \frac{M}{r^{n+1}}\left\{1 - \frac{n+1}{2}\frac{l^2}{r^2} + \ldots\right\}.$$

Thus

$$\frac{\tan\phi}{\tan\psi} = n\left\{1 + \frac{(n+1)(n+5)}{6}\frac{l^2}{r^2} + \ldots\right\}.$$

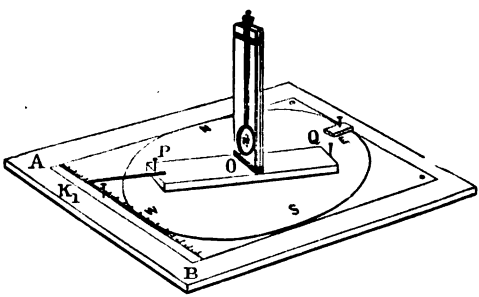
pins being at a greater height above the board than the mirror.

The mirror and magnet are enclosed in a wooden or brass case, with glass windows back and front, through which the pins can be seen ; a mica vane is attached to the back of the mirror to damp the oscillations.

The whole is mounted on a drawing-board, carrying a sheet of paper, on which a circle of about 20 cm. radius, with O as centre, is drawn. A horizontal scale, A B, divided to millimetres, is adjusted, as described below, to lie in the magnetic meridian, and fixed to the board.

On looking at the mirror an image of the pin P can be

FIG. xxxviii.



seen, and by turning the board round O carefully this image can be made to coincide with Q. In this case the line P Q is normal to the mirror, and, therefore, if there are no other magnets near, points east and west. Draw the east and west line, E W, on the paper, and through O draw N O S perpendicular to it. Adjust the scale A B to be perpendicular to E W ; the scale then lies in the magnetic meridian. Note the point W in which the east and west line cuts the scale. This is most readily done by holding a piece of fine wire vertically in a small clip, and moving it until the wire, the pin P, and the image of P in the mirror appear in one line ; or it may be

done by having a pointer attached to the stand Q P, the direction of the pointer being that of Q P produced.

Now place the disturbing magnet with its centre on the circle at E and its north pole pointing east, so that it is in the 'end-on' position; the mirror will be deflected. Turn the stand P Q until the line P Q is again normal to the mirror, and read the position K_1 of the pointer on the scale.

Reverse the position of the deflecting magnet at E so that the south pole may point east. The mirror will be deflected to the other side of the meridian, and another position (K_2) found for K. If we call the deflexions ϕ_1 and ϕ_2, and the corresponding distances measured on the scale c_1, c_2, we have

$$\tan\phi_1 = \frac{WK_1}{OW}; \ \tan\phi_2 = \frac{WK_2}{OW}.$$

Thus the distances c_1, c_2 are respectively proportional to $\tan\phi_1$ and $\tan\phi_2$. If the deflecting magnet is perfectly symmetrical, the two distances will be equal. Now place the magnet with its centre at W, and observe again; let the distances be c_1', c_2'. Take the mean of the four c_1, c_2, c_1', c_2'; let it be x. It will correspond to a value of $\tan\phi$, corrected for want of symmetry in the deflecting magnet, and for the fact that the deflected magnet may not be exactly at the centre of the circle N E S W.

Move the deflecting magnet, still with its axis pointing east and west, until its centre is at S, and afterwards at N (it is then in the 'broadside-on' position), and observe as before the four distances, d_1, d_2, d_1', d_2'; let the mean of these be y. Then y is proportional to $\tan\psi$, the corrected deflexion in the 'broadside-on' position; thus

$$\frac{x}{y} = \frac{\tan\phi}{\tan\psi} = n\left\{1 + \frac{(n+1)(n+5)}{6}\frac{l^2}{r^2}\right\}.$$

From this equation n can be found. If the experiments are conducted with care, we obtain $n = 2$ very approximately

as the result. To solve the equation for n, we may first omit the terms involving l^2/r^2, which will be small. We thus get an approximate value n_1. Then substitute this value in the small terms, and we have

$$n = \frac{x}{y}\left\{1 - \frac{(n_1+1)\ (n_1+5)}{6}\frac{l^2}{r^2}\right\}.$$

To obtain an estimate of the value of the correcting term, we may remember that n_1 is nearly 2; thus the value of the term in l^2/r^2 is $7l^2/2r^2$. Suppose $l = 2$ cm., so that the magnet is 4 cm. long, and $r = 20$ cm., then $7l^2/2r^2 = 7/200 = 1/30$ approximately.

By making observations in a similar manner with the deflecting magnet at different distances from O, we can verify the fact that $\tan\phi$ is inversely proportional to r^3. These experiments were first carried out by Gauss. He found that, provided l/r were less than $\frac{1}{4}$, the results of his own observations were represented by the formulæ

$$\tan\phi = {\cdot}086870\,r^{-3} - {\cdot}002185\,r^{-5},$$
$$\tan\psi = {\cdot}043435\,r^{-3} + {\cdot}002449\,r^{-5},$$

which afford a double verification of the law.

Experiment.—Verify the law of the inverse square in Gauss method.

Enter the results thus :—

Value of x	. . .	4·56
" y	. . .	2·20
$l = 3.$	$r = 20.$	

Approximate value of $n = 2{\cdot}08$
Corrected value of $n = 1{\cdot}99$

U. Magnetic Induction due to the Earth.

A piece of soft iron placed in a magnetic field becomes magnetised by induction. If the intensity of the field be small, such as that due to the earth, the magnetic moment induced by it in the iron will be proportional to the com-

ponent of the intensity of the field in the direction of the magnetic axis of the bar. A bar of soft iron may be thus magnetised by induction, and by measuring the strength of either pole of the bar we may obtain a measure of the strength of the inducing field.

Thus, take a rod of soft iron about 1 metre long and 1 centimetre in diameter. Hold it in a vertical position, and hit it three or four sharp blows with a hammer, or allow one end to fall vertically on to a flag-stone from about 25 cm. three or four times. The rod will be magnetised along its length, under the action of V, the vertical component of the earth's magnetic force, and the strength of each pole will be proportional to V, and may be written λ V. Since the rod is very thin, the effect of the horizontal force in magnetising it is negligible.

Now bring the rod carefully, still holding it vertical, until the lower end (the north pole) is in some definite position with regard to a compass needle—e.g. let it be at the same level as the needle, and 10 cm. to the east of its centre. Call the distance between the two r, and let ϕ be the deflexion of the compass. Then, since the south pole of the bar is so far off, the magnetic force at each pole of the compass needle is $\lambda \mathrm{V}/r^2$, and if the compass needle is small the forces on the two poles are nearly parallel, so that

$$\lambda \mathrm{V} = r^2 \mathrm{H} \tan \phi,$$

H being the horizontal component of the earth's magnetism.

Now place the bar with its axis horizontal and north and south, and magnetise it by striking it as before; the strength of the poles will in this case be λ H, and if the bar be moved carefully, being kept horizontal, and with its axis north and south all the time, until the north pole comes into the same position as before, and the deflexion now observed in the compass needle be ψ, then

$$\lambda \mathrm{H} = r^2 \mathrm{H} \tan \psi.$$

Now, if i be the magnetic dip,

$$\tan i = \frac{\text{V}}{\text{H}} = \frac{\tan \phi}{\tan \psi}.$$

Thus the dip can be found from observations of ϕ and ψ.

To obtain an accurate result the experiment must be repeated, care being taken to strike the bar sufficiently in each position to ensure its receiving the maximum amount of magnetisation which the horizontal and vertical forces, respectively, are capable of inducing.

Experiment.—Determine the dip[1] by observations on the magnetism induced in a rod by the horizontal and vertical components of the earth's magnetic field.

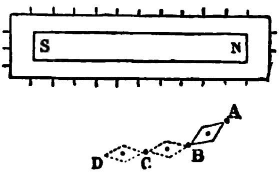
70. Exploration of the Magnetic Field due to a given Magnetic Distribution.

Place a bar magnet on a large sheet of paper on a table. In the neighbourhood of the magnet there will be a field of magnetic force due to the joint action of the earth and the bar magnet, and if a small compass needle be placed with its centre at any point of the field, the direction of the needle, when in equilibrium, will indicate, very approximately indeed, the direction of the line of magnetic force which passes through its centre. Draw a line on the paper round the bar magnet at a distance of 2 or 3 cm. from it, and mark off points along this line at intervals of 2 cm. Take a small compass needle and lay it so that its centre is above the first of the points so marked; it will then set itself in the direction tangential to the line of force which passes through the point. Make marks on the paper exactly opposite to the points at which the ends of the

[1] The student should notice that this experiment merely illustrates the proportionality between the small magnetising forces and the corresponding magnetisation. It is not a standard method of determining the dip.

compass rest, and as close to them as possible. Let A B (fig. 55) be the ends of the compass. Move the compass on in the direction in which it points, and place it so that the end A comes exactly opposite the mark against the old position of B, while the end B moves on to position C, so that B C is the new position of the compass. Make a mark opposite the point C in its new position. Again move the compass on until the end at B comes into the position C, and so on. A series of points will thus be drawn on the paper, and a line which joins them all will very nearly coincide with a line of force due to the given distribution. The line of force can thus be traced until it either cuts the line drawn round the magnet or goes off the paper. Repeat the operations, starting from the second of the points on the line drawn round the bar magnet, and then from the third, and so on, until the lines of force for all the points are drawn, thus giving a complete map of the *directions* of the lines of force due to the combination.[1]

Fig. 55.

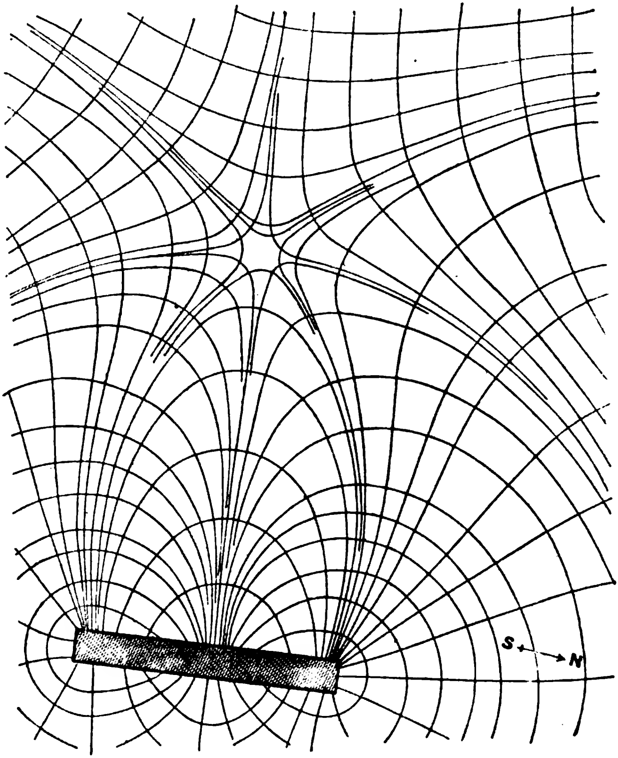


It is convenient to have the compass needle mounted, as is often done for trinkets, between two pieces of glass. The dots on the paper can then be seen through the glass, and the compass set so that the end of the needle may be accurately over the dot. If, further, the compass have a

[1] However the bar magnet be placed, there will generally be found two points in the field at which the resultant force is zero. These points can be very accurately identified by carefully drawing the lines of force in their neighbourhood. When they have been determined their distances from the poles of the bar magnet can be measured by a scale; the angles between the lines joining one of the points of zero force with the poles can be determined, and from these observations an estimate can be made of the strength of either pole of the bar magnet in terms of the strength of the earth's field. The positions of the poles are very well indicated by the convergence of the lines of force.

non-magnetic arm fixed at right angles to the needle, then the direction of this arm gives the direction of the equipotential surface at the point, and by making dots under

FIG. xxxix.



the ends of this arm, and working with it in the same way as with the needle itself, we can draw the equipotential surfaces. Fig. xxxix is a set of such lines drawn in this way.

Experiment.—Draw a map of the directions of the lines of force due to the combined action of the earth and the given bar magnet.

V. Magnetic Induction in Iron.

The magnetic force at a point has been defined as the force on a unit pole placed at that point. Now if the point be in the middle of a magnet, such as a mass of iron or steel in a magnetic field, we must suppose a small cavity removed in order to place the unit pole there. We can shew that the force on the pole depends on the shape of the cavity (see Ewing, 'Magnetic Induction in Iron and other Metals,' pp. 1–22), for the magnetic forces induce on the walls of the cavity magnetism, which acts on the pole, and the effect of the magnetism so induced depends on the shape of the cavity. The iron or steel is magnetised by the external field. Let us suppose the cavity takes the form of a long narrow cylinder, with its length along the lines of magnetisation. Then the force on the pole is defined as the magnetic force inside the cavity; we denote it by H. If, on the other hand, the cavity is a very narrow crevasse at right angles to the direction of magnetisation, then the force on unit pole in such a cavity defines the magnetic induction; we denote it by B. The ratio of B to H is generally denoted by μ, and is called the permeability. The permeability is not a constant, but depends on the value of H and on the past history of the iron. When the iron is subject to magnetic force each small element of volume v becomes a magnet; let us denote the moment of that element by $\mathsf{I}\, v$, so that I is the magnetic moment per unit volume of the iron. I is called the intensity of magnetisation. The ratio of I to H is the susceptibility, and is denoted by κ. The susceptibility, like the permeability, is not constant, but depends on H and on the past history of the iron.

Now we may shew[1] that

$$\begin{aligned} \mathsf{B} &= \mathsf{H} + 4\pi\,\mathsf{I} \\ &= (1 + 4\pi\,\kappa)\mathsf{H} \\ &= \mu\,\mathsf{H} \text{ by definition;} \end{aligned}$$

$$\therefore\ \mu = 1 + 4\pi\,\kappa.$$

[1] See Ewing, *loc. cit.*

The induced magnetisation produces magnetic force inside the magnetised body, which acts in the opposite direction to the magnetising force. The amount of this induced force depends on the shape and material of the magnetised body. Thus if a long rod be magnetised by a force H_0, one end becomes a north pole, the other a south pole, and within the rod we have, in addition to the force H_0, the opposing force due to the ends. In any calculation, then, the effect of this must be allowed for; but if we make the length of the rod very long compared with its diameter (say 400 times the diameter), the effect of the ends is negligible except near the ends, and we may treat the problem as though the magnetic force in the rod were the impressed force H_0.

Now if a current be allowed to circulate in a long coil of insulated wire wound into the form of a close straight helix, the lines of force inside the helix, except near its ends, are straight lines parallel to the axis of the helix (see Searle,[1] 'Determination of Currents in Absolute Measure'), and it can be shewn that if γ be the current in absolute electro-magnetic measure, and n the number of turns per unit length of the helix, then inside the helix $H_0=4\pi n\gamma$.[2]

If then a thin rod of soft iron be placed inside the helix, we can subject it to a known magnetising force, and examine in the following way the effects.

Place the helix horizontally, with its axis east and west, in such a position that the axis produced passes through the centre of a magnetometer needle. A small mirror, with a magnet at its back, suspended by a silk fibre, and a lamp and scale are arranged in the usual manner. The coil may conveniently be about 50 cm. in length and 1 cm. in diameter, wound with two or more layers of insulated wire.

The ends of the wires are connected through a tangent

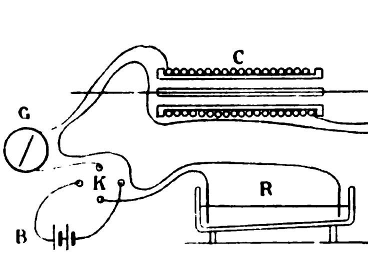
[1] Mr. Searle's papers appeared in the *Electrician* for 1891, and are being republished in book form.

[2] See Ewing, *loc. cit.*

galvanometer G (fig. xl), or a direct-reading ammeter and an adjustable resistance R, to a battery and a reversing key K.

On passing a current through the coil C the magnetometer is affected by the direct action of the coil. This action may be compensated by a permanent magnet. It is better, however, to pass the same current through a second coil C′ of larger area with a few turns of wire, placed

FIG. xl.



near the magnetometer, this coil can be adjusted so that its effect on the magnetometer is exactly opposite to that of the main coil. Make this adjustment for the largest current which is to be used, and secure the coil C′ in position with a clamp. Then the currents in the coils will not affect the magnetometer, and any action which takes place is due to the magnetism induced in the soft iron rod when it is put in. The leading-wires should be kept close together and not moved.

(1) *To find the Magnetic Moment of a Soft Iron Rod.*

The rod may be 40 cm. long by ·1 cm. thick. See, in the usual way, that the rod is free from permanent magnetism. If not, heat it to a red heat and allow it to cool in an east and west position.

Insert it in the helix, and let its centre be r cm. distant from the magnetometer; let the length of the rod

be $2l$ cm., and let M be the induced magnetic moment, m the strength of either pole, assuming the magnetisation uniform. The rod should be distinctly shorter than the helix. Then the magnetic force in the direction of its axis at distance r from its centre is

$$\frac{m}{(r-l)^2}-\frac{m}{(r+l)^2},$$

and this is equal to

$$\frac{4mrl}{(r^2-l^2)^2},$$

or to

$$\frac{2Mr}{(r^2-l^2)^2}.$$

If ϕ be the deflexion of the magnetometer, then this magnetic force is equal to H tan ϕ. Thus we have

$$M=H\tan\phi\,\frac{(r^2-l^2)^2}{2r}.$$

In making the observations it is desirable to tap the rod lightly when in position ; this helps the magnetisation.

The value of M should be measured for different rods. By taking rods of the same thickness, but of different lengths, we can examine the effect of the ends ; if this effect be inappreciable the values found for M will be proportional to the respective lengths. In order to secure this the ratio diameter to length should not be greater than 1/400.

(2) *To find the Magnetic Susceptibility.*

Take a rod in which the effect of the ends is known to be small, and measure its magnetic moment M. Let $2l$ be its length, and a the radius of a section which we suppose is circular ; then its volume is $2\pi la^2$, and if I is the intensity of magnetisation, I is the magnetic moment per unit volume.

$$\therefore I=\frac{M}{2\pi la^2}.$$

Thus I can be found. Since we may neglect the ends of the rod, the magnetic force inside it is

$$H = 4 \pi n \gamma,$$

and κ, the susceptibility, is the ratio of I to H.

$$\therefore \kappa = \frac{M}{8\pi^2 a^2 l n \gamma}$$

$$= \frac{H \tan\phi \, (r^2 - l^2)^2}{16 \pi^2 a^2 r l n \gamma}.$$

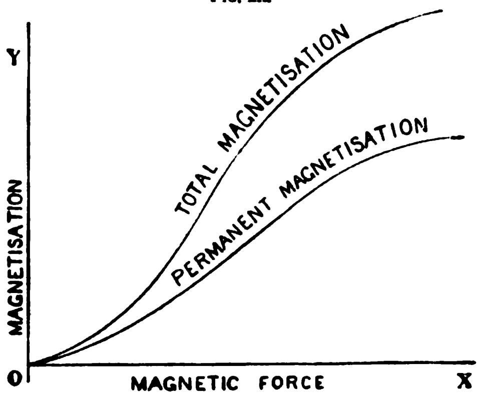
Let G be the galvanometer constant of the galvanometer used to measure the current (see p. 503), and θ the deflexion of the magnet; then

$$\gamma = \frac{H}{G} \tan\theta ;$$

$$\therefore \kappa = \frac{G \tan\phi}{\tan\theta} \frac{(r^2 - l^2)^2}{16 \pi^2 a^2 r l n}.$$

The same observations give us μ, the permeability, for $\mu = 1 + 4\pi\kappa$. Now break the battery circuit. The rod

FIG. xli.



will remain magnetised, though to a less extent than before, the amount of residual magnetisation depending largely on the method adopted for breaking the current. Measure the

residual moment M in the same way, and calculate the residual susceptibility, viz. by the ratio of the residual magnetisation to the maximum magnetising force. Now free the rod from magnetisation, and repeat the experiment, using a different magnetising current. Plot the results on a curve, taking the values of the magnetising force H as abscissæ and the corresponding magnetisations as ordinates. The curves will have the form shewn in fig. xli.

(3) *Magnetic Cycles. Hysteresis.*

The behaviour of iron in a magnetic field can be more completely investigated if the magnetic force be carried through a complete cycle of changes in the following manner.

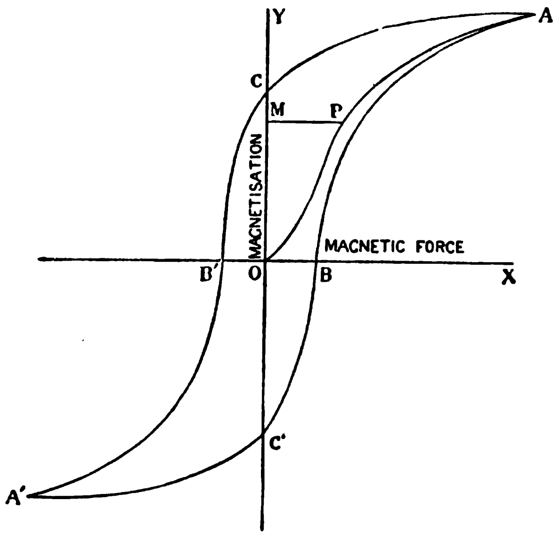
Include in the battery circuit a variable resistance. This may consist either of an adjustable wire rheostat, or, better, of a liquid resistance, such as copper sulphate. This may be contained in a rectangular trough. A fixed copper plate dips into one end of the trough, while a second movable plate can be inserted in any other position. The trough, which is shown at R, fig. xl, is tilted, so that the depth of the liquid is much greater near the fixed plate than at the other end, where it only just covers the base. If the movable plate be inserted at this end, a very large resistance is in circuit; as the plate is moved towards the other end the resistance decreases. The battery circuit should also contain a reversing key.

We wish to investigate the magnetisation of the rod as the magnetising force gradually increases from zero up to a maximum, and then decreases again through zero to an equal negative maximum, from which it is again increased through zero up to the same positive maximum as before.

The adjustments are made as already described, the movable plate being placed so that the resistance in circuit at starting is very great, and the current made. A series of simultaneous readings of the galvanometer and magneto-

meter are then taken, the resistance being gradually decreased. When the current has reached its maximum value the resistance is again gradually increased and the current reduced to zero; if the results be plotted it will be found that the descending curve is much less steep than the ascending, and when the current is zero there will be a considerable amount of residual magnetism left. The battery

FIG. xlii.



commutator is then reversed, and the resistance again diminished until the current reaches a maximum negative value. It will be found that during this process the magnetisation does not at first alter much, but that after the current has attained a not very large negative value there is a sudden large change in the magnetisation from a considerable positive amount to an equally large negative value. After this, as the current increases the magnetisation in-

creases, but more gradually. When the current has reached its maximum negative value it is again decreased by increasing the resistance, and afterwards, passing through zero, reversed and increased again up to the same positive maximum as before.

If the magnetisation curve for this process be drawn, it will be a closed curve,[1] resembling in form that given in figure xlii.

Again, it has been shewn that the area measured on a proper screen of the closed cycle is the total energy required to carry unit volume of the iron through the magnetic changes. This energy is dissipated as heat.

Moreover, Prof. Ewing has shewn that whenever iron is taken through any cyclic process of magnetising force, the magnetisation changes, but in such a way as always to lag behind the magnetising force; there is a tendency for the existing state of magnetisation to persist. To this tendency he has given the name hysteresis, and it is in consequence of this hysteresis that energy is required to produce a cycle of magnetic changes.

Experiments.

(1) Determine the magnetic moment of the given pieces of soft iron under a given magnetic force.

(2) Find the susceptibility and permeability of soft iron for various values of the magnetising force, and determine also the residual magnetisation when the force is suddenly removed.

(3) Draw the hysteresis curve for the given specimen of soft iron, and calculate the energy dissipated as heat in carrying it round a complete cycle.

Enter in parallel columns the values of H, I, κ, B, μ, and draw the curve.

[1] For a discussion of the properties of this curve, and the variations in its form for various specimens of iron, see Ewing, *Magnetic Induction in Iron*, &c., chaps. iv. and v., from which much of the above is taken.

CHAPTER XVIII.

ELECTRICITY—DEFINITIONS AND EXPLANATIONS OF ELECTRICAL TERMS.

IN the last chapter we explained various terms relating to magnetism. Just as in the neighbourhood of a magnet we have a field of magnetic force, so, too, in the neighbourhood of an electrified body there is a field of electric force. We proceed to consider certain facts, and to explain some of the terms connected with the theory of electricity, a clear comprehension of which will be necessary in order to understand rightly the experiments which follow.

Most bodies can by friction, chemical action, or by various other means, be made to exert forces on other bodies which have been similarly treated. The phenomena in question are classed together as *electrical*, and the bodies are said to have been *electrified*. By experiments with Faraday's ice-pail among others (*vide* Maxwell's 'Elementary Electricity,' p. 16, &c.), it has been shewn that these effects can be accounted for by supposing the bodies to be charged with certain *quantities* of one of two opposite kinds of *electricity*, called respectively positive and negative, and such that equal quantities of positive and negative electricity completely annihilate each other.

An electrified body exerts force on other electrified bodies in its neighbourhood—in other words, produces a field of electrical force—and the force at any point depends on the position of the point, on the form and dimensions of the electrified body, and on the quantity of electricity on the body. By doubling the charge we can double the force. We are thus led to look upon electricity as a quantity which can be measured in terms of a unit of its own kind, and we may speak of the quantity of electricity on a body, in somewhat the same way as we use the term quantity of magnetism for the strength of a magnetic pole. The magnetic forces

produced by a magnetic pole are due to a quantity of magnetism concentrated at the pole. The electrical forces produced by an electrified body are due to a quantity of electricity distributed over the body. By supposing the body to become very small while the quantity of electricity on it still remains finite, we may form the idea of an electrified point or a point charged with a given quantity of electricity.

With regard to the transmission of electrical properties bodies may be divided into two classes, called respectively conductors and non-conductors. To the latter the name 'dielectric' is also applied.

DEFINITIONS OF CONDUCTORS AND NON-CONDUCTORS.—If a quantity of electricity be communicated to a conductor or conducting body at one point, it distributes itself according to certain laws over the body; if, on the other hand, it be communicated to a non-conductor, it remains concentrated at the point where it was first placed. Quantities of electricity pass freely through the substance of a conductor; they cannot do so through a non-conductor.

Quantities of electricity are of two kinds, having opposite properties, and are called positive and negative respectively. Two bodies each charged with the same kind of electricity repel each other; two bodies charged with opposite kinds attract each other. To move an electrified body in the field of force due to an electrified system, against the forces of the system requires work to be done, depending partly on the forces of the system and partly on the quantity of electricity on the body moved.—We shall see shortly how best to define the unit in terms of which to measure that quantity.—Moreover, owing to the action between the electrified body and the rest of the system, alterations will generally be produced in the forces in consequence of the motion.

DEFINITION OF RESULTANT ELECTRICAL FORCE.—The resultant electrical force at a point is the force which would be exerted on a very small body charged with unit quantity of positive electricity placed at the point, it being supposed

that the presence of the body does not disturb the electrification of the rest of the system.

Hence if R be the resultant electrical force at a point, and e the number of units of electricity at that point, the force acting on the body thus charged is R e.

If the body so charged be moved by the forces acting on it, work is done.

DEFINITION OF ELECTROMOTIVE FORCE. — The work done in moving a unit quantity of positive electricity from one point to another is called the electromotive force between those points.

Hence, if the electromotive force (denoted by the symbols E.M.F.), *between two points* be E, the work done in moving a quantity e of positive electricity from the one point to the other is E e. Electromotive force is sometimes defined as the *force* which tends to move electricity; the definition is misleading. The name itself is perhaps ambiguous, for the electromotive force *between two points* is not force, but work done in moving a unit of positive electricity; it, therefore, has the dimensions of work divided by electrical quantity (see p. 20). The term electromotive force *at a point*, however, is sometimes used as equivalent to the resultant electrical force. We shall avoid the term.

Suppose that a single body charged with positive electricity is being considered, then it is found that the force which this body exerts on any electrified body decreases very rapidly as the distance between the two bodies is increased, becoming practically insensible when the distance is considerable. We may define as the field of action of an electrified system of bodies that portion of space throughout which the electrical force which arises from the action of those bodies has a sensible value. If a quantity of positive electricity be moved from any point of the field to its boundary by the action of the electrical forces, work is done.

DEFINITION OF ELECTRICAL POTENTIAL.—The electrical potential at a point is the work which would be done by the

electrical forces of the system in moving a unit quantity of positive electricity from the point to the boundary of the field, supposing this could be done without disturbing the electrification of the rest of the bodies in the field.

We may put this in other words, and say that the electrical potential at a point is the E.M.F. between that point and the boundary of the field.

It is clear from this definition that the potential at all points of the boundary is zero.

The work done by the forces of the system, in moving a quantity e of positive electricity from a point at potential V to the boundary, is clearly Ve, and the work done in moving the same quantity from a point at potential V_1 to one at potential V_2 is $e(V_1 - V_2)$.

Hence, it is clear that the E.M.F. between two points is the difference of the potentials of the points.

We are thus led to look upon the electric field as divided up by a series of surfaces, over each of which the potential is constant. The work done in moving a unit of positive electricity from any point on one of these to any point on another is the same.

When two points are at different potentials there is a tendency for positive electricity to flow from the point at the higher to that at the lower potential. If the two points be connected by a conductor, such a flow will take place, and unless a difference of potential is maintained between the two points by some external means, the potential will become equal over the conductor; for if one part of the conductor be at a higher potential than another, positive electricity immediately flows from that part to the other, decreasing the potential of the one and increasing that of the other until the two become equalised.

Now the earth is a conductor, and all points, not too far apart,[1] which are in metallic connection with the earth are at the same potential.

[1] If the points are far apart, electro-magnetic effects are produced by the action of terrestrial magnetism.

It is found convenient in practice to consider this, the potential of the earth, as the zero of potential; so that on this assumption we should define the potential at a point as the work done in moving a unit of positive electricity from that point to the earth. If the work done in moving a unit of positive electricity from the earth to the boundary of the field be zero, the two definitions are identical; if this be not the case, the potential at any point measured in accordance with this second definition will be less than its value measured in accordance with the first definition by the work done in moving the unit of positive electricity from the earth to the boundary of the field; but since electrical phenomena depend on difference of potential, it is of no consequence what point of reference we assume as the zero of potential, provided that we do not change it during the measurements. In either case the E.M.F. between two points will be the difference of their potentials. Potential corresponds very closely to level or pressure in hydrostatics. The measure of the level of the water in a dock will depend on the point from which we measure it, *e.g.* high water-mark, or the level of the dock-sill below high water-mark; but the flow of water from the dock if the gates be opened will depend not on the actual level, but on the difference between the levels within and without the dock, and this will be the same from whatever zero we measure the levels.

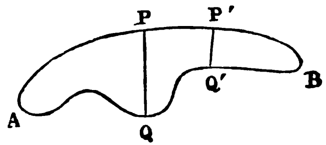
Various methods have been discovered for maintaining a difference of potential between two points connected by a conductor, and thus producing between those points a continuous flow of electricity; the most usual are voltaic or galvanic batteries.

For the present, then, let us suppose that two points A and B are connected with the poles of a battery, A and B being points on a conductor, and let us further suppose that the pole of the battery connected with A is at a higher potential than that connected with B. The pole connected with A is said to be the positive pole. A continuous transfer

of positive electricity will take place along the conductor from A to B. Such a transfer constitutes an electric current.

Let P Q (fig. 56) be any cross-section of the conductor between the points A and B, dividing it into two parts. Then it is found that during the same interval *the quantity of electricity which in a given time (say one second) flows across the section* P Q *is the same for all positions of* P Q, *provided only that* A *and* B *are on opposite sides of the section.* Thus, if in the figure P′ Q′ be a second section, then at each instant the same quantity of electricity crosses P Q and P′ Q′ per second.

FIG. 56.



The laws of the flow of electricity in conductors resemble in this respect those which regulate the flow of an incompressible fluid, such as water, in a tube; thus, if the conductor were a tube with openings at A and B, and if water were being poured in at A and flowing out at B, the tube being kept quite full, then the quantity of water which at any time flows in one second across any section of the tube, such as P Q, is the same for all positions of P Q, and as in the case of the water the quantity which flows depends on the difference of pressure between A and B, so with the electricity, the quantity which flows depends on the E.M.F., or difference of potential between the points.[1]

DEFINITION OF A CURRENT OF ELECTRICITY.—A current of electricity is the quantity of electricity which passes in one second across any section of the conductor in which it is flowing.

Thus, if in one second the quantity which crosses any section is the unit quantity, the measure of the current is unity.

A unit current is said to flow in a conductor when unit

[1] Maxwell's *Elementary Electricity*, § 64.

quantity of electricity is transferred across any section in one second.

But as yet we have no definition of the unit quantity of electricity. To obtain this, we shall consider certain other properties of an electric current.

A current flowing in a conductor is found to produce a magnetic field in its neighbourhood. Magnetic force is exerted by the current, and the pole of a magnet placed near the conductor will be urged by a force definite in direction and amount. If the conductor be in the form of a long straight wire, a north magnetic pole would tend to move in a circle round the wire, and the direction of its motion would be related to the direction of the current in the same way as the direction of rotation is related to that of translation in a right-handed screw.

If instead of a magnetic pole we consider a compass needle placed near the wire, the needle will tend to set itself at right angles to the wire, and if we imagine a man to be swimming with the current and looking at the needle, then the north end will be turned towards his left hand.

As to the intensity of the force, let us suppose that the length of the wire is l centimetres, and that it is wound into the form of an arc of a circle r centimetres in radius; then when a current of intensity i circulates in the wire, it is found that the magnetic force at the centre is proportional to li/r^2 and acts in a direction at right angles to the plane of the circle, and if i be measured in proper units, we may say that the magnetic force is equal[1] to li/r^2.

If we call this F, we have

$$F = \frac{li}{r^2}.$$

Let the length of the wire be one centimetre, and the radius one centimetre, and let us inquire what must be the strength of the current in order that the force on a unit magnetic pole may be one dyne.[2]

[1] See p. 500. [2] See chap. ii. p 18.

We have then in the equation

$$F = 1, l = 1, r = 1,$$

and it becomes therefore

$$i = 1;$$

that is, the strength of the current is unity, or the current required is the unit current. Thus, in order that the equation

$$F = \frac{l i}{r^2}$$

may be true, it is necessary that the unit current should be that current which circulating in a wire of unit length, bent into the form of an arc of a circle of unit radius, exerts unit force on a unit magnetic pole placed at the centre.

But we have seen already that the unit current is obtained when unit quantity of electricity crosses any section of the conductor. We have thus arrived at the definition of unit quantity of electricity of which we were in search.

This definition is known as the definition of the electro-magnetic unit of quantity.

DEFINITION OF C.G.S. ELECTRO-MAGNETIC UNIT QUANTITY AND UNIT CURRENT.—Consider a wire one centimetre in length bent into an arc of a circle one centimetre in radius. Let such a quantity of electricity flow per second across any section of this wire as would produce on a unit magnetic pole placed at its centre a force of one dyne. This quantity is the electro-magnetic unit of quantity of electricity, and the current produced is the electro-magnetic unit of current.

With this definition understood then, we may say that if a current of strength i traverse a wire of length l bent into an arc of a circle of radius r, the force on a magnetic pole of strength m placed at the centre of the circle will be $m i l / r^2$ dynes in a direction normal to the circle, and the strength of the magnetic field *at the centre* is il/r^2.

The magnetic field will extend throughout the neigh-

bourhood of the wire, and the strength of this field at any point can be calculated. Accordingly, a magnet placed in the neighbourhood of the wire is affected by the current, and disturbed from its normal position of equilibrium.

It is this last action which is made use of in galvanometers. Let the wire of length l be bent into the form of a circle of radius r, then we have

$$l = 2 \pi r,$$

and the strength of the field, at the centre of the circle, is $2 \pi i/r$.

Moreover, we may treat the field as uniform for a distance from the centre of the circle, which is small compared with the radius of the circle. If then we have a magnet of moment M, whose dimensions are small compared with the radius of the circle, and if it be placed at the centre of the circle so that its axis makes an angle θ with the lines of force due to the circle, and therefore an angle of $90° - \theta$ with the plane of the circle, the moment of the force on it which arises from the magnetic action of the current is $2 \pi \mathrm{M} i \sin \theta/r$.

If, at the same time, ϕ be the angle between the axis of the magnet and the plane of the meridian, the moment of the force due to the horizontal component H of the earth's magnetic force is $\mathrm{M H} \sin \phi$; if the small magnet be supported so as to be able to turn round a vertical axis, and be in equilibrium under these forces, we must have the equation

$$\frac{2 \pi \mathrm{M} i \sin \theta}{r} = \mathrm{M H} \sin \phi,$$

or

$$i = \frac{\mathrm{H} r \sin \phi}{2 \pi \sin \theta};$$

if then we know the value of H, and can observe the angles ϕ and θ, and measure the distance r, the above equation gives us the value of i.

Two arrangements occur usually in practice. In the first the plane of the coil is made to coincide with the magnetic meridian; the lines of force due to the coil are then at right angles to those due to the earth, and

$$\theta = 90° - \phi$$

Hence

$$\sin \theta = \cos \phi,$$

and we have

$$i = \frac{\text{H} r \tan \phi}{2 \pi}.$$

The instrument is then called a tangent galvanometer. In the second the coil is turned round a vertical axis until the axis of the magnet is in the position of equilibrium in the same plane as the circle; the lines of force due to the coil are then at right angles to the axis of the magnet, so that the effect of the current is a maximum, and $\theta = 90°$. In these circumstances, therefore, we have, if ψ be the deflection of the magnet,

$$i = \frac{\text{H} r \sin \psi}{2 \pi}.$$

The instrument is in this case called a sine-galvanometer.

We shall consider further on, the practical forms given to these instruments. Our object at present is to get clear ideas as to an electric current, and the means adopted to measure its strength.

The current strength given by the above equation will, using C.G.S. units of length, mass, and time, be given in absolute units. Currents, which in these units are represented even by small numbers, are considerably greater than is convenient for many experiments. For this reason, among others, which will be more apparent further on, it is found advisable to take as the *practical unit of current*, one-tenth of the C.G.S. unit. This practical unit is called an ampère.

DEFINITION OF AN AMPÈRE.—A current of one ampère is one-tenth of the C.G.S. absolute unit of current.

Thus, a current expressed in C.G.S. units may be reduced to ampères by multiplying by 10.

CHAPTER XIX.

EXPERIMENTS ON THE FUNDAMENTAL PROPERTIES OF ELECTRIC CURRENTS—MEASUREMENT OF ELECTRIC CURRENT AND ELECTROMOTIVE FORCE.

71. Absolute Measure of the Current in a Wire.

THE wire in question is bent into the form of a circle, which is placed approximately in the plane of the magnetic meridian. This is done by using a long magnet mounted as a compass-needle and placing the plane of the wire by eye parallel to the length of this magnet. The two ends of the wire are brought as nearly into contact as is possible, and then turned parallel to each other at right angles to the plane of the circle; they are kept separate by means of a small piece of ebonite, or other insulating material. A small magnet is fixed on to the back of a very light mirror, and suspended, by a short single silk fibre, in a small metal case with a glass face in front of the mirror, just as in a Thomson's mirror galvanometer. The case is only just large enough to allow the mirror to swing freely, so that the air enclosed damps the vibrations rapidly. The case is fixed to an upright stand and rests on levelling screws in such a way that the centre of the magnet can be brought into the centre of the circle. A scale parallel to the plane of the circle is fixed some little distance in front of the mirror, the level of the scale being very slightly above that of the mirror. Below the scale is a slit, and behind that a lamp, the light from which shines through the slit on to the

mirror, and is reflected by it, throwing a bright spot of light on to the scale, if the scale and lamp be properly adjusted.

The mirror is usually slightly concave, and by adjusting the distance between the scale and the mirror, a distinct image of the slit can be formed on the scale, and its position accurately determined. In some cases it is convenient to stretch a thin wire vertically across the middle of the slit, and read the position of its image. If an image cannot be obtained by simply varying the distance, through the mirror not being concave, or from some other defect, a convex lens of suitable focal length may be inserted between the slit and the mirror; by adjusting the lens the image required can be obtained. When there is no current passing through the wire the image should coincide with the division of the scale which is vertically above the slit. To determine whether or not the scale is parallel to the mirror, mark two points on the scale near the two ends, and equidistant from the middle point, and measure with a piece of string the distances between each of these two points and a point on the glass face of the mirror-case exactly opposite the centre of the mirror. If these two distances be the same, the scale is rightly adjusted; if they be not, turn the scale, still keeping the image of the slit vertically above the slit, until they become equal. Then it is clear that the scale is at right angles to the line which joins its middle point to the mirror, and that this line is also at right angles to the mirror. The scale, therefore, is parallel to the mirror. If now the ends of the wire be connected with the poles of a Daniell's battery, or with some other apparatus which maintains a difference of potential between them, a current will flow in the wire. The magnet and mirror will be deflected, and the spot of light will move along the scale, coming to rest after a short time in a different position. Note this position, and suppose the distance between it and the original resting-point to be x_1 scale divisions—it will be convenient when possible to use a scale divided into

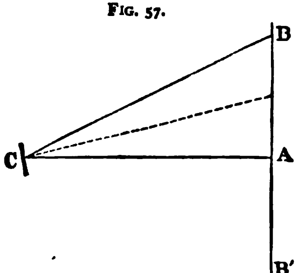
centimetres and millimetres.—Reverse the direction of the current in the circuit, either by using a commutator or by actually disconnecting it from the battery, and connecting up in the opposite way. The spot will be deflected in the opposite direction through, let us suppose, x_2 scale divisions. If the adjustments were perfect, we ought to find that x_1 and x_2 were equal; they will probably differ slightly. Let their mean be x. Then it can be shewn that, if the difference between x_1 and x_2 be not large, say about 5 scale divisions, when the whole deflexion is from 100 to 200 divisions, we may take x as the true value of the deflexion which would have been produced if the scale and mirror had been perfectly adjusted. Let us suppose further that a large number of scale divisions—say 500—occupies l cm. Then the number of centimetres in x scale divisions is $xl/500$. Measure the distance between the centre of the mirror and the scale, and let it be a cm. Measure also the diameter of the circle in centimetres, estimating it by taking the mean of measurements made in five different directions across the centre. Allow for the thickness of the wire, and so obtain the mean diameter of the core of the circle formed by the wire; let it be $2r$ centimetres.

Let BAB′ (fig. 57) be the scale, A the slit, and B the point at which the image is formed; let C be the centre of the mirror; the ray of light has been turned through the angle ACB, and if ϕ be the angle through which the magnet and mirror have moved, then

$$ACB = 2\phi,$$

for the reflected ray moves through twice the angle which the mirror does (see § 48). Moreover, the distances CA and AB have been observed, and we have $AB = xl/500$, $CA = a$.

FIG. 57.



Thus

$$\frac{x\,l}{500} \times \frac{1}{a} = \frac{AB}{CA} = \tan 2\,\phi.$$

From this equation then $2\,\phi$ can be found, using a table of tangents, and hence $\tan\phi$, by a second application of the table.

But the circle was placed in the magnetic meridian, parallel, therefore, to the magnet, and the force due to the current is consequently at right angles to that due to the earth. We have, therefore, from the last section, if i represent the current,

$$i = \text{H}r \tan\phi/2\pi.$$

We have shewn in § 69 how H is to be found, and the values of r and $\tan\phi$ have just been determined; the value of π is, of course, 3·142, and H may be taken as ·180. Thus we can measure i in C.G.S. absolute units. To find i in ampères we have to multiply the result by 10, since the C.G.S. unit of current contains 10 ampères.

The repetition of this experiment with circles of different radii would serve to demonstrate the accuracy of the fundamental law of the action of an electric current on a magnet. The experiment may, by a slight modification, be arranged with the more direct object of verifying the law in the following manner. Set up two coils concentrically, in the magnetic meridian, with a needle at their common centre. Let the one coil consist of a single turn of wire and the other of two turns, and let the radius of the second be double that of the first. Then on sending the same current through either coil the deflexion of the needle will be found to be the same; the best way, however, of demonstrating the equality is to connect the two coils together so that the *same* current passes through both, but in *opposite* directions; the effect on the needle for the two coils respectively being equal and opposite, the needle will remain undeflected. We are indebted to Professor Poynting, of Birmingham, for the

suggestion of this method of verifying the fundamental electro-magnetic law.

It should be noticed that the formula for the deflexion does not contain any factor which depends on the magnetism of the suspended needle; in other words, the deflexion of a galvanometer is independent of the magnetic moment of its needle. This fact may also be experimentally verified by repeating the experiment with different needles and noticing that the deflexion is always the same for the same current.

Experiment.—Determine the strength of the current from the given battery when flowing through the given circle.

Enter results thus:—

Observations for diameter, corrected for thickness of the wire—

32 cm. 32·1 cm. 31·9 cm. 32 cm. 32·1 cm.

Mean value of r, 16·01 cm.

x = 165 divisions of scale.
l = space occupied by 500 divisions = 31·7 cm.
a = 60·7 cm.
tan 2ϕ = ·1723 tan ϕ = ·0855
i = ·03925 C.G.S. unit = ·3925 ampère.

GALVANOMETERS.

The galvanometer already described, as used in the last section, was supposed to consist of a single turn of wire, bent into the form of a circle, with a small magnet hanging at the centre. If, however, we have two turns of wire round the magnet, and the same current circulates through the two, the force on the magnet is doubled, for each circle producing the same effect, the effect of the two is double that of one; and if the wire have n turns, the force will be n times that due to a wire with one turn. Thus the force which is produced by a current of strength

i, at the centre of a coil of radius r, having n turns of wire, is $2n\pi i/r$.

But we cannot have n circles each of the same radius, having the same centre; either the radii of the different circles are different, or they have different centres, or both these variations from the theoretical form may occur. In galvanometers ordinarily in use, a groove whose section is usually rectangular is cut on the edge of a disc of wood or brass, and the wire wound in the groove.

The wire is covered with silk or other insulating material, and the breadth of the groove parallel to the axis of the disc is such that an exact number of whole turns of the wire lie evenly side by side in it.

The centre of the magnet is placed in the axis of the disc symmetrically with reference to the planes which bound the groove. Several layers of wire are wound on, one above the other, in the groove. We shall call the thickness of a coil, measured from the bottom of the groove outwards along a radius, its depth.

Let us suppose that there are n turns in the galvanometer coil. The mean radius of the coil is one n^{th} of the radius of a circle, whose circumference is the sum of the circumferences of all the actual circles formed by the wire; and if the circles are evenly distributed, so that there are the same number of turns in each layer, we can find the mean radius by taking the mean between the radius of the groove in which the wire is wound and the external radius of the last layer. Let this mean radius be r; and suppose, moreover, that the dimensions of the groove are so small that we can neglect the squares of the ratios of the depth or breadth of the groove to the mean radius r, then it can be shewn [1] that the magnetic force, due to a current i in the actual coil, is n times that due to the same current in a single circular wire of radius r, so that it is equal to $2n\pi i/r$.

[1] Maxwell, *Electricity and Magnetism*, vol. ii. § 711.

And if the magnet be also small compared with r, and the plane of the coils coincide with the meridian, the relation between the current i and the deflection ϕ is given by

$$i = \text{H}\, r \tan\phi \,/\, (2\, n\, \pi).$$

Unless, however, the breadth and depth of the coil be small compared with its radius, there is no such simple connection as the above between the dimensions of the coil and the strength of the magnetic field produced at its centre. The strength of field can be calculated from the dimensions, but the calculation is complicated, and the measurements on which it depends are difficult to make with accuracy.

DEFINITION OF GALVANOMETER CONSTANT. — The strength at the centre of a coil of the magnetic field produced by a unit current flowing in it, is called the galvanometer constant of the coil.

Hence, if a current i be flowing in a coil of which the galvanometer constant is G, the strength of the field at the centre of the coil is G i, and the lines of force are at right angles to the coil.

Let us suppose that a coil, of which the galvanometer constant is G, is placed in the magnetic meridian, with a magnet at its centre, and that the dimensions of the magnet are so small that, throughout the space it occupies, we may treat the magnetic field as uniform ; then, if the magnet be deflected from the magnetic meridian, through an angle ϕ by a current i, the moment of the force on it due to the coil is G i M $\cos\phi$, M being the magnetic moment of the magnet, while the moment of the force, due to the earth, is H M $\sin\phi$; and since these must be equal, the magnet being in equilibrium, we have

$$i = \text{H} \tan\phi / \text{G}.$$

In using a tangent galvanometer it is not necessary that the earth's directing force alone should be that which retains the magnet in its position of equilibrium when no

current passes round the coil. All that is necessary is that the field of force in which the magnet hangs should be uniform, and that the lines of force should be parallel to the coils. This may be approximately realised by a suitable distribution of permanent magnets.

If the coil of wire can be turned round a vertical axis through its centre, parallel to the plane of the circles, the instrument can be used as a sine galvanometer. For this purpose place the coils so that the axis of the magnet lies in their plane before the current is allowed to pass. When the current is flowing, turn the coils in the same direction as the magnet has been turned until the axis of the magnet again comes into the plane of the coils, and observe the angle ψ through which they have been turned. Then we can shew, as in chap. xviii., that

$$i = \text{H} \sin \psi / \text{G}.$$

To obtain these formulæ, we have supposed that the dimensions of the magnet are small compared with those of the coil. If this be not the case, the moment of the force produced by the magnetic action of the coil when used as a tangent galvanometer is not $\text{MG} \cos \phi$, as above, but involves other terms depending on the dimensions of, and distribution of magnetism in, the magnet.

In order to measure the deflexions, two methods are commonly in use. In the first arrangement there is attached to the magnet, which is very small, a long pointer of glass, aluminium, or some other light material. This pointer is rigidly connected with the magnet, either parallel to or at right angles to its axis, and the two, the magnet and pointer, turn on a sharp-pointed pivot, being supported by it at their centre, or are suspended by a fine fibre free from torsion. A circle, with its rim divided to degrees, or in good instruments to fractions of a degree, is fixed in a horizontal plane so that the axis of rotation of the magnet passes through its centre, and the position of the

magnet is determined by reading the division of this circle with which the end of the pointer coincides. In some cases the end of the pointer moves just above the scale, in others the pointer is in the same plane as the scale, the central portion of the disc on which the graduations are marked being cut away to leave space for it, and the graduations carried to the extreme inner edge of the disc. With the first arrangement it is best to have a piece of flat mirror with its plane parallel to the scale, beneath the pointer, and, when reading, to place the eye so that the pointer covers its own image formed by reflexion in the mirror. The circle is usually graduated, so that when the pointer reads zero, the axis of the magnet is parallel to the plane of the coils if no current is flowing.

In order to eliminate the effects of any small error in the setting, we must proceed in the following manner :— Set the galvanometer so that the pointer reads zero, pass the current through it, and let θ be the deflexion observed. Reverse the direction of the current so that the needle may be deflected in the other direction ; let the deflexion be θ' If the adjustments were perfect—the current remaining the same—we should have θ and θ' equal; in any case, the mean, $\frac{1}{2}(\theta+\theta')$, will give a value for the deflection corrected for the error of setting.

To obtain a correct result, however, the position of both ends of the pointer on the scale must be read. Unless the pointer is in all positions a diameter of the circle, that is, unless the axis of rotation exactly coincides with the axis of the circle, the values of the deflexions obtained from the readings at the two ends will differ. If, however, we read the deflexions θ, θ_1, say, of the two ends respectively, the mean $\frac{1}{2}(\theta+\theta_1)$, will give a value of the deflexion corrected for errors of centering.[1] Thus, to take a reading with a galvanometer of this kind, we have to observe four values of the deflexions, viz. two, right and left of the zero respectively,

[1] See Godfray's *Astronomy*, § 93.

for each end of the needle. This method of reading should be adopted whether the instrument be used as a tangent or a sine galvanometer.

The second method of measuring the deflexion has been explained at full length in the account of the last experiment (p. 497). A mirror is attached to the magnet, and the motions of the magnet observed by the reflexion by it of a spot of light on to the scale. The following modification of this method is sometimes useful.[1] A scale is fixed facing the mirror, (which should in this case be plane) and parallel to it. A virtual image of this scale is formed by reflexion in the mirror, and this image is viewed by a telescope which is pointed towards the mirror from above or below the scale. The telescope has cross-wires, and the measurements are made by observing the division of the scale, which appears to coincide with the vertical cross-wire, first without, and then with a current flowing in the coil. For details of the method of observation see § 23.

In the best tangent galvanometers[2] there are two coils, of the same size and containing the same number of turns, placed with their planes parallel and their centres on the same axis. The distance between the centres of the coils is equal to the radius of either, and the magnet is placed with its centre on the axis midway between the two coils. It has been shewn[3] that with this arrangement the field of force near the point at which the magnet hangs is more nearly uniform than at the centre of a single coil. It has also been proved that in this case, if G be the galvanometer constant, n the number of turns in the two coils, r the mean radius, and ξ the depth of the groove filled by the wire, then

$$G = \frac{16\pi n}{5\sqrt{5}}\frac{1}{r}\left(1 - \frac{1}{60}\frac{\xi^2}{r^2}\right).$$

[1] See § 23, p. 191.

[2] Helmholtz's arrangement, Maxwell, *Electricity and Magnetism*, ii. § 715.

[3] Maxwell, *Electricity and Magnetism*, vol. ii. § 713.

Various other forms of galvanometers have been devised for special purposes. Among them we may refer to those which are adapted to the measurement of the large currents required for the electric light. An account of Lord Kelvin's galvanometers arranged for this purpose will be found in Professor Gray's book on 'Absolute Measurements in Electricity and Magnetism;' while a paper by Professor Ayrton and Dr. Simpson, 'Phil. Mag.,' July 1890, contains valuable information about other instruments.

On the Reduction Factor of a Galvanometer.

The deflexion produced in a galvanometer needle by a given current depends on the ratio H/G, H being the strength of the field in which the needle hangs when undisturbed, and G the strength of the field due to a unit current in the coil. This ratio is known as the *reduction factor* of the galvanometer. Let us denote it by k, then

$$k = \mathrm{H}/\mathrm{G};$$

and if the instrument be used as a tangent galvanometer we have

$$i = k \tan \phi;$$

if it be used as a sine galvanometer

$$i = k \sin \psi,$$

ϕ and ψ being the deflexions produced in either case by a current i.

It must be remembered that the reduction factor depends on the strength of the magnetic field in which the magnet hangs as well as on the galvanometer constant. There is generally attached to a reflecting galvanometer a controlling magnet capable of adjustment. The value of k will accordingly depend on the position of this control magnet, which in most instruments is a bar, arranged to slide up and down a vertical axis above the centre of the coils, as well as to rotate about that axis. The sensitiveness

of the instrument can be varied by varying the position of this magnet.

On the Sensitiveness of a Galvanometer.

The sensitiveness of a galvanometer will depend on the couple which tends to bring the needle back to its position of equilibrium, and is increased by making that couple small. The couple is proportional to the magnetic moment of the needle and to the strength of the field in which the magnet hangs. Two methods are employed to diminish its value.

If the first method be adopted two needles are employed. They are mounted, parallel to each other, a short distance apart, so that they can rotate together as a rigid system about their common axis. Their north poles are in opposite directions, and their magnetic moments are made to be as nearly equal as possible. If the magnetic moments of the two be exactly the same, and the magnetic axes in exactly opposite directions, such a combination when placed in a uniform magnetic field will have no tendency to take up a definite position. In practice this condition of absolute equality is hardly ever realised, and the combination, if free to move, will be urged to a position of equilibrium by a force which will be very small compared with that which would compel either magnet separately to point north and south. It will take, therefore, a smaller force to disturb the combination from that position than would be required for either magnet singly. Such a combination is said to be *astatic.*

When used for a galvanometer the coils are made to surround one needle only; the other is placed outside them, either above or below as the case may be.

The magnetic action of the current affects mainly the enclosed magnet; the force on this is the same as if the other magnet were not present, and hence, since the controlling force is much less, the deflexion produced by a given current is much greater. This deflexion is still further

increased by the slight magnetic action between the current and the second magnet.

In some cases this second magnet is also surrounded by a coil, in which the current is made to flow in a direction opposite to that in the first coil, and the deflexion is thereby still further augmented.

In the second method the strength of the field in which the needle hangs is reduced by the help of other magnets; if this method be adopted, the advantages of an astatic combination may be partly realised with an ordinary galvanometer by the use of control magnets placed so as to produce a field of force opposite and nearly equal to that of the earth at the point where the galvanometer needle hangs. The magnetic force tending to bring the needle back to its equilibrium position can thus be made as small as we please—neglecting for the moment the effect of the torsion of the fibre which carries the mirror—and the deflexion produced by a given current will be correspondingly increased.

The increase in sensitiveness is most easily determined, as in § 69, by observations of the time of swing, for if H represent the strength of the field in which the magnet hangs, we have seen (§ 69) that $H = 4\pi^2 K/M T^2$, M being the magnetic moment, K the moment of inertia, and T the time of a complete period. But, being small, the deflexion produced by a given current, on which, of course, the sensitiveness depends, is inversely proportional to H; that is, it is directly proportional to the square of T.

The method of securing sensitiveness thus by the use of a control magnet is open to the objection that the small variations in the direction and intensity of the earth's magnetic force, which are continually occurring, become very appreciable when compared with the whole strength of the field in which the magnet hangs. The sensitiveness, and, at the same time, the equilibrium position of the magnet, are, therefore, continually changing.

On the Adjustment of a Reflecting Galvanometer.

In adjusting a reflecting galvanometer, we have first to place it so that the magnet and mirror may swing quite freely. This can be attained by the adjustment of the levelling screws on which the instrument rests. There is generally a small aperture left in the centre of the coils opposite to that through which the light is admitted to the mirror. This is closed by a short cylinder of brass or copper which can be withdrawn, and by looking in from behind, it is easy to see if the mirror hangs in the centre of the coils as it should do.

The lamp and scale are now placed in front of the mirror, the plane of the scale being approximately parallel to the coils, and the slit through which the light comes rather below the level of the mirror.

The magnet and mirror are adjusted, by the aid of the control magnet, until the light is reflected towards the scale. The position of the reflected beam can easily be found by holding a sheet of paper close to the mirror so as to receive it, moving the paper about without intercepting the incident beam. By moving the control magnet, and raising or lowering the scale as may be required, the spot may be made to fall on the scale.

The distance between the galvanometer and scale must now be varied until the image formed on the scale is as clear and distinct as possible; and, finally, the control magnet must be adjusted to bring the spot to the central part of the scale, and to give the required degree of sensitiveness.

As we have seen, the sensitiveness will largely depend on the position of the control magnet. Its magnetic moment should be such that when it is at the top of the bar which supports it, as far, that is, as is possible from the needle, the field which it alone would produce at the needle should be rather weaker than that due to the earth If this

be the case, and the magnet be so directed that its field is opposite to that of the earth, the sensitiveness is increased at first by bringing the control magnet down nearer to the coils, becoming infinite for the position in which the effect of the control magnet just balances that of the earth, and then as the control magnet is still further lowered the sensitiveness is gradually decreased.

The deflexion observed when a reflecting galvanometer is being used is in most cases small, so that the value of ϕ measured in circular measure will be a small fraction; and if this fraction be so small that we may neglect ϕ^3, we may put $\sin\phi = \phi = \tan\phi$ (see p. 45) and we get $i = k\phi$.

With a sensitive galvanometer in which the coils are close to the magnet the ratio of the length of the magnet to the diameter of the coil is considerable, and the galvanometer constant is a function of the deflexion; so that k is not constant for all deflexions in such an instrument, but depends on the angle ϕ. If, however, the deflexions employed be small we may without serious error use the formula $i = k\phi$, and regard k as a constant.

---

72. Determination of the Reduction Factor of a Galvanometer.

If the dimensions and number of turns of the galvanometer and the value of H can be measured accurately the reduction factor can be calculated. We shall suppose, however, that these data cannot be directly measured, and turn to another property of an electric current for a means of determining the reduction factor.

Let i be a current which produces a deflexion ϕ in a galvanometer of which the reduction factor is k; then if it be used as a tangent instrument we have

$$i = k\tan\phi,$$

and therefore,

$$k = i/\tan\phi.$$

If we can find by some other means the value of i, we can determine k by observing the deflexion ϕ which it produces.

Now it has been found that when an electric current is allowed to pass through certain chemical compounds which are known as electrolytes, the passage of the current is accompanied by chemical decomposition. The process is called *Electrolysis*; the substance is resolved into two components called *Ions*; these collect at the points at which the current enters and leaves the electrolytes respectively.

The conductors by which the current enters or leaves the electrolyte are known as the *Electrodes*[1]; that at which the current enters the electrolyte is called the *Anode*, and the component which appears there is the *Anion*. The conductor by which the current leaves the electrolyte is the *Kathode*, and the ion which is found there is the *Kathion*. An apparatus arranged for collecting and measuring the products of electrolytic decomposition is called a *Voltameter*.

Moreover, it has been shewn by Faraday ('Exp. Res.' ser. vii.) that the quantities of the ions deposited either at the kathode or the anode are proportional to the quantity of electricity which has passed. If this quantity be varied the quantity of the ions deposited varies in the same ratio. This is known as Faraday's law of electrolysis.

DEFINITION OF ELECTRO-CHEMICAL EQUIVALENT.—The electro-chemical equivalent of a substance is the number of grammes of the substance deposited by the passage of a unit quantity of electricity through an electrolyte in which the substance occurs as an ion. Thus, if in a time t a current i deposits m grammes of a substance whose electro-chemical equivalent is γ, it follows from the above definition, in conjunction with Faraday's law, that

$$m = \gamma i t,$$

and hence

$$i = m/\gamma t.$$

[1] The term 'electrode' was originally applied by Faraday in the sense in which it is here used. Its application has now been extended, and it is employed in reference to any conductor by which electricity enters or leaves an electrical apparatus of any sort.

If, then, we observe the amount of a substance, of known electro-chemical equivalent, deposited in time t, we can find the current, provided it has remained constant throughout the time t. If a current be allowed to pass between two plates of copper immersed in a solution of sulphate of copper, the sulphate is electrolysed and copper deposited on the kathode. The acid set free by the electrolysis appears at the anode, and combines with the copper. The quantity of copper deposited on the kathode in one second by a unit current has been found to be ·00328 gramme. This is the electro-chemical equivalent of copper. The loss of weight of the anode is for various reasons found to be somewhat in excess of this.

We proceed to describe how to use this experimental result to determine the reduction factor of a galvanometer.

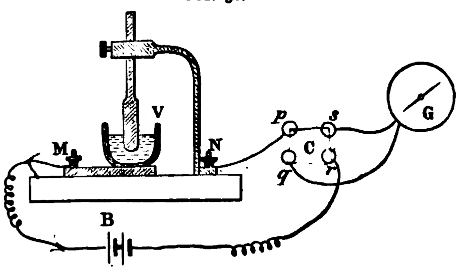
Two copper plates[1] are suspended in a beaker containing a solution of copper sulphate, by wires passing through a piece of dry wood or other insulating material which forms a covering to the beaker. The plates should be well cleaned before immersion by washing them with nitric acid, and then rinsing them with water, or by rubbing them with emery cloth, and then rinsing them with water. They must then be thoroughly dried. One of the plates must be carefully weighed to a milligramme. On being put into the solution this plate is connected to the negative pole—the zinc—of a constant battery, preferably a Daniell's cell, by means of copper wire; the other plate is connected with one electrode of the galvanometer. The positive pole of the battery is connected through a key with the other pole of the galvanometer, so that on making contact with the key the current flows from the copper of the battery round the galvanometer, through the electrolytic cell, depositing copper on the weighed plate, and finally passes to the zinc or negative pole of the battery. Since the galvanometer

[1] For details as to precautions see Gray, *Absolute Measurements in Electricity and Magnetism*, p. 169.

reading is most accurate when the deflexion is 45° (see p. 47), the battery should if possible be chosen so as to give about that deflexion. For this purpose a preliminary experiment may be necessary. It is also better if possible to attach the copper of the battery and the anode of the cell to two of the binding screws of a commutator, the other two being in connection with the galvanometer. By this means the current can easily be reversed in the galvanometer without altering the direction in which it flows in the cell, and thus readings of the deflexion on either side of the zero can be taken.

The connections are shewn in fig. 58. B is the battery, the current leaves the voltameter[1] V by the screw M,

FIG. 58.



entering it at the binding screw N from the commutator C. This consists of four mercury cups, *p*, *q*, *r*, *s*, with two ∩-shaped pieces of copper as connectors. If *p* and *s*, *q* and *r* respectively be joined, the current circulates in one direction round the galvanometer; by joining *p* and *q*, *r* and *s*, the direction in the galvanometer is reversed. The cup *r* is connected with the positive pole of the battery B.

Now make contact, and allow the current to flow through the circuit for fifteen minutes, observing the value of the deflexion at the end of each minute. If there be a commutator in the circuit as in the figure, adjust it so that

[1] See next page.

the current flows in opposite directions during the two halves of the interval. Let ϕ be the mean of the deflexions observed. If the battery has been quite constant the deflexions observed will not have varied from minute to minute; in any case the deflexion must not have changed much during the interval. If any great variation shews itself, owing to changes in the battery or voltameter, the experiment must be commenced afresh.

At the end of the fifteen minutes the weighed plate must be taken out of the solution, washed carefully, first under the tap, and then by pouring distilled water on it, and finally dried by being held in a current of hot dry air. It is then weighed carefully as before. It will be found to have increased in weight; let the increase be m grammes. Then the increase per second is $m/(15 \times 60)$, and since the electro-chemical equivalent of copper is ·00329, the average value of the current in C.G.S. units (electro-magnetic measure) is

$$m/(60 \times 15 \times \cdot 00329).$$

But if $\phi_1\ \phi_2\ \ldots\ \phi_{15}$ be the readings of the deflexion, this average value of the current is also

$$\tfrac{1}{15} k(\tan\phi_1 + \tan\phi_2 + \ \ldots\ \tan\phi_{15}).$$

And if $\phi_1\ \phi_2$, &c., are not greatly different, this expression is very nearly equal to $k \tan \phi$, where ϕ is the average value of $\phi_1, \ldots \phi_{15}$. We thus find

$$k = \frac{m}{60 \times 15 \times \cdot 00329 \times \tan\phi}$$

If the factor is so small that the copper deposited in fifteen minutes—m grammes—is too little to be determined accurately, the experiment must be continued in the same way for a longer period. It must be remembered that the mass m is to be expressed in grammes.

Instead of using a glass beaker to hold the sulphate, it is sometimes convenient to make the containing vessel

itself one of the electrodes. Thus a copper crucible may be used as cathode, like the platinum one in Poggendorff's voltameter; in this the sulphate is placed, and the anode may be a rod of copper which hangs down into it. This form is shewn in the figure.

We have already said that if the dimensions of the galvanometer coil, and the number of turns of the wire of which it is composed can be determined, the value of k can be calculated, provided that the value of H be known; or, on the other hand, H can be found from a knowledge of the dimensions, and of the value of k determined by experiment. For if G be the galvanometer constant, r the mean radius, and n the number of turns, we have $G = 2\pi n/r$. Also $k = H/G$. Whence $H = Gk = 2\pi n k/r$.

The current, which is determined by the observations given above, is measured in C.G.S. units. The value of k gives the current which deflects the needle 45°, measured also in the same units. To obtain the value in ampères we must multiply the result by 10, since the C.G.S. unit of current contains 10 ampères.

[NOTE.—We have supposed a copper voltameter to be used. A silver voltameter is more accurate. Directions for its use are given in § X, p. 579.]

Experiment.—Determine the reduction factor of the given galvanometer by electrolysis, comparing your result with that given by calculation.

Enter the results thus—

Battery	3 Daniells
Gain of kathode	·2814 gm.
Deflexion, greatest	46°
" least . . .	45° 30′
" mean of 15 . . .	45° 50′
Time during which experiment lasted	15 minutes
Value of k	·0923 C.G.S. unit
Radius of wire	16·2 cm.

Number of turns	5
Value of H	·180
Value of k calculated . . .	·0928

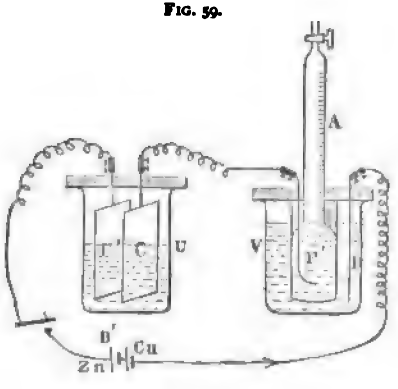
73. Faraday's Law. Comparison of Electro-Chemical Equivalents.

The electro-chemical equivalent of an element or radicle *in absolute measure* is the number of units of mass of the element or radicle separated from one of its compounds by the passage of an absolute unit of electricity.

The *ratio* of the electro-chemical equivalents of two elements may thus be found by determining the mass of each element deposited by the *same quantity of electricity.* In order to ensure that the same quantity of electricity passes through two solutions we have only to include both in one circuit with a battery. This plan is to be adopted in the following experiment to compare the electro-chemical equivalents of hydrogen and copper.

Arrange in circuit with a battery (fig. 59) (the number of cells of which must be estimated from the resistance[1] to be overcome, and must be adjusted so as to give a supply of bubbles in the water voltameter that will form a measurable amount of gas in one hour) (1) a beaker U of copper sulphate, in which dip two plates of copper C, C′, soldered to copper wires passing through a piece of wood which acts as a support on top of the beaker, and (2) a water voltameter[2] V.

FIG. 59.



[1] See p. 527.

[2] An arrangement which is easily put together is shewn in the

Mount over the platinum plate P′, by which the current is to *leave* the voltameter, a burette to be used for measuring the amount of hydrogen generated during the experiment, taking care that all the hydrogen must pass into the burette. Place a key in the circuit, so that the battery may be thrown in or out of circuit at will.

The zinc of the battery must be in connection with the plate C′ on which copper is to be deposited. The copper or platinum is in connection with the platinum plate P, on which oxygen will be deposited.

About three Grove's cells will probably be required for a supply of gas that can be measured in a convenient time; and as this will correspond to a comparatively large current, the plates of copper should be large, say 6 in. × 3 in., or the deposit of copper will be flocculent and fall off the plate.

When the battery has been properly adjusted to give a current of the right magnitude, the apparatus will be in a condition for commencing the measurements. Accordingly, take out, dry, and carefully weigh the copper plate on which the metal will be deposited during the experiment. This of course is the plate which is connected with the negative pole of the battery. Let its weight be W.

After weighing the copper plate no current must be sent through the voltameter containing it, except that one which is to give the required measurement.

Read the position of the water in the burette—the height in centimetres of the water in the burette above the level of the water in the voltameter. Let this be h. Read the barometer; let the height be H. Read also a thermometer in the voltameter; let the temperature be $t°$ C., Make the battery circuit by closing the key and allow the

figure. The plate P′ is inside a porous pot, such as is used in a Leclanché battery, and the open end of the burette is sealed into the top of the pot by means of pitch or some kind of insulating cement. The hydrogen is formed inside the pot and rises into the burette. A graduated Hofmann voltameter is of course better, but the above can be made in any laboratory with materials which are always at hand.

current to pass until about twenty centimetres of the burette have been filled by the rising gas. Shut off the current, and dry and weigh the same plate of copper again; let the weight be w′.

Then the amount of copper deposited by the current is w′ − w.

Read again the position of the water in the burette. From the difference between this and the previous reading we may obtain the volume of the gas generated. Let the difference in volume actually observed be v cubic centimetres, and let the height of the water in the burette above that in the voltameter at the end of the experiment be h'.

Before using v to find the mass of hydrogen deposited we have to apply several corrections.

There was some gas above the water in the burette before the experiment began. The pressure of the gas above the water has been increased by the experiment, and this gas has in consequence decreased in volume. We require to find what the decrease is.

Let the original volume of the gas be v. The graduations on the burette are generally not carried to the end, and to find v we require to know the volume between the last graduation and the tap of the burette. For this purpose a second burette is needed. This is filled with water to a known height. The burette to be used in the experiment is taken and inverted, being empty. Water is run into it from the second burette until it is filled up to the first graduation; the quantity of water so run in is found by observing how far the level in the second burette has fallen. Or, if it be more convenient, the method may be reversed; the second burette being partly filled as before, the first burette is also filled up to some known graduation, and all the water which it contains is run out into the second; the rise in level in this gives the quantity of water which has run out, and from this we can find the volume required

between the bottom of the burette and the first graduation; knowing this we find the volume v easily.

Now this gas of volume v was at the commencement under a pressure equal to the difference between the atmospheric pressure and the pressure due to a column of water of height h; if δ be the specific gravity of mercury, the pressure due to a column of water of height h is the same as that due to a column of mercury of height h/δ; so that H being the height of the barometer, the pressure of the gas will be measured by the weight of a column of mercury of height $\mathrm{H}-h/\delta$, while at the end of the experiment the pressure is that due to a column $\mathrm{H}-h'/\delta$. Therefore the volume which the gas now occupies is

$$v\frac{\mathrm{H}-\frac{h}{\delta}}{\mathrm{H}-\frac{h'}{\delta}},$$

so that the decrease required is

$$v\left(1-\frac{\mathrm{H}\delta-h}{\mathrm{H}\delta-h'}\right)=v\frac{h-h'}{\mathrm{H}\delta-h'};$$

and h' being small compared with $\mathrm{H}\delta$, we may write this:—

$$v\frac{h-h'}{\mathrm{H}\delta}.$$

This must be added in the observed volume V to find the volume occupied by the gas electrolysed, at a pressure due to a column of mercury of height $\mathrm{H}-h'/\delta$, giving us thus as the volume,

$$\mathrm{V}+v\frac{h-h'}{\mathrm{H}\delta}.$$

It is sometimes more convenient to avoid the necessity for this correction by filling the burette with water before beginning, so that v, the space, initially filled with gas is

zero. If this plan be adopted we shall still require to know the volume between the end of the burette and the graduations, and this must be obtained as described above.

Correction for aqueous vapour.—The solution of sulphuric acid used in the voltameter is exceedingly dilute, and it may be supposed without error that the hydrogen gas comes off saturated with aqueous vapour; the pressure of this vapour can be found from the table (34), for the temperature of the observation, $t°$ C. Let it be e. Then if e be expressed as due to a column of mercury of e centimetres in height, the pressure of the hydrogen will be measured by

$$\mathrm{H} - h'/\delta - e,$$

and its volume at this pressure and temperature t is

$$\mathrm{V} + v\frac{h-h'}{\mathrm{H}\delta}.$$

Thus its volume at a pressure due to 76 centimetres and temperature 0° C. is

$$\left\{\mathrm{V} + v\frac{(h-h')}{\mathrm{H}\delta}\right\} \times \left\{\frac{\mathrm{H} - h'/\delta - e}{76}\right\} \times \left\{\frac{273}{273+t}\right\} \text{ c.c.}$$

Let this be V'. The weight required is $\mathrm{V}' \times {\cdot}0000896$ gm., ·0000896 being the density of hydrogen.

But according to Faraday's fundamental law of electrolysis, the weights of two elements deposited by the same current in the same time are proportional to their chemical equivalents. We must, therefore, have

$$\frac{\mathrm{W}' - \mathrm{W}}{\mathrm{V}' \times {\cdot}0000896} = \text{chemical equivalent of copper.}$$

The value of the equivalent, as deduced from chemical experiments, is 31·75.

Experiment.—Determine by the use of voltameters the chemical equivalent of copper.

Enter results thus:

$W = 61{\cdot}0760$ gms.	$h = 20$ cm.
$W' = 61{\cdot}1246$ gms.	$h' = 5$ cm.
$V = 18{\cdot}5$ c.c.	$e = 1{\cdot}9$ cm.
$v = 1{\cdot}25$ c.c.	$t = 15°$ C.
$H = 75{\cdot}95$ cm.	$V' = 17{\cdot}0$ c.c.

Chemical equivalent $= 31{\cdot}13$

74. Joule's Law—Measurement of Electromotive Force.

We have seen that work is done when a quantity of electricity passes from a point at one potential to a second point at a different one. If Q be the quantity of electricity which passes thus, and E the difference of potential, or electromotive force, maintained constant between the points while Q passes, then the work done is $Q \times E$. If the electricity pass as a steady current of strength C, for a time t seconds, then, since the strength of a current is measured by the quantity which flows in a unit of time, we have $Q = Ct$, and if W be the work done,

$$W = ECt.$$

If this current flow in a wire the wire becomes heated, and the amount of heat produced measures the work done, for the work which the electricity does in passing from the point at high to that at low potential is transformed into heat. If H be the amount of heat produced and J the mechanical equivalent of heat, that is, the number of units of work which are equivalent to one unit of heat, then the work required to produce H units of heat is JH. Hence we have

$$JH = W = ECt;$$

whence

$$E = JH/(Ct).$$

Now J is a known constant, H can be measured by immersing the wire in a calorimeter (see § 39) and noting the rise of temperature of a weighed quantity of

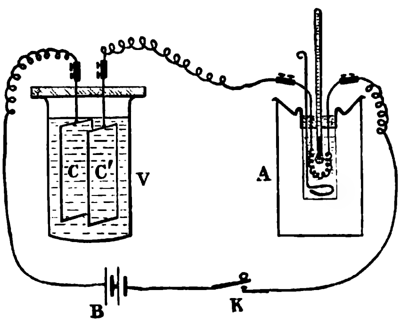
water which is contained therein; if a copper-voltameter be included in the circuit C *t* is obtained, knowing the electro-chemical equivalent of copper, by determining the increase in weight of the cathode. We can thus find E, the difference of potential between the two points at which the current respectively enters and leaves the wire in the calorimeter. For the calorimeter we use a small vessel of thin sheet copper polished on the outside and suspended in another copper vessel, as in § 39. The water equivalent of this must be determined, as is explained in that section, either experimentally or by calculation from the weight of the vessel and the known specific heat of copper, which for this purpose may be taken as ·1. A small stirrer made of thin copper wire coiled into a spiral may be included in the estimate with the calorimeter determination. The outer vessel of the calorimeter is closed by a copper lid with a hole in the middle, through which a cork passes. The end of the stirrer passes through a hole in this cork, and through two other holes pass two stout copper wires, to the ends of which the wire to be experimented on is soldered. The thermometer is inserted through a fourth hole. The bulb of the thermometer should be small, and the stem should be divided to read to tenths of a degree. The wire should be of German-silver covered with silk and coiled into a spiral. Its length and thickness will depend on the nature of the source of electromotive force used. If we take a battery of three Grove's cells of the usual pint size, it will be found that the electrical resistance of the wire (see chap. xx.) should be about 4 ohms. The two ends are soldered on to the copper electrodes and the wire completely immersed in the water of the calorimeter. It must be carefully remembered that the quantity which we are to determine is the difference of potential between the two points at which the wire cuts the surface of the water.

Some of the heat developed in the wire will of course remain in it, and in our calculations we ought strictly to

allow for this. It will be found, however, that in most instances the correction is extremely small, and may, for the purposes of the present experiment, be safely neglected. We may assume that the whole of the heat produced goes into the water and the calorimeter. But the experiment lasts for some time, and meanwhile the temperature of the calorimeter is raised above that of the surrounding space, so that heat is lost by radiation. We shall shew how to take the observations so as to compensate for this.

The apparatus is arranged as follows (fig. 60):—The cathode C of the voltameter V is carefully weighed and connected to the negative pole of the battery B, the anode C′ being connected by means of a piece of copper wire with one of the ends of the wire in the calorimeter A; the other end of this wire is joined through a key K to the positive pole of the battery.

FIG. 60



The plates of the voltameter must be so large and so close together that its resistance may be very small indeed compared with that of the wire in the calorimeter: otherwise the rise of temperature in the calorimeter may be hardly large enough for convenient measurement without using a considerable number of battery cells.

To perform the experiment, note the temperature of the water and allow the current to flow, keeping the water well stirred; the temperature will gradually rise. After two minutes stop the current; the temperature may still rise slightly, but if the stirring has been kept up, the rise, after the current has ceased flowing, will be very small. Let the

total rise observed be τ_1 degrees. Keep the circuit broken for two minutes; the temperature will probably fall. Let the fall be τ_2 degrees. This fall during the second two minutes is due to loss of heat by radiation; and since during the first two minutes the temperature did not differ greatly from that during the second two, we may suppose that the loss during the first two minutes was approximately the same as that during the second two; so that, but for this loss, the rise of temperature during those first two minutes would have been $\tau_1 + \tau_2$ degrees.

We thus find the total rise of temperature produced in the mass of water in two minutes by the given current by adding together the rise of temperature during the first two minutes and the fall during the second two minutes. Take six observations of this kind, and let the total rise of temperature calculated in the manner above described be τ degrees; let the mass of water, allowing for the water equivalent of the calorimeter and stirrer, be m grammes, then the quantity of heat given out by the current in twelve minutes is $m\tau$ units.

Let M grammes of copper be deposited by the same current; then since the passage of a unit of electricity causes the deposition of ·00328 gramme of copper, the total quantity of electricity which has been transferred is M/·00328 units, and this is equal to Ct in the equation for E. Hence

$$E = J m \tau \times \cdot 00328 / M.$$

Now the value of J in C.G.S. units is 42×10^6, so that we have

$$E = 420 \times 328 \times m \times \tau / M.$$

The value of E thus obtained will be given in C.G.S. units; the practical unit of E.M.F. is called a volt, and one volt contains 10^8 C.G.S. units; hence the value of E in volts is

$$420 \times 328 \times m \times \tau / (M \times 10^8).$$

We have used the results of the experiment to find E. If, however, E can be found by other means—and we shall

see shortly how this may be done—the original equation, $JH = ECt$, may be used to find J or C. It was first employed by Joule for the former of the two purposes, i.e. to calculate the mechanical equivalent of heat, and the law expressed by the equation is known as Joule's law.

Experiment.—Determine the difference of potential between the two ends of the given wire through which a current is flowing.

Enter results thus:—

Mass of water	24·2 gms.
Water equivalent of the calorimeter .	4·2 gms.
m	28·4 gms.
M	·222 gm.

Total rise of temperature for each two minutes:—

4°	4°·4	4°·4	4°·2	4°	3°·8

τ . . . . . 24°·8

$$E = 4{\cdot}37 \times 10^8 = 4{\cdot}37 \text{ volts.}$$

CHAPTER XX.

OHM'S LAW—COMPARISON OF ELECTRICAL RESISTANCES AND ELECTROMOTIVE FORCES.

WE have seen that if two points on a conductor be at different potentials, a current of electricity flows through the conductor. As yet we have said nothing about the relation between the difference of potential and the current produced. This is expressed by Ohm's law, which states that the current flowing between any two points of a conductor is directly proportional to the difference of potential between those points so long as the conductor joining them remains the same and in the same physical state. Thus, if C be the current, and E the electromotive force, C is proportional to E, and we may write

$$C = \frac{E}{R}$$

where R is a quantity which is known as the resistance of the conductor. It depends solely on the shape and temperature of the conductor, and the nature of the material of which it is composed, being constant so long as these remain unaltered.

DEFINITION OF ELECTRICAL RESISTANCE.—It is found by experiment that the ratio of the E.M.F. between two points to the current it produces, depends only on the conductor which connects the two points, and is called the resistance of the conductor.

The reciprocal of the resistance—that is, the ratio of the current to the electromotive force—is called the conductivity of the conductor.

Thus between any two points on a conductor there is a certain definite resistance: a metal wire, for example, has an electrical resistance of so many units depending on its length, cross-section, material, and temperature. Resistance coils are made of such pieces of wire, covered with an insulating material, cut so as to have a resistance of a certain definite number of units and wound on a bobbin. The ends of the coil are fastened in some cases to binding screws, in others to stout pieces of copper which, when the coil is in use, are made to dip into mercury cups, through which connection is made with the rest of the apparatus used. We refer to § 78 for a description of the method of employing such coils in electrical measurements.

Standards of resistance have the advantages of material standards in general. The resistance is a definite property of a piece of metal, just as its mass is. The coil can be moved about from place to place without altering its resistance, and so from mere convenience electrical resistance has come to be looked upon as in some way the fundamental quantity in connection with current electricity. We have defined it by means of Ohm's law as the ratio of electromotive force to the current. Whenever difference of potential exists between two points of a conductor, a current

of electricity is set up, and the amount of that current depends on the E.M.F. and the resistance between the points.

We may say that electrical resistance is that property of a conductor which prevents a finite electromotive force from doing more than a finite quantity of work in a finite time. Were it not for the resistance, the potential would be instantaneously equalised throughout the conductor; a finite quantity of electricity would be transferred from the one point to the other, and therefore a finite quantity of work would be done instantaneously.

The work actually done in time t is, we have seen, equal to $E\,C\,t$, and by means of the equation $C = E/R$ expressing Ohm's law, we may write this

$$W = CEt = E^2 t/R = C^2 R\, t.$$

Moreover the E.M.F. between two points is given if we know the resistance between them and the current, for we have $E = C\,R$. Further the resistance of a wire is evidently equal to the rate of expenditure of energy required to maintain unit current in the wire.

On the Resistance of Conductors in Series and Multiple Arc.

If $A\,B$, $B\,C$ be two conductors of resistances R_1 and R_2, the resistance between A and C is $R_1 + R_2$. For let the potentials at A, B, C be V_1, V_2, V_3 respectively, and suppose that owing to the difference of potential a current i is flowing through the conductors. This current is the same in the two conductors (see p. 492), and if R be the resistance between A and C, we have from Ohm's law

$$V_1 - V_2 = R_1\, i$$
$$V_2 - V_3 = R_2\, i$$
$$V_1 - V_3 = R\;\; i.$$

But by adding the first two equations we have

$$V_1 - V_3 = (R_1 + R_2)\, i\,;$$
$$\therefore\; R = R_1 + R_2.$$

By similar reasoning it may be shewn that the resultant resistance of any number of conductors placed end to end is equal to the sum of the resistances of the several conductors. Conductors connected in this manner are said to be in series.

Again, let there be two conductors of resistances R_1, R_2, joining the same two points A and B, and let R be the equivalent resistance of the two, that is, the resistance of a conductor, which, with the same E.M.F. would allow the passage of a current of electricity equal to the sum of those which actually flow in the two conductors. Hence, if V_1, V_2 be the potentials at A and B, we have

$$\frac{V_1 - V_2}{R_1} = i_1; \quad \frac{V_1 - V_2}{R_2} = i_2;$$

$$\frac{V_1 - V_2}{R} = i = i_1 + i_2;$$

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2},$$

and

$$R = \frac{R_1 R_2}{R_1 + R_2}.$$

Also

$$i_1 / i_2 = R_2 / R_1.$$

Conductors joined up in the above manner are said to be connected in multiple arc; thus, remembering that the reciprocal of the resistance is called the conductivity, we may shew by reasoning precisely similar to that given above that the conductivity of a system of any number of conductors in multiple arc is the sum of the conductivities of the several conductors.

Let B A C be a circuit including a battery B, and suppose that we wish to send between the two points, A and C, only $1/n$th part of the current produced by the battery. Let R be the resistance between A and C. Connect these two points by a second conductor of resistance, $R/(n-1)$.

Let i_1 be the current in the original conductor between A and C, i_2 the current in the new conductor, i the current in the rest of the circuit. Then we have

$$i_2/i_1 = \text{R}(n-1)/\text{R};$$
$$\therefore\ i_2 = (n-1)\,i_1,$$

and

$$i = i_1 + i_2 = n\,i_1$$

So that

$$i_1 = i/n.$$

The second conductor, connected in this manner with the two points, is called a shunt, and the original circuit is said to be shunted.

Shunts are most often used in connection with galvanometers. Thus we might require to measure a current by the use of a tangent galvanometer, and, on attempting to make the measurement, might find that the galvanometer was too sensitive, so that the deflexion produced by the current was too large for measurement. By connecting the electrodes of the galvanometer with a shunt of suitable resistance we may arrange to have any desired fraction of the current sent through the galvanometer.

This fraction can be measured by the galvanometer, and the whole current is obtained from a knowledge of the resistances of the shunt and galvanometer. A galvanometer is often fitted with a set of shunts, having resistances 1/9, 1/99, and 1/999 of its own resistance, thus enabling ·1, ·01, or ·001 of the whole current to be transmitted through it.

In applying Ohm's law to a circuit in which there is a battery of electromotive force E, it must be remembered that the battery itself has resistance, and this must be included in the resistance of the circuit. Thus, if we have a circuit including a resistance R, a battery of E.M.F. E and resistance B, and a galvanometer of resistance G, the total resistance in the circuit is R + B + G, and the current is

$$\text{E}/(\text{R}+\text{B}+\text{G}).$$

The normal E.M.F. of the battery is taken to be the difference of potential between its poles when they are insulated from each other. If they be connected together, the difference of potential between them will depend on the resistance of the conductor joining them. In the case in point this is R+G; and since the difference of potential is found by multiplying together the current and the resistance, it will in that case be

$$E(R+G)/(R+G+B).$$

On the Absolute Measurement of Electrical Resistance. Electrical resistance is measured in terms of its proper unit defined by the equation

$$R = \frac{E}{C}.$$

For let a conductor be such that unit difference of potential between its two ends produces unit current; then in the above equation E and C are both unity; so that R is also unity and the conductor in question has unit resistance.

DEFINITION OF AN ABSOLUTE UNIT RESISTANCE.—The unit of resistance is the resistance of a conductor in which unit electromotive force produces unit current.

This is a definition of the absolute unit. Now it is found[1] that on the C.G.S. system of units the unit of resistance thus defined is far too small to be convenient. Therefore, just as was the case for E.M.F., a practical unit of resistance is adopted, and this contains 10^9 absolute C.G.S. units, and is called an 'ohm'; so that 1 ohm contains 10^9 absolute units.

We have already seen that the volt or practical unit of E.M.F. is given by the equation

$$1 \text{ volt} = 10^8 \text{ absolute units.}$$

[1] See F. Jenkin, *Electricity and Magnetism*, chap. x.; Maxwell, *Electricity and Magnetism*, vol. ii. § 629.

Let us suppose that we have a resistance of 1 ohm and that an E.M.F. of 1 volt is maintained between its ends; then we have for the current in absolute units

$$C = \frac{E}{R} = \frac{10^8}{10^9} = \frac{1}{10} \text{ absolute unit} = 1 \text{ ampère.}$$

Thus an ampère, the practical unit of current, is that produced by a volt when working through an ohm.

But electrical resistance is, as we have seen, a property of material conductors. We can, therefore, construct a coil, of German-silver or copper wire suppose, which shall have a resistance of 1 ohm. The first attempt to do this was made by the Electrical Standards Committee of the British Association, and the standards constructed by them are now at the National Physical Laboratory.

More recent experiments have shewn, however, that these standards have a resistance somewhat less than 1 ohm. They have for some time past been in use as ohms and numbers of copies have been made and circulated among electricians. The resistances of these standards are now known as British Association Units.

In accordance with the resolutions of the Committee of the British Association passed at Edinburgh in 1892, it has been decided to define the ohm in terms of the resistance of a certain column of mercury at the temperature of melting ice. The length of the column is 106·3 centimetres; for practical purposes the area of its cross-section is one square millimetre, but the area of such a column would always be determined by finding the mass of a known length, and dividing this by the density of mercury in grammes per c.c. and by the length. Now the specific gravity of mercury is known with all the necessary accuracy, but the density of water is still a little uncertain. To avoid the difficulty caused by this, the mass of mercury in the column is stated.

The specific gravity of mercury at 0° is 13·5956; if we assume that the mass of one c.c. of water at 4° C. is one gramme, then the mass of a column of mercury 106·3 cm.

long, one square mm. in section, is $13{\cdot}5956 \times 1{\cdot}063$ grammes, and this comes to $14{\cdot}4521$ grammes.

Thus one ohm is defined to be the resistance of $14{\cdot}4521$ grammes of mercury in the form of a column of uniform cross-section $106{\cdot}3$ cm. in length at $0°$ C. Moreover it has been shown by experiment that

$$1 \text{ B.A. unit} = {\cdot}9866 \text{ ohm}.$$

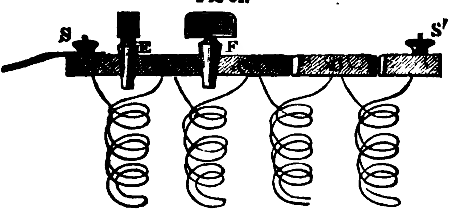
Thus

$$1 \text{ ohm} = 1{\cdot}01358 \text{ B.A. unit}.$$

On Resistance Boxes.

For practical use resistance coils are generally grouped together in boxes. The top of the box is made of non-conducting material, and to it are attached a number of stout brass pieces shewn in fig. 61 at A, B, C, D. A small space is left between the consecutive brass pieces, and the ends of these pieces are ground in such a way that a taper plug

FIG 61.



of brass can be inserted between them and thus put the two consecutive pieces into electrical connection. The coils themselves are made of German-silver or platinum-silver wire. The wire is covered with silk or some other insulating material. A piece of wire of the required resistance is cut off and bent double. It is then wound on to a bobbin of ebonite or other insulating material. The bobbins are not drawn in the figure. The two ends are soldered to two consecutive brass pieces in the box, the bobbin being fixed to the under side of the lid of the box. The coils when complete are covered with paraffin to maintain a good insulation.

Let A, B be the two brass pieces, and suppose a current flowing from A to B; if the plug is in its place, the current

can pass through it, and the resistance between A and B is infinitesimally small, provided always that the plug fits properly. If, however, the plug be removed, the current has to flow through the coil itself; so that by removing the plug the resistance of the coil may be inserted in the circuit between A and B.

The coils in a box are generally arranged thus:—

1	2	2	5	
10	10	20	50	
100	100	200	500	units, &c.

Thus, if there be the twelve coils as above, by taking out suitable plugs we can insert any desired integral number of units of resistance between 1 and 1000, like weights in the balance. Binding screws, S, S′, are attached to the two extreme brass pieces, and by means of these the box can be connected with the rest of the circuit.

The coils are wound double, as described, to avoid the effects which would otherwise arise from self-induction,[1] and also to avoid direct magnetic action on the needle of the galvanometer.

On the Relation between the Resistance and Dimensions of a Wire of given Material.

We have seen that if two conductors be joined in series the resistance of the combination is the sum of the resistances of the parts. Let the conductor be a long wire of uniform material and cross-section. Then it follows from the above (p. 528) that the resistance is proportional to the length; for if we take two pieces of the same length they will have the same resistance, and if connected end to end the resistance of the double length is double that of the single. Thus the resistance is proportional to the length.

Again, we may shew that the resistance is inversely proportional to the area of the cross-section. For suppose two points, A and B, are connected by a single wire, the

[1] See S. P. Thompson's *Elec. and Mag.*, § 404; Jenkin. *Elec. and Mag.*, pp. 74, 232.

resistance of which is R. Introduce a second connecting wire of the same length and thickness, and therefore of the same resistance as the former. The resistance will now be $\frac{R}{2}$, and since it was found by Ohm that the resistance depends on the area of the cross-section and not on its form, we may without altering the result suppose the two wires, which have been laid side by side, welded into one, having a cross-section double of that of either wire.

Thus, by doubling the cross-section the resistance is halved. The resistance, therefore, varies inversely as the area of the cross-section.

DEFINITION OF SPECIFIC RESISTANCE.—Consider a cube of conducting material having each edge one centimetre in length. Let two opposite faces of this be maintained at different potentials, a current will be produced through the cube, and the number of units in the resistance of the cube is called the specific resistance of the material of which the cube is composed.

Let ρ be the specific resistance of the material of a piece of wire of length l and cross-section a, and let R be the resistance of the wire. Then

$$R = \rho\, l/a.$$

For, suppose the cross-section to be one square centimetre, then the resistance of each unit of length is ρ and there are l units in series; thus the whole resistance is $\rho\, l$. But the resistance is inversely proportional to the cross-section, so that if this be a square centimetres, the resistance R is given by the equation

$$R = \rho\, l/a.$$

Again, it is found that the resistance of a wire depends on its temperature, increasing in most cases uniformly with the temperature for small variations, so that if R_0 be the resistance at a temperature zero and R that at temperature t, we have

$$R = R_0(1 + a\, t),$$

where a is a constant depending on the nature of the material of the wire; this constant is called the temperature coefficient of the coil. For most materials the value of a is small. German-silver and platinum-silver alloy are two substances for which it is specially small, being about ·00032 and ·00028 respectively.

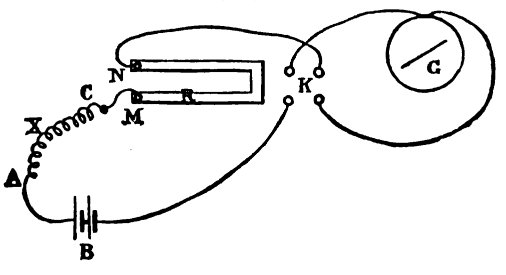
Its value for copper is considerably greater, being about ·003, and this is one reason why resistance coils are made of one of the above alloys in preference to copper. Another reason for this preference is the fact that the specific resistance of the alloys is much greater than that of copper, so that much less wire is necessary to make a coil than is required if the material be copper.

75. Comparison of Electrical Resistances.

Ohm's law forms the basis of the various methods employed to compare the electrical resistance of a conductor with that of a standard coil.

In the simplest arrangement of apparatus for making the measurements the connections are made in the following

FIG. 62



manner (fig. 62):—One pole of a battery B of constant E.M.F. is connected to one end A of the conductor whose resistance is required; the other end C of this conductor is in connection with a resistance box M N. N is in connection with a key or, better, a commutator K, from which

the circuit is completed through a galvanometer G to the other pole of the battery.

Let X be the resistance to be measured, B the battery resistance, G that of the galvanometer, and suppose a resistance R is in circuit in the box.

Make contact with the commutator. A current passes through the galvanometer. Observe the deflexion when the needle has become steady. Reverse the commutator; the galvanometer needle is deflected in the opposite direction, and if the adjustments were perfect, the two deflexions would be the same. They should not differ by more than 0°·5.

Adjust R, the resistance in the box, if it be possible, until the deflexion observed is about 45°. Of course it may be impossible to do this with the means at hand. If when R is zero the deflexion observed be small, the electromotive force of the battery will require to be increased; we must use more cells in series. If, on the other hand, with as great a resistance in the box as is possible, the deflexion be too large, then either the galvanometer must be shunted or the E.M.F. of the battery reduced by reducing the number of cells, or by connecting its poles through a shunt. In any case the deflexion should be between 30° and 60°.

Let E be the E.M.F. and k the reduction factor of the galvanometer, which, we shall suppose, is a tangent instrument. Then, if i be the current, and a the mean of the two deflexions in opposite directions, we have

$$\frac{\mathrm{E}}{\mathrm{B}+\mathrm{G}+\mathrm{X}+\mathrm{R}} = i = k \tan a.$$

Hence

$$\mathrm{B}+\mathrm{G}+\mathrm{X}+\mathrm{R} = \mathrm{E}/k \tan a \quad . \quad . \quad . \quad (1)$$

and if B, G, E, and k be known, R and a being observed, this equation will give us X.

If E and k be not known, while B and G are, we proceed thus. Take the unknown resistance X out of the circuit, connecting one pole of the battery with the electrode M

of the resistance box. Take a resistance R′ out of the box and observe the deflexion, which, as before, should lie between 30° and 60°, reversing the current and reading both ends of the needle; let the mean deflexion be α'. Then we have, as before, if the battery have a constant E.M.F.,

$$\frac{E}{B+G+R'} = k \tan \alpha' ;$$

$$\therefore E/k = (B+G+R') \tan \alpha' \quad . \quad . \quad . \quad (2)$$

so that the original equation (1) becomes

$$B+G+X+R = (B+G+R') \tan \alpha' / \tan \alpha, \quad . \quad . \quad . \quad (3)$$

and from this X can be found.

But in general B and G will not be known. We can easily find the sum B+G as follows:—

Make two sets of observations exactly in the same manner as the last were made, with two different resistances R_1, R_2 out of the box, and let the deflexions be α_1 and α_2; α_1 may be just over 30°, α_2 just under 60°.

[There should be a large difference between α_1 and α_2, for we have to divide, in order to find the result, by $\tan \alpha_2 - \tan \alpha_1$, and, if this be small, a large error may be produced.]

Then, assuming as before that the E.M.F. of the battery does not alter, we have

$$\frac{E}{B+G+R_1} = k \tan \alpha_1, \quad . \quad . \quad . \quad (4)$$

and

$$\frac{E}{B+G+R_2} = k \tan \alpha_2. \quad . \quad . \quad . \quad (5)$$

Hence

$$(B+G+R_1) \tan \alpha_1 = \frac{E}{k} = (B+G+R_2) \tan \alpha_2,$$

and

$$B+G = \frac{R_1 \tan \alpha_1 - R_2 \tan \alpha_2}{\tan \alpha_2 - \tan \alpha_1} \quad . \quad . \quad . \quad (6)$$

Having thus found B+G, we may use either of the equations (4) or (5) in combination with (1) to give us X.

If we wish to find B and G separately we may proceed as follows :—

Shunt the galvanometer with a shunt of resistance S; then the resistance between the poles of the galvanometer is equivalent to GS/(S+G). Make two more observations like those from which equations (4) and (5) are deduced, we thus find a value for B+GS/(S+G).

Suppose we find

$$B+\frac{GS}{S+G}=z,$$

having already obtained

$$B+G=y,$$

when y is written for the right-hand side of equation (6). Hence

$$G-\frac{GS}{S+G}=y-z;$$

thus

$$G^2=(S+G)(y-z),$$

or

$$G^2-G(y-z)-S(y-z)=0;$$

$$\therefore\ G=\tfrac{1}{2}[y-z+\sqrt{\{(y-z)^2+4S(y-z)\}}].$$

Thus, G having been found, B is given from the equation

$$B=y-G=\tfrac{1}{2}[y+z-\sqrt{\{(y-z)^2+4S(y-z)\}}].$$

The methods here given for measuring resistance, involving, as they do, the assumption that the E.M.F. of the battery remains the same throughout, cannot be considered as completely satisfactory. Others will be given in §§ 77–79, which are free from the objections which may be urged against these. Various modifications of the above methods have been suggested for measuring more accurately the resistance of a battery or galvanometer. For an account of these the reader is referred to Kempe's 'Handbook of

Electrical Testing,' chapters v. and vi. In practice much is gained by a little judgment in the choice of the resistances taken from the box. Thus, in finding B+G as above it might happen that when R_2 is 19, a_2 is 59° 30′, and when R_2 is 20, a_2 is 58° 45′. Now the tangent of either of these angles can be looked out equally easily in the tables, but the multiplication involved in finding $R_2 \tan a_2$ is much more easily done if R_2 be 20 than if it be 19.

Experiment.—Determine the resistance of the given coil X.

Enter results thus:—

Observations to find B + G.

$$R_1 = 20 \text{ ohms.} \qquad a_1 = 57^\circ$$
$$R_2 = 50 \;\;\text{,,} \qquad a_2 = 34^\circ$$

Whence B + G = 3·37 ohms.

Observations to find X.

$$R = 10 \text{ ohms.} \qquad a = 46^\circ{\cdot}52$$
$$R_1 = 20 \;\;\text{,,} \qquad a_1 = 57^\circ$$

Whence X = 20·75 ohms.

N.B.—If a large number of resistances have to be determined by the use of the same galvanometer, it will be best to calculate the value of B + G, and the ratio of the E.M.F. to the reduction factor once for all, checking the results occasionally during the other observations. These are both given by the observations just made, for we have found B + G, and we have

$$\frac{E}{B+G+R_1} = k \tan a_1$$

$$\therefore \frac{E}{k} = (B+G+R_1) \tan a_1.$$

With the numbers in the above example,

$$B + G = 3{\cdot}37; \qquad R_1 = 20;$$
$$a_1 = 57^\circ;$$

and we find

$$\frac{E}{k} = 35{\cdot}97.$$

So that, if we find, with an unknown resistance X in circuit and a resistance R out of the box, that the deflexion is a, we obtain

$$B + G + X + R = \frac{E}{k \tan a} = \frac{35{\cdot}97}{\tan a}.$$

76. Comparison of Electromotive Forces.

We may moreover use Ohm's law to compare the electromotive forces of batteries.[1] For suppose we have two batteries; let B, B′ be their resistances, E, E′ their electromotive forces. Pass a current from the two batteries in turn through two large resistances, R and R′ and the galvanometer, and let the deflexions observed be a, a'. Suppose the galvanometer to be a tangent instrument. Then, if k be its reduction factor, G its resistance, we have

$$E = k(B + G + R) \tan a,$$
$$E' = k(B' + G + R') \tan a'.$$

Hence

$$\frac{E}{E'} = \frac{(B + G + R) \tan a}{(B' + G + R') \tan a'},$$

and B+G, B′+G being determined as in the last section, the quantities on the right-hand side are all known.

In practice there are some simplifications. A Thomson's reflecting galvanometer is used, and this is so sensitive that R and R′ will need to be enormously large to keep the spot of light on the scale. The values will be probably from eight to ten thousand ohms if only single cells of the batteries in ordinary use be employed. Now the resistance of such a cell will be very small compared with these; an ordinary quart Daniell should be under one ohm; a Leclanché from one to three ohms; and hence we may neglect B and B′ as compared with R and R′, and we have

$$\frac{E}{E'} = \frac{(R + G) \tan a}{(R' + G) \tan a'}.$$

[1] See p. 531.

This equation is applied in two ways :—

(1) *The Equal Resistance Method.*—The resis... made equal to R, i.e. the two batteries are worked the same external circuit, and we have then

$$\frac{E}{E'} = \frac{\tan \alpha}{\tan \alpha'}.$$

But if the angles α, α' be not too large, the scale-d flexions of the spot of light are very nearly proportional t tan α and tan α'. Let these deflexions be δ and δ' respe tively, then

$$\frac{E}{E'} = \frac{\delta}{\delta'}.$$

For this method we do not need to know the galvanometer resistance, but we suppose that the galvanometer is such that the displacement of the spot is proportional to the current.

(2) *The Equal Deflexion Method.*—In this method of working α' is made equal to α, and we have

$$\frac{E}{E'} = \frac{R+G}{R'+G}.$$

For this method we require to know G, or, at any rate, to know that it is so small compared with R and R′ that we may neglect it. The method has the advantage that we do not assume any relation between the current in the galvanometer and the deflexion produced, except that the same current produces the same deflexion ; and this is obviously true whatever be the form of the instrument.

Both methods are open to the objection that the E.M.F. of a battery which is actually producing a current changes from time to time. We shall see in § 80 how to compare the E.M.F. of batteries without allowing them to produce a current.

Experiments.

Compare the E.M.F. of the given batteries by the equal resistance and the equal deflexion methods, and taking the E.M.F. of

'l's cell as 1·08 volts, find the E.M.F. of the others in

results thus :—

Resistance Method.—Resistance used, 10,000 ohms.
resistance of cells, small.

Battery	Deflexions in scale divisions	E.M.F. in volts
Daniell . . .	46	1·08
Sawdust Daniell . .	35	·82
Leclanché . . .	52	1·22
Bichromate . . .	68	1·60

Equal Deflexion Method.—Deflexion, 83 scale divisions. Galvanometer resistance, small.

Battery	Resistance	E.M.F. in volts
Daniell . . .	8000	1·08
Sawdust Daniell . .	6020	·81
Leclanché . . .	9040	1·22
Bichromate . . .	11980	1·61

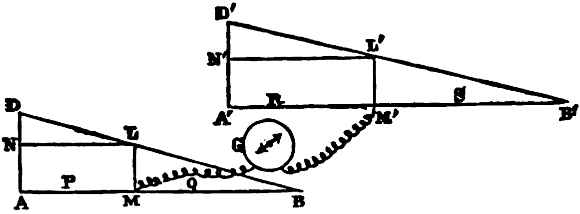
77. Wheatstone's Bridge.

The method of comparing electrical resistances which has been already described depends on the measurement of the deflexion produced in a galvanometer, and we make the assumptions that the E.M.F. of the battery remains constant during the experiment, and that the relation between the current flowing through the galvanometer and the deflexion it produces is known. The disadvantages which thus arise are avoided in the Wheatstone bridge method, the principles of which we proceed to describe.

It follows from Ohm's law (p. 526) that, if a steady current be flowing through a conductor, then the electromotive force between any two points of the conductor is proportional to the resistance between those points. We can express this graphically thus. Let the straight line A B (fig. 63) represent the resistance between the two points A and B of a conductor, and let the line A D, drawn at right angles to A B, represent the electromotive force or difference of

potential between A and B. Join D B, and let M be a point on the line A B, such that A M may represent the resistance between A and another point of the conductor. Draw M L

FIG. 63.



at right angles to A B to meet B D in L, then L M represents the E.M.F. between M and B.

For if C represent the current flowing through the conductor, then, by Ohm's law,

$$\frac{DA}{AB} = C;$$

and since M L is parallel to D A,

$$\frac{DA}{AB} = \frac{LM}{MB}.$$

$$\therefore\ LM = C \times MB.$$

But since M B represents the resistance and C the current between two points M and B, it follows from Ohm's law that L M represents the E.M.F. between those points.

Now let A′ B′ represent the resistance between two points on another conductor, between which the E.M.F. is the same as that between A and B, and let A′ D′ represent this E.M.F.; then

$$A'D' = AD.$$

Join D′ B′, and in it take L′ M′, such that L′ M′ shall be equal L M.

Then M′ will represent a point on the second conductor,

such that the difference of potential between it and B' is equal to the difference of potential between M and B.

Thus if B, B' be at the same potential, A, A' and M, M' respectively are at the same potentials. Hence, if $M\,M'$ be joined through a galvanometer G, no current will flow through the galvanometer, and no deflexion, therefore, will be observed.

We can now express the condition for this in terms of the four resistances $A\,M$, $M\,B$, $A'\,M'$, $M'\,B'$. Let these resistances respectively be denoted by P, Q, R, and S.

Draw $L\,N$, $L'\,N'$ parallel to $A\,B$ and $A'\,B'$.

Then clearly $D\,N = D'\,N'$, and we have

$$\frac{P}{Q} = \frac{A\,M}{M\,B} = \frac{N\,L}{M\,B} = \frac{D\,N}{L\,M} = \frac{D'\,N'}{L'\,M'} = \frac{N'\,L'}{M'\,B'} = \frac{A'\,M'}{M'\,B'} = \frac{R}{S}.$$

Thus the condition required is

$$\frac{P}{Q} = \frac{R}{S}.$$

If, then, we have four conductors, $A\,M$, $M\,B$, $A'\,M'$, $M'\,B'$, and we connect together B and B', and so keep them at the same potential, and also connect A and A', thus keeping them at any other common potential, then, provided the above condition holds, we may connect M and M' through a galvanometer without producing a deflexion; and conversely if, when $M\,M'$ are thus connected, no deflexion be observed, we know that the above condition holds. Hence, if P and Q be any two known resistances, R any unknown resistance, and S an adjustable known resistance, and we vary S, the other connections being made as described, until no deflexion is observed in the galvanometer, R can be found, for we then have

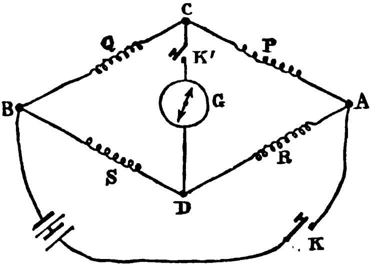
$$R = S \times \frac{P}{Q},$$

and P, Q, S are known.

In practice, to secure that B and B' should be at the same potential, they are connected together, and to o pole of a battery, A and A' being connected through a to the other pole.

Fig. 64 shews a diagram of the connections. A C, C B correspond to the two conductors A M, M B of fig. 63, while A D, D B correspond to A′ M′, M′ B′. A key K′ is placed in the galvanometer circuit and a second key K in the battery circuit. On making contact with the key K a difference of potential is established between A and B, and a current flows through the two conductors A C B and A D B. If on making contact with K′ no deflexion is observed in the galvanometer, it follows that C and D are at the same potential, and therefore that

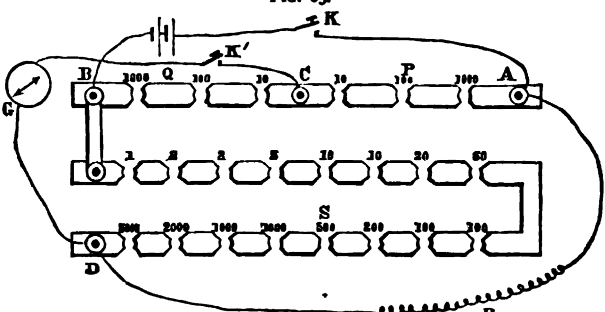
FIG. 64.



$$R = S \times \frac{P}{Q}.$$

In practice P, Q, and S are resistance coils included in the same box, which is arranged as in fig. 65 for the purposes of the experiment, and is generally known as a Wheatstone-bridge box, or sometimes as a Post-Office box.[1] The

FIG. 65.



[1] But see next page.

resistances P and Q, which are frequently spoken of as the arms of the bridge, are taken, each from a group of three coils of 10, 100, and 1000 units. Thus, by taking the proper plugs out we may give to the ratio P/Q any of the values

100, 10, 1, ·1, or ·01.

The resistance S is made up of 16 coils from 1 to 5,000 ohms in resistance, and by taking the proper plugs out it may have any integral value between 1 and 10,000 units. Thus the value of R may be determined to three figures if it lie between 1 and 10, or to four figures if it be between 10 and 1,000,000, provided, that is, the galvanometer be sufficiently sensitive.

At A, B, C, and D are binding screws, those at A and D being double. By means of these the electrodes of the battery, galvanometer, and conductor whose resistance is required, are connected with the box. In some boxes the two keys, K and K′, are permanently connected with the points A and C, being fixed on to the insulating material of the cover. The arrangement is then technically known as a Post-Office box. The galvanometer to be employed should be a sensitive reflecting instrument; the method of adjusting this has been already described (p. 510), while for a battery, one or two Leclanché or sawdust Daniell cells are generally the most convenient. The number of cells to be used depends, however, on the magnitude of the resistance to be determined and the sensitiveness of the galvanometer. The key K is inserted in the battery circuit in order that the battery may be thrown out, except when required for the measurement. The continual passage of a current through the coils of the box heats them, and if the current be strong enough may do damage.

It will be noticed that at each of the points A, B, C, D, three conductors meet, and that including the galvanometer and battery there are six conductors in all, joining the four points A, B, C, D. When the resistances are such that the

current in the conductor joining two of the points is independent of the E.M.F. in the conductor joining the other two, then those two conductors are said to be conjugate.

In the Wheatstone's bridge method of measuring resistances the battery and galvanometer circuits are made to be conjugate; the current through the galvanometer is independent of the E.M.F. of the battery. If the equation

$$P/Q = R/S$$

hold, the galvanometer is not deflected whatever be the E.M.F. of the battery; there is no need, therefore, to use a constant battery. Moreover, since we only require to determine when no current flows through the galvanometer circuit, and not to measure a steady current, a sensitive galvanoscope is all that is necessary; we do not need to know the relation between the current and the deflexion produced by it.

FIG. 66.

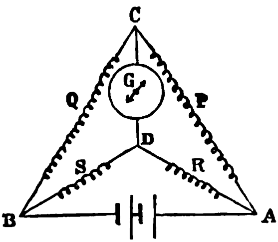


Fig. 66 is another diagram of the connections, which shews more clearly the conjugate relation. The conductors A B and C D are conjugate if the equation

$$P/Q = R/S \text{ holds.}$$

It follows from this that we may interchange the galvanometer and battery without affecting the working of the method. The galvanometer may be placed between A and B, and the battery between C and D. The sensitiveness of the measurements will, however, depend on the relative positions of the two, and the following rule is given by Maxwell, 'Electricity and Magnetism,' vol. i. § 348, to determine which of the two arrangements to adopt. Of the two resistances, that of the battery and that of the galvanometer, connect the greater resistance, so as to join the two greater to the two less of the other four.

As we shall see directly, it will generally happen when

making the final measurements, that Q and S are greater than P and R; thus, referring to fig. 65, the connections are there arranged to suit the case in which the resistance of the battery is greater than that of the galvanometer.

To measure a Resistance with the Wheatstone-bridge Box.

Make the connections as shewn in fig. 65. Be sure that the binding screws are everywhere tight and that the copper wires are clean and bright at all points where there are contacts. This is especially necessary for the wires which connect R to the box. Any resistance due to them or their contacts will of course be added to the value of R. For delicate measurements contacts must be made by means of thick copper rods amalgamated with mercury, and dipping into mercury cups. The bottoms of the cups should be covered with discs of amalgamated copper, and the wires must press on to these with a steady pressure throughout the experiment; it is not sufficient to make the contact through the mercury by letting the wires drop into it without touching the copper bottom. The cups themselves are conveniently made of pill boxes, covered with a good thick coat of varnish.

See that all the plugs are in their places in the box, and press them firmly in with a screw motion to ensure efficient contact.

Bring the control magnet of the galvanometer down near the coils, and if the resistance to be measured be not even approximately known, it generally saves time to shunt the galvanometer, using the shunt, provided there be one, if not, a piece of thin German-silver wire. Take two equal resistances out of the arms P and Q. Since it is probable that the galvanometer will be somewhat too sensitive even when shunted, it is better to take out the two 100 ohm plugs rather than the two 10 ohms. Then, since $P = Q$, R will be equal to S.

Take 1 ohm out from S. Make contact first with the

battery key K, and then with the galvanometer key K′, and note the direction of the deflexion—suppose it be to the right.

Take out 1000 ohms from S, and note the deflexion—suppose it be to the left. The resistance is clearly between 1 and 1000 ohms.

Now take out 500 ohms—let the deflexion be to the left—R is less than 500. Proceed thus, and suppose that with 67 ohms the deflexion is to the left, and that with 66 ohms it is to the right. The resistance R is clearly between 66 and 67 ohms.

Now make P 10 ohms and Q 100, and at the same time remove the shunt, and raise the galvanometer magnet to increase the sensitiveness. Since Q is ten times P, S must be ten times R to obtain a balance. Thus S must be between 660 and 670. Suppose that it is found that with 665 ohms the deflexion is to the left, and with 664 it is to the right, the true value of S is between 664 and 665, and since $R = PS/Q$, the true value of R is between 66·4 and 66·5. We have thus found a third figure in the value of R.

Now make Q 1000 ohms and P 10 ohms. Then, since Q is 100 times P, S must be 100 times R to secure the balance; and it will be found that when S is 6640 the deflexion is to the right; when it is 6650 it is to the left. The galvanometer may now be made as sensitive as possible; and it will probably be found that with a value of S, such as 6646, there is a small deflexion to the right, and with S equal to 6647 a small deflexion to the left. Thus the value of R is between 66·46 and 66·47.

If the fourth figure be required correctly, we may find it by interpolation as follows:—

When S is 6646 let the deflexion to the right be a scale divisions, and when it is 6647 let it be b divisions to the left. Then since an addition of 1 ohm to the value of R alters the reading by $a+b$ scale divisions, it will require an addition of $a/(a+b)$ ohms to alter it by a divisions.

Thus the true value of R is $6646+a/(a+b)$ ohms, and the value of S is

$$66{\cdot}46+a/100(a+b) \text{ ohms.}$$

The exactness to which the determination can be carried will depend on the accuracy with which the small outstanding deflexions a and b can be read, and on the constancy of the battery.

If it be found that the resistance R is less than 1 ohm, make P 100 ohms, and Q 10; then the value of S will be ten times that of R, and if we find that S lies between 5 and 6, it follows that R is between ·5 and ·6; then make P 1000 ohms, and Q 10, and proceed similarly.

After making the determination the connecting wires must all be removed from the box and the plugs replaced.

Experiment.—Determine the values of the resistances in the given box.

Enter results thus :—

Nominal value	Real value
10 ohms	10·03 ohms
20 "	20·052 "
50 "	50·005 "
100 "	100·13 "

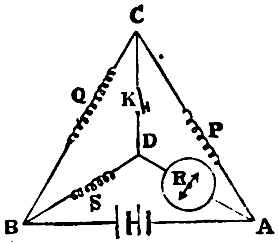
Measurement of a Galvanometer Resistance—Thomson's Method.

It has been shewn that if, in the Wheatstone's bridge arrangement, two of the conductors, as A B, C D (fig. 66, p. 548), are conjugate, then the current through the one due to an E.M.F. in the other is zero. It follows from this that the current through the other conductors is independent of the resistance in C D, and is the same whether C D be connected by a conductor or be insulated; for the condition that the two should be conjugate is that C and D should be at the same potential, and if this condition be satisfied there will never be any tendency for a current to flow along C D;

the currents in the rest of the circuit will, therefore, not depend on C D.

Suppose, now, a galvanometer is placed in the branch D A, and a key in C D (fig. 67), there will be a deflexion produced in the galvanometer. Adjust the resistance S until *the galvanometer deflexion is unaltered by making or breaking contact in the branch* C D. When this is the case it follows that A B and C D are conjugate, and, therefore, that

FIG. 67.

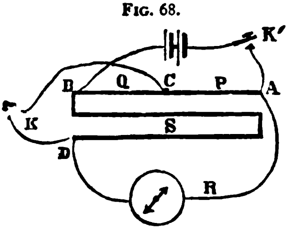


$$R = \frac{P}{Q} \times S.$$

But R is the resistance of the galvanometer, which is thus measured by a null method without the use of a second galvanometer.

Fig. 68 shews the connections, using the Wheatstone-bridge box. A considerable portion of the current from the battery flows through the galvanometer, and the needle is thereby deflected. If a Thomson's galvanometer be used in the ordinary manner, the spot of light will be quite off the scale. In order to ascertain if the adjustment of the resistances is correct the

FIG. 68.



mirror must be brought back to near its zero position by the aid of permanent magnets; it is probable that the control magnet will be too weak to do this alone, and others must be employed in addition. This constitutes one of the defects of the method; the field of magnetic force in which the needle hangs thus becomes very strong, and the sensitiveness of the galvanometer is thus diminished. By using a very weak electromotive force we may dispense with the

additional magnets; the control magnet itself may be sufficient. We may attain this end by shunting the battery with a German-silver wire. The resistance suitable will depend on many conditions, and must be found by trial. A more economical method of diminishing the electromotive force between the points A and B is to introduce resistance into the battery circuit between point A or B and the pole. By making this interpolated resistance sufficiently great we may make the E.M.F. between A and B, what fraction we please of the total E.M.F. of the battery. And by increasing the resistance of the circuit we diminish the current which flows, and therefore diminish the consumption of zinc in the battery, whereas if the E.M.F. between A and B be reduced by shunting, the total current supplied by the battery is increased, and a larger expenditure of zinc is the result.

The battery used should be one of fairly constant E.M.F., for, if not, the current through the galvanometer will vary, and it will be difficult to make the necessary observations.

The method of proceeding is the same as that employed in the last section; the arms P and Q are first made equal, and two values found, differing by one ohm, between which S lies. The ratio P/Q is then made ·1, and the first decimal place in the value of R obtained, and so on.

Experiment.—Determine, by Thomson's method, the resistance of the given galvanometer.

Enter result thus:—

Galvanometer No. 6 . Resistance 66·3 ohms.

Measurement of a Battery Resistance—Mance's Method.

If we recollect that electromotive forces can be superposed, and that the resultant effect is simply the sum of the individual effects produced by each, it is clear that the condition that two conductors in a Wheatstone bridge, such as A B and C D (fig. 66), may be conjugate is not altered by the

introduction of a second battery into any of the arms of the bridge. Such a battery will of course send a current through the galvanometer, and produce a deflexion, which will be superposed on that due to the battery in A B. Let a battery be put in the arm A D (fig. 69), R being its resistance, and let the galvanometer needle be brought back to its zero position by the use of external magnets. Adjust the resistance S until making or breaking contact in the battery circuit A B produces no effect on the galvanometer; that is, until the circuits A B and C D are conjugate. When this is the case we have

$$R = PS/Q;$$

and P, S, and Q being known, we can find R, the resistance of the battery.

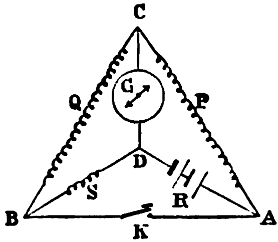
There is, however, no need for a second battery in A B; for the effect on the galvanometer due to this battery is zero when the conjugate condition is satisfied, whatever be its E.M.F. Take then the case when the E.M.F. is zero, i.e. connect A and B directly through a conductor. If the conjugate condition be satisfied this will produce no effect on the galvanometer; the deflexion due to the battery in A D will not be altered.

Again take the case in which the E.M.F. produced between A and B by the battery in A B is exactly equal and opposite to that produced between those points by the battery in A D. The galvanometer deflexion will still, if the conjugate condition hold, be unaltered. But in this case no current flows along A B; the conditions are the same as if A and B were insulated.

Thus the battery in A B may be supposed removed and replaced by a key. If the resistance S be adjusted until no effect is produced on the galvanometer by making contact with this key, it follows that the conjugate condition holds, and therefore $R = PS/Q$, so that R is determined. This is the principle of Mance's method.

Fig. 69 gives a diagram of the arrangement. Fig. 70 shews how the connections are made with the Wheatstone-bridge box.

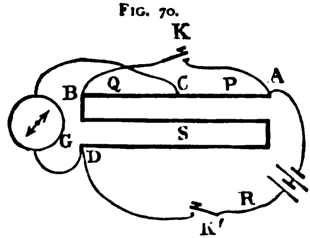
FIG. 69.



The method of procedure is as follows :—

Make the arms P and Q equal. Make contact in the battery circuit with the key K′. Since any resistance which may exist in this key will of necessity be included in the measurement of the resistance R, it is important that its resistance should be small enough to be neglected. It is advisable to have a key in the circuit, for, as we have said already, it is always best to allow the current to flow through the coils only when actually required for the experiment.

FIG. 70.



Bring the spot of light back to the centre of the scale by the use of the control magnet and, *if requisite, by shunting the galvanometer.*

Determine thus two values of S differing by 1 ohm, between which R lies. It must be remembered that any variation in S alters the permanent current through the galvanometer, and therefore the control magnet may require readjustment each time S is changed.

Make the ratio P/Q ·1 and proceed in the same way to find the first decimal place in the value of R. Then make the ratio ·01 and find a second decimal.

One difficulty requires special notice. It is true that making or breaking contact in the circuit A B will, if the conjugate condition hold, have no direct effect on the current in C D. It does, however, alter the total amount of

current which is being produced by the battery. When AD is closed an additional circuit is open for this current; now with most batteries the E.M.F. depends somewhat on the current which the battery is producing, that is, on the rate at which chemical changes are going on in it; so that when the battery is called upon to do more work by the closing of the circuit AB, its E.M.F. is gradually altered and the permanent deflexion is thereby changed. On making contact with the key the spot of light may move, not because the conjugate condition is not satisfied, but because of this change in the E.M.F. of the battery. This is a fundamental defect in the method, and prevents the attainment of results of the highest accuracy. The difficulty may be partially obviated as follows:—It will be found that the displacement produced through the conjugate condition not being satisfied is a somewhat sudden jerk, while that which arises from variation in the E.M.F. is more gradual in its nature. A little practice is all that is required to recognise the difference between the two. Now it will always be possible to arrange the resistances so that the two displacements are in opposite directions. Let us suppose that it is found that when S is too large on making contact the jerk is to the right; the gradual deflexion to the left. Gradually decrease S until the jerk appears to be zero, and the spot seems to move steadily to the left, and take the value of S thus found as the one required. The results thus obtained will be found fairly consistent.

A more exact method for overcoming the difficulty, due to Professor O. J. Lodge, was described by him in the 'Philosophical Magazine' of 1876. This, however, involves the use of a specially constructed key, and for an account of it the reader must be referred to the original paper.

Experiment.—Determine by Mance's method the resistance of the given battery.

Enter results thus:—

1 Leclanché cell (a) . . . . 1·21 ohm

1 Leclanché cell (*b*)	1·09 ohm
1 Sawdust Daniell	10·95 „
1 Cylinder Daniell	·58 „

78. The British Association Wire Bridge.—Measurement of Electrical Resistance.

The apparatus used for measuring resistances by the Wheatstone-bridge method frequently takes another form. The theory of the method is of course the same as when the box is employed, but instead of varying the resistance S, the ratio P/Q is made capable of continuous alteration.

The conductors B C, C A of figure 64 are two portions of a straight wire of platinum-silver or German-silver, or some other material of a high specific resistance, which is carefully drawn so as to have a uniform cross-section, the resistance of any portion of such a wire being proportional to its length. The ratio of the resistances P/Q will be the ratio of the two lengths A C/B C.

A sliding-piece or jockey moves along this wire, and by pressing a spring attached to it electrical connection with the galvanometer can be made at any desired point C of A B. Thus the ratio of A C to B C can be made to have any value by altering the position of the point C along this wire.

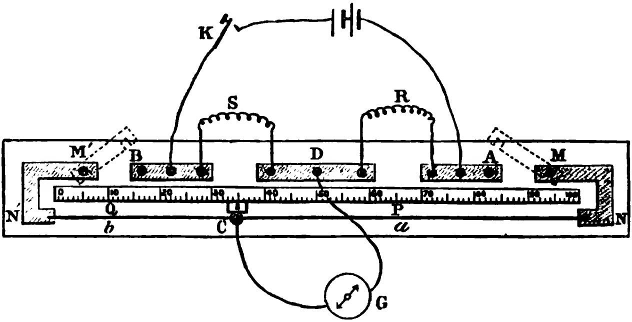
A scale, usually divided to millimetres, is fixed parallel to the wire; the ends of the wire A and B coincide with the extremities of the scale; and the position of the point C, at which the contact is made, can be read by means of a mark on the sliding-piece. The ends of this wire are fixed to stout copper pieces, by means of which connection is made with the resistances R and S. These copper strips are so thick that for many purposes their resistance may be neglected when compared with that of the wire A C B.

The apparatus usually takes the form shewn in fig. 71.

The strips N M A, N′ M′ B are the stout copper pieces just referred to. It will be noticed that there are gaps left between M and A, M′ and B; their purpose will be explained

shortly (p. 560). When the bridge is used as described above, these two gaps are closed by two strips of copper, shewn by dotted lines in the figure, which are screwed tightly down to the fixed copper pieces. The wire R, whose resistance is required, and S, the standard, are electrically connected with the apparatus, either by means of binding screws or of mercury cups, as may be most convenient;

FIG. 71.



binding screws are also provided for the battery and galvanometer wires.

To make a determination of the value of R, close the gaps A M and B M′ and connect the resistances, battery, and galvanometer, as shewn in the figure. Close the battery circuit by the key K. Move the jockey C until a position is found for it, such that no deflexion is produced in the galvanometer on making contact at C. Let a and b be the lengths of the two pieces of the bridge wire on either side of C. Then we have

$$R/S = P/Q = a/b,$$

and

$$R = S\,a/b$$

The apparatus may conveniently be used to find the specific resistance of the material of which a wire is composed. For if R be the resistance, and ρ the specific resistance of a wire of length l and uniform circular cross-

section of diameter d, then the area of the cross-section is $\frac{1}{4}\pi d^2$, and we have

$$R = \frac{4\rho l}{\pi d^2};$$

so that

$$\rho = \frac{R\pi d^2}{4l}.$$

The value of R can be found by the method just described. The length of the wire may be measured with a steel tape, or other suitable apparatus, and the diameter d can be determined by the aid of the screw gauge. For great accuracy this method of finding the diameter may not be sufficient. It may be more accurately calculated from a knowledge of the mass, length, and density of the wire (see § 8).

The determination of R by the method just described is not susceptible of very great accuracy. The position of C cannot be found with very great exactness, and an error in this will produce very considerable error in the result.

It can be shewn as follows that the effect of an error x in the position of C produces least effect in the result when C is the middle point of the wire.

For let c be the whole length of the wire; then we have found that

$$R = S\frac{a}{c-a}$$

Suppose that an error x has been made in the position of C, so that the true value of a is $a+x$. Then the true value of R is R+X, say, where

$$R+X = S\frac{a+x}{c-a-x}.$$

Hence if we neglect terms involving x^2 we have

$$R+X = S\frac{a}{c-a}\left\{1+\frac{xc}{a(c-a)}\right\} = R\left\{1+\frac{xc}{a(c-a)}\right\}.$$

Hence

$$\frac{X}{R} = \frac{xc}{a(c-a)}.$$

Now it is shewn in books on Algebra that $a(c-a)$ is greatest when $a=c-a$, that is, when $a=\frac{1}{2}c$, or C is at the middle point of the bridge-wire; and in this case the ratio of X to R, that is, the ratio of the error produced by an error x in a to the resistance measured, is least when C is at the middle point. Thus the standard chosen for S should have approximately the same value as R. This may be conveniently arranged for by using a resistance-box for S and taking out plugs until the adjusted position of C is near the middle of the wire.

But even with this precaution the method is far from sensitive; the resistance of the wire N N′ is probably very small compared with the resistances R and S. Nearly all the current flows directly through the wire, and very little through the coils R and S. The greatest possible difference of potential between C and D is small, and the deflexion of the galvanometer will always be small.

To remedy this two other resistance coils are inserted in the gaps A M and B M′, the copper strips being removed. Suppose their resistances respectively are P′ and Q′, and suppose that the value of R is known approximately, or has been found from a rough observation as above. The values of P′, Q′ must be such the ratio of P′ to Q′ does not differ much from that of R to S.

Suppose that when the position of equilibrium is found the lengths of wire on either side of C are a and b, and that the resistance of a unit length of the wire is known to be σ. Then, if we neglect the resistances of the copper strips M N and M′ N′—these will be exceedingly small, and may be neglected without sensible error—the value of P will be $P'+a\sigma$, and that of Q, $Q'+b\sigma$, and we have

$$\frac{R}{S}=\frac{P'+a\sigma}{Q'+b\sigma}.$$

The value of R is thus determined, and it can be shewn that the error in the result produced by a given error in the position of C is much less than when there is no resistance between A and M, B and M′.

This method involves a knowledge of σ, the resistance of a centimetre of the bridge-wire. To find this the resistance of the whole wire may be measured with a Post-Office box, or otherwise, and the result divided by the length of the wire in centimetres. Another method of determining σ will be given in the next section.

Moreover, since $a\sigma$ and $b\sigma$ are small compared with P' and Q', it follows that, as stated above, the ratio R/S must not differ much from the ratio P'/Q'.

Experiments.

(1) Measure by means of a resistance box and the wire bridge the resistance of the given coils.

(2) Determine accurately the length of the given wire which has a resistance of 1 ohm.

(3) Determine also the specific resistance of the material of the wire.

Enter results thus :—

(1)

Nominal values	Observed values
1 ohm.	1·013 ohm.
10 "	10·22 "
20 "	20·18

(2) Length of wire given, 250 cm.

$$P' = 1 \text{ ohm.}$$
$$Q' = 2 \text{ "}$$
$$S = 1 \text{ "}$$
$$a = 43{\cdot}2$$
$$b = 56{\cdot}8$$
$$\sigma = {\cdot}0018 \text{ ohm.}$$
$$\therefore\ R = {\cdot}5129 \text{ "}$$

Length of wire having a resistance of 1 ohm = 487·4 cm.

(3) Same wire used as in (2). Diameter (mean of ten observations with screw gauge) = ·1211 cm.

Specific resistance, 23,640 abs. units

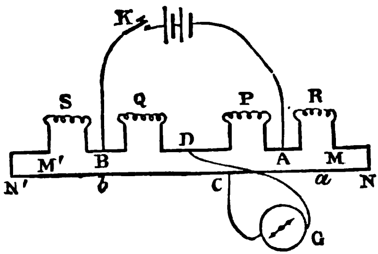
$$= 23640 \times 10^{-9} \text{ ohms.}$$

79. Carey Foster's Method of Comparing Resistances.

The B.A. wire bridge just described is most useful when it is required to determine the difference between two

nearly equal resistances of from one to ten ohms in value. The method of doing this, which is due to Professor Carey Foster, is as follows. Let R and S be the two nearly equal resistances to be compared; P and Q two other nearly equal resistances, which should, to give the greatest accuracy, not differ much from R and S.

FIG. 72.

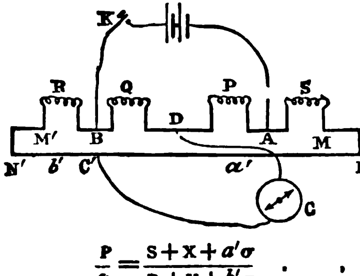


We do not require to know anything about P and Q except that they are nearly equal. It is convenient to have them wound together on the same bobbin, for then we can be sure that they are always at the same temperature.

Place R and S in the gaps A M, B M′ of the bridge, and P and Q in the gaps A D and D B respectively. Let a and b, as before, be the lengths of the bridge-wire on either side of C when the galvanometer needle is in equilibrium. Let X, Y be the unknown resistances of the two strips M N and M′ N′. Fig. 72 shews the arrangement. Then, if σ be the resistance of one centimetre of the bridge-wire, we have

$$\frac{P}{Q}=\frac{R+X+a\sigma}{S+Y+b\sigma} \quad . \quad . \quad . \quad . \quad (1)$$

FIG. 73.



Interchange the position of R and S and determine another position C′ (fig. 73), for the galvanometer contact in which there is no deflexion. Let a', b' be the corresponding values of a and b. Then

$$\frac{P}{Q}=\frac{S+X+a'\sigma}{R+Y+b'\sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

And by adding unity to each side we have, from equations (1) and (2)

$$\frac{\text{R}+\text{X}+a\sigma+\text{S}+\text{Y}+b\sigma}{\text{S}+\text{Y}+b\sigma}=\frac{\text{P}+\text{Q}}{\text{Q}}$$

$$=\frac{\text{S}+\text{X}+a'\sigma+\text{R}+\text{Y}+b'\sigma}{\text{R}+\text{Y}+b'\sigma} \quad . \quad . \ (3)$$

Also

$$a+b = \text{whole length of bridge wire} = a'+b' \quad . \quad . \ (4)$$

$$\therefore \ \text{R}+\text{X}+a\sigma+\text{S}+\text{Y}+b\sigma=\text{S}+\text{X}+a'\sigma+\text{R}+\text{Y}+b'\sigma \quad . \ (5)$$

Hence from (3)

$$\text{S}+\text{Y}+b\sigma=\text{R}+\text{Y}+b'\sigma;$$

$$\therefore \ \text{R}-\text{S}=(b-b')\sigma=(a'-a)\sigma, \text{ by } (4) \quad . \quad . \ (6)$$

Now $(a'-a)\sigma$ is the resistance of a portion of the bridge wire equal in length to the distance through which the sliding-piece has been moved. This distance can be measured with very great accuracy, and thus the difference of the resistances of the two coils can be very exactly determined.

To obtain all the accuracy of which the method is capable, it is necessary that the contacts should be good, and should remain in the same condition throughout. Mercury cups should generally be employed to make contact, and it is necessary that the electrodes of the various coils should be pressed firmly on to the bottoms of these either by weights, or, if convenient, by means of spring clamps.

At the three points C, N, N′, we have contacts of two dissimilar metals. These points are probably at different temperatures—the observer's hand at C tends to raise its temperature—and a difference of temperature in a circuit of different metals will, it is known, produce a thermo-electric current in the circuit. This current will, under the circumstances of the experiment, be very small; still, it may be a source of error.

The best method of getting rid of its effects is to place a commutator in the battery circuit, and make two observations of each of the lengths a and a', reversing the battery between the two. It can be shewn that the mean of the two observations gives a value free from the error produced by the thermo-electric effect.

Again, a variation in the temperature of a conductor produces an alteration in its resistance. For very accurate work it is necessary to keep the coils R and S at known temperatures. This is generally done by means of a water-bath, in which the coils are immersed.

It has been found that for most of the metals, at any rate within ordinary limits of temperature, the change of resistance per degree of temperature is very nearly constant, so that if R be the resistance of a coil at temperature t° C., R_0 its resistance at 0°, and a the coefficient of increase of resistance per degree of temperature, we have

$$R = R_0 (1 + a t).$$

Carey Foster's method is admirably adapted for finding this quantity a. The standard coil S is kept at one definite temperature, and the values of the difference between its resistance and that of the other coil are observed for two temperatures of the latter. Let these temperatures be t_1 and t_2, and the corresponding resistances R_1 and R_2; then we have

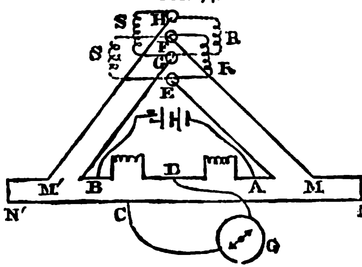
$$a = (R_1 - R_2) / R_0 (t_1 - t_2).$$

The observations have given us the values of $R_1 - S$ and $R_2 - S$ with great accuracy, and from them we can get $R_1 - R_2$; an approximate value of R_0 will be all that is required for our purpose, for it will be found that a is a very small quantity, and we have seen (p. 44) that we may without serious error employ an approximate value in the denominator of a small fraction.

Whenever precautions are requisite to maintain the coils at a uniform temperature, the interchanging of the

coils R, S is a source of difficulty with the ordinary arrangements. Time is lost in moving the water-jackets in which the coils are immersed, and the temperature may vary. The contacts, moreover, are troublesome to adjust. To obviate this, among other difficulties, a special form of bridge was devised by Dr. J. A. Fleming, and described in the 'Proceedings of the Physical Society of London,' vol. iii. The ordinary bridge may be easily adapted to an arrangement similar to Fleming's, as follows. E G F H (fig. 74) are four mercury cups; E and F are connected by stout copper rods with A and M, G and H with B and M′ respectively.

FIG. 74.



For the first observation the electrodes of R are placed in E and F being held in their position by weights or spring clamps, while the electrodes of S are in G and H.

For the second observation the electrodes of R are placed in G and H, those of S in E and F, as shewn by the dotted lines. This interchange is easily effected. The water jackets need not be displaced; the coils can readily be moved in them.

The connections A E, M F, &c., may conveniently be made of stout copper rod, fastened down to a board of dry wood, coated with paraffin. To make the mercury cups the ends of these rods are turned up through a right angle and cut off level. They are then amalgamated and short pieces of india-rubber tubing are slipped over them, and tied round with thin wire; the india-rubber tubing projects above the rod, and thus forms the cup. The other ends of the rods are made to fit the binding screws of the ordinary bridge.[1]

[1] For a fuller account of this and other similar contrivances, see *Philosophical Magazine*, May 1884.

Calibration of a Bridge-wire.

The method gives us also the best means of calibrating a bridge-wire. Make an observation exactly as above Alter the value of P slightly by inserting in series with it a short piece of German-silver wire. The only effect will be to shift somewhat the positions of C and C′ along the scale, and thus the difference between R and S is obtained in terms of the length of a different part of the bridge-wire. If the wire be of uniform section the two lengths thus obtained will be the same. If they are not the same, it follows that the area of the cross-section, or the specific resistance of the wire, is different at different points, and a table of corrections can be formed as for a thermometer (p. 242).

If the difference between the two coils be accurately known we can determine from the observations the value of the resistance of a centimetre of the bridge-wire. This is given by equation (6); for the values of $R-S$ and $a'-a$ are known, and we have

$$\sigma = (R-S)/(a'-a).$$

For this purpose the following method is often convenient. Take two 1-ohm coils and place in multiple arc with one of them a 10-ohm coil. Let the equivalent resistance of this combination be R; then the value of R is 10/11 ohms. Instead of interchanging the coils place the ten in multiple arc with the other single ohm and make the observation as before; then in this case we have

$$R - S = 1 - \frac{10}{11} = \frac{1}{11} \text{ ohm.}$$

and if l be the distance through which the jockey has been moved we obtain

$$\sigma = \frac{\cdot 09091}{l}.$$

Experiments.

(1) Calibrate the bridge-wire.

(2) Determine the average resistance of one centimetre of it.

(3) Determine accurately the difference between the resistance of the given coil and the standard 1-ohm at the temperature of the room.

Enter results thus :—

(1) Value of R – S for calibration, ·009901—being the difference between 1 ohm and 1 ohm with 100 in multiple arc—

Position of C	Value of $a' - a$
Division 20	5·48
" 40	5·49
" 60 . . .	5·51
" 80 . . .	5·52

(2) R – S = ·09091 ohm. l (mean of 5 observations) = 50·51 cm.

$$\sigma = ·00179 \text{ ohm.}$$

(3) Difference between the given coil and the standard at temperature of 15° C., observed three times.

Values ·0037, ·0036, ·00372 ohm. Mean ·00367 ohm.

80. Poggendorff's Method for the Comparison of Electromotive Forces. Latimer-Clark's Potentiometer.

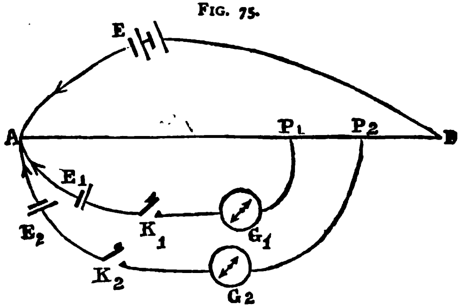
The method given in § 76 for the comparison of electromotive forces is subject to a defect similar to that mentioned in § 77, on the measurement of resistance; that is, it depends upon measuring the deflexion of a galvanometer needle, and assumes that the E.M.F. of the batteries employed remain constant throughout the experiment.

The following method, first suggested by Poggendorff, resembles the Wheatstone-bridge method for measuring resistances, in being a null method; it depends, that is to say, on determining when no current passes through a galvanometer, not on measuring the deflexion. We have seen

(p. 528) that if a current C be flowing through a conductor, the E.M.F. or difference of potential between any two points, separated by a resistance R, is C R.

Let A B (fig. 75) be a conductor of considerable resistance, through which a current is flowing from A to B; let P_1 be a point on this conductor, E_1 the difference of potential between A and P_1. If A and P_1 be connected by a second wire A G_1 P_1, including a galvanometer G_1 in its circuit, a current will flow from A to P_1 through this wire also. Let a second battery be placed in this circuit in such a way as to tend to produce a current in the direction P_1 G_1 A_1; the current actually flowing through the galvanometer G_1 will depend on the difference between E_1 and the E.M.F. of this

FIG. 75.



battery. By varying the position of P_1 along the wire A B, we can adjust matters so that no current flows through the galvanometer G_1; when this is the case it is clear that the E.M.F. E_1 of the battery is equal to the difference of potential between A and P_1 produced by the first battery. Let the resistance A P_1 be R_1, and let R be the resistance of A B, and ρ that of the battery which is producing the current through A B, including, of course, any connecting wires, E being the E.M.F. of this battery. Then, if C be the current in A B, we have

$$E_1 = C R_1 = E R_1/(R+\rho) \quad \text{(p. 528)},$$

or,

$$\frac{E_1}{E} = \frac{R_1}{R+\rho}.$$

This equation gives us, if we know ρ, the ratio E_1/E; for R and R_1 can be observed.

This method will be satisfactory in practice if R is very great compared with ρ, for then an approximate value of ρ will be sufficient; or if R is sufficiently large, ρ may be entirely neglected, and we may write $E_1/E = R_1/R$.

This is Poggendorff's method of comparing the E.M.F. of two batteries.

The following arrangement, suggested by Latimer-Clark, obviates the necessity for knowing ρ.

Let E_1, E_2 be the two E.M.F. to be compared, E that of a third battery, producing a current between the two points A and B; E must be greater than E_1 or E_2. Connect the three positive poles of the three batteries to A, the negative pole of E to B, and the negative poles of E_1 and E_2, through two galvanometers G_1 and G_2, to two points P_1, P_2 on AB; adjust the positions of P_1 and P_2 separately until no current flows through either galvanometer. It will be found convenient to have two keys, K_1, K_2, in the circuits for the purposes of this adjustment. Thus, positions are to be found for P_1 and P_2, such that on making contact simultaneously with the two keys there is no deflexion observed in either galvanometer. Let R_1, R_2 be the resistances of AP_1, AP_2 respectively, when this is the case. Then, C being the current in AB, we have

$$E_1 = C R_1, \quad E_2 = C R_2.$$

$$\therefore \frac{E_1}{E_2} = \frac{R_1}{R_2}.$$

By this method of procedure results are obtained entirely independent of the battery used to give the main current through AB.

The differences of potential actually compared are those between the two poles of the batteries respectively, when neither is producing a current.

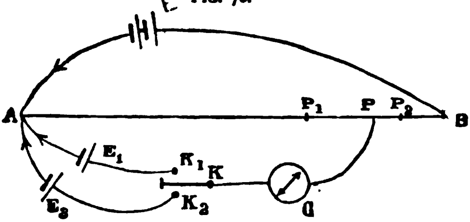
A convenient experimental arrangement for carrying out the comparison of electromotive forces on this method

as described by Latimer-Clark, has been called a 'potentiometer.'

The use of the two galvanometers is sometimes inconvenient, as it involves considerable complication of apparatus. In practice the following method may be adopted :—

Connect the three positive poles of the batteries to A and the negative pole of E to B (fig. 76). Choose for the battery E one which will give a fairly constant current through a large resistance, such as A B. Connect the two negative poles of E_1 and E_2 respectively to K_1, K_2, two of the binding screws of a switch. Connect K, the third screw of this switch, to one pole of the galvanometer G, and the other pole of the galvanometer to P, some point on A B. Make contact

FIG. 76.



between K and K_1, and find a position P_1 for P, such that the galvanometer is not deflected. Turn the switch across to make contact between K and K_2, and find a second position P_2, such that the galvanometer is again not deflected. Then, if we assume that E has not altered during the measurement R_1, R_2, being the resistances of A P_1 and A P_2, we have $E_1/E_2 = R_1/R_2$.

To eliminate the effect of any *small* change which may have occurred in E, reverse the switch again, putting K and K_1 into connection, and observe a second position P_1' for P_1; the two will differ very slightly if the apparatus be correctly set up. Let R_1' be the corresponding value of R_1; the mean $\frac{1}{2}(R_1 + R_1')$ will give a value corrected for the assumed small alteration in E.

For the resistance A B a long thin wire is sometimes used. This is either stretched out straight or coiled in a screw-thread cut on a cylinder of some insulating material. Contact is made at P by means of a sliding piece of metal. If this plan be adopted, it is somewhat difficult to get sufficient resistance between A and B for very accurate work. It is preferable, if possible, to use resistance boxes. Since the resistance A B is to be kept the same throughout the observations, two boxes are necessary. One of these forms the portion A P, the other the portion P B, the point P being the junction of the two. Having settled the total resistance A B, plugs are taken out of the two boxes to make up this total. The required adjustment is then attained by taking plugs, as may be needed, out of the one box A P, and putting plugs of the same value into the other box P B, or *vice versâ*, by putting plugs into A P and removing them from P B. In this way the total resistance A B remains unchanged.

In order to ascertain if the measurement be possible with the three given batteries, it is best to begin by making A P large and noting the direction of the deflexion; then make it small; the deflexion should be in the opposite direction. If this be the case, a value can be found for the resistance A P, such that the deflexion will be zero.

Experiment.—Compare by means of the last arrangement given above the E.M.F. of the two given batteries.

Enter results thus :—

Battery used for main current, two Daniell cells.

E_1 = E.M.F. of a Leclanché.
E_2 = E.M.F. of a Daniell.

Total resistance of A B, 2,000 ohms.

$R_1 = 1{,}370$ „
$R_2 = 1{,}023$ „
$R_1' = 1{,}374$ „

$$\frac{E_1}{E_2} = 1 \cdot 342.$$

W. The Clark Cell.

The cell devised by Mr. Latimer Clark has been shewn by numerous experiments to have a very constant electromotive force, which it retains, if properly treated, for a long time without serious change.

The positive pole of the cell is mercury. Contact is made with this by means of a platinum wire. The negative pole of the cell is zinc. This dips into a saturated solution of neutral zinc sulphate, which is also saturated with pure mercurous sulphate.

The mercury usually is placed at the bottom of a test-tube. The mercurous sulphate forms a paste above this, and on the top rests the saturated neutral zinc sulphate, which must contain visible crystals at all temperatures at which the cell is to be used. The liquid is also saturated with mercurous sulphate in solution. The zinc dips into the liquid, and the tube is closed with a cork, through which the zinc passes, being secured with marine glue, so as to be practically air-tight.

(1) *To set up a Clark Cell.*

The materials should all be chemically pure.

The Mercury.—To secure purity this should be first treated with acid and then distilled in vacuo (see note, p. 89). Pure vacuum-distilled mercury may, however, be purchased.

The Zinc.—Take a portion of a rod of pure (pure re-distilled) zinc. Solder to one end a piece of copper wire. Clean the whole with glass-paper, carefully removing any loose pieces of the zinc. Just before making up the cell, dip the zinc into dilute sulphuric acid, wash it with distilled water, and dry it with a clean cloth or filter-paper.

The Zinc Sulphate Solution.—Make a neutral saturated solution of pure zinc sulphate by mixing in a flask distilled water with about two-and-a-half times its weight of crystals of pure zinc sulphate. These crystals may be recrystallised before use ; it is generally, however, sufficient to employ

'pure recrystallised' zinc sulphate as purchased. To neutralise the solution, if slightly acid, a little zinc oxide may be added; the amount necessary depends on the acidity to be neutralised, and as zinc oxide must not be left in solution care is required. A method of avoiding the error to which the presence of the zinc oxide may lead is given below. The crystals of zinc sulphate should be dissolved by the aid of gentle heat, but the temperature should not be raised above 30°, and the solution should be filtered while still warm into a stock-bottle, any crystals which remain undissolved being thus removed. This solution is afterwards to be mixed with the mercurous sulphate, treated as described below. If the solution contains zinc oxide, when the mercurous sulphate is introduced mercurous oxide and zinc sulphate are formed; if this takes place in the cell, the mercurous oxide may be a source of error. To avoid it it is well to mix with the zinc sulphate solution, before it is filtered and while still warm, some of the mercurous sulphate which has been washed and treated as described below. If mercurous oxide is formed, it is removed from the solution, together with the undissolved mercurous sulphate, by the filtration, and the liquid in the stock-bottle is pure zinc sulphate containing mercurous sulphate in solution.

The Mercurous Sulphate.—Pure mercurous sulphate is a white powder. The salt as purchased is frequently greyish. This is due to the presence of metallic mercury in a finely divided state, and this is rather an advantage than otherwise, for it preserves the basicity of the salt. But it often contains large quantities of mercuric sulphate also, and this may be a source of considerable error. The mercuric sulphate is a dead white in colour, so that the colour of the mercurous sulphate is not a guide to its purity. The object of the following treatment is to get rid of this impurity. Mercuric sulphate when washed with water decomposes into an acid and a basic mercuric sulphate. The latter is a yellow substance (turpeth mineral) insoluble in water, and its presence (at any rate, to a moderate amount) is nc

seriously harmful. The acid sulphate is soluble in water, and if present the E.M.F. of the cell will not be normal.

The rationale of the following process for preparing the mercurous sulphate will now be intelligible.

Wash the mercurous sulphate with cold distilled water by agitation in a bottle; drain off the water. If the sulphate has turned yellow, the presence of acid mercuric sulphate is shewn. If a large quantity of the yellow turpeth mineral is formed, it means that there is a great deal of acid sulphate present, and it may be desirable to obtain a fresh supply of the mercurous sulphate. If the quantity of yellow salt formed is small, drain off the water, and repeat the process of washing twice at least. After the last washing drain off as much of the water as possible. This washing removes the soluble acid mercuric sulphate, and leaves only mercurous sulphate and the insoluble turpeth mineral. Mix the washed mercurous sulphate with the zinc sulphate solution, adding sufficient crystals of zinc sulphate from the stock-bottle to ensure saturation, and a small quantity of pure mercury; shake these up well together to form a paste of the consistency of cream. Heat the paste, but not above a temperature of 30°. Keep the paste for an hour at this temperature (this is best done by putting the bottle containing the paste into a water-bath kept at about 30°), and shake it well from time to time; then allow the paste to cool, but continue to shake the bottle occasionally, thus securing that the materials are all uniformly mixed. Crystals of zinc sulphate form as the paste cools; these should be distinctly visible and should be distributed throughout the mass; if this be not the case, add more crystals from the stock-bottle, and repeat the process of heating and mixing.

Contact is made with the mercury by means of a platinum wire about No. 22 gauge. This is protected from contact with the other materials of the cell by being sealed into a glass tube; the ends of the wire project from the tube; one end forms the terminal, the other end and a portion of the tube dip into the mercury.

The cell may conveniently be set up in a small test-tube of about 2 cm. diameter and 6 or 7 cm. deep. Place the mercury in the bottom of this tube, filling it to a depth of, say, 1·5 cm. Cut a cork about 0·5 cm. thick to fit the tube; at one side of the cork bore a hole, through which the zinc rod can pass tightly; at the other side bore another hole for the glass tube which covers the platinum wire; at the edge of the cork cut a nick through which the air can pass when the cork is pushed into the tube. Pass the zinc rod about 1 cm. through the cork.

Clean the glass tube and platinum wire carefully, then heat the exposed end of the platinum red hot, and insert it in the mercury in the test-tube, taking care that the whole of the exposed platinum is covered.

Shake up the mercurous sulphate paste and introduce it without contact with the upper part of the walls of the test-tube, filling the tube above the mercury to a depth of rather more than 2 cm.

Then insert the cork and zinc rod, passing the glass tube through the hole prepared for it. Push the cork gently down until its lower surface is nearly in contact with the liquid. The air will thus be nearly all expelled, and the cell should be left in this condition for at least twenty-four hours before sealing, which should be done as follows:—

Melt some marine glue until it is fluid enough to pour by its own weight, and pour it into the test-tube above the cork, using sufficient to cover completely the zinc and soldering. The glass tube should project above the top of the marine glue.

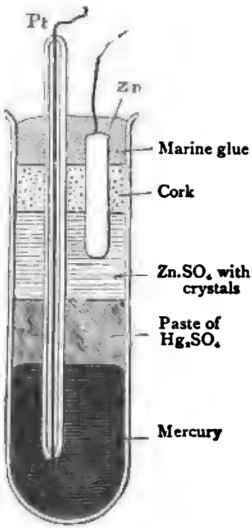
The cell thus set up may be mounted in any desired manner. It is convenient to arrange the mounting so that the cell may be immersed in a water-bath up to the level of, say, the upper surface of the cork. Its temperature can then be determined more accurately than is possible when the cell is in air.

Fig. xliii gives a drawing of the cell thus set up. The cell thus prepared should have, at a temperature of 15° C.,

an E.M.F. of 1·434 volts. The E.M.F. decreases as the temperature rises by ·00077 of its value per 1° C., so that at $t°$ the E.M.F. is

$$1{\cdot}434\{1 - {\cdot}00077(t-15)\}.$$

FIG. xliii.



If the cell is carefully set up in accordance with these instructions, its E.M.F. at the end of a week or so should be within 2 or 3 in 10,000 of this value. Some cells shew considerable changes of E.M.F. when first made. This is usually due to one of two causes: either the solution is acid (in this case the free acid attacks the zinc, and the evil cures itself), or there is zinc oxide in the solution. In this case mercurous oxide is formed, and may be deposited as a grey powder on the zinc. The E.M.F. falls greatly, and remains too low. Neither of these defects should be present if care has been taken to use neutral zinc sulphate, free from zinc oxide.

The cell in the form just described[1] is not suitable for use as a source of current. It is intended as a standard of E.M.F., though in many cases it may conveniently be employed to measure a current in the manner described in (3) below.

[1] The cell may be set up in various other forms. For standard purposes the H form devised by Lord Rayleigh is probably best. To secure portability, Professor Carhart introduces between the mercury and the paste a thin disc of cork, which fits the tube tightly; the cork must be well washed with warm water and left to soak in zinc sulphate solution before being used. Thus the mercury cannot become impure through contact with the zinc. Professor Carhart also coats the top of the marine glue with a thin layer of silicate of soda, which forms a hard and lasting glaze. The Berlin Reichsanstalt now use the H form with some modifications.

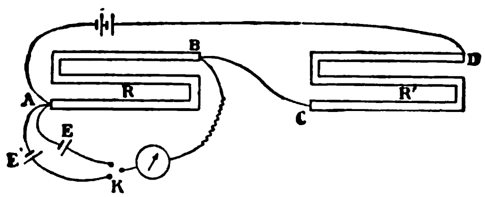
(2) *To use the Clark Cell as a Standard of E.M.F*

Poggendorff's method of comparing electromotive forces has been described in § 80. If one of the two cells employed be a Clark's standard, any other E.M.F. can be compared with this.

For many purposes it is desirable to use higher resistances than can be conveniently employed on a stretched wire bridge, and then the following method may be adopted :—

Connect up in series two resistance boxes A B, C D

FIG. xliv.



(fig. xliv) with a suitable battery. Let us suppose that we can take 10,000 ohms out of each box. The E.M.F. of the battery will depend on the value of the E.M.F. to be measured. If this be comparable with the E.M.F. of a Clark's cell, two Leclanché cells will be convenient. The boxes are to be used in such a way that the total resistance in circuit remains constant, and equal to say 10,000 ohms. Thus, if 4,500 be out in one box, 5,500 will be out in the other; and if an additional plug, say 5 ohms, is inserted in the first, the plug of the same value is taken out of the second.

The connexions are made as in fig. 76, the resistance from A to P in that figure being represented by one box, AB, that from P to B by the second box, CD. It is desirable to put a high resistance in with the galvanometer when commencing the experiment. If increased sensitiveness is

required, the resistance can be reduced or removed as the state of balance is approached. Fig. xliv shews the practical arrangement of the connexions for comparing two unequal electromotive forces E, E_1. If R be the resistance in one box, R' that in the second when there is no current through the galvanometer and the battery E is in circuit, and if R_1, R_1' are the corresponding resistances for E_1, then

$$\frac{E}{E_1} = \frac{R}{R_1}, \text{ and } R + R' = R_1 + R_1'.$$

If the two electromotive forces are nearly equal, as, for example, those of two Clark cells, a better method is to connect the two in opposite directions and compare the difference of their electromotive forces with the electromotive force of one of them or of a third standard cell.

(3) *To use a Clark Cell to Measure a Current.*

This is effected by passing the current through a wire of known resistance, and comparing the E.M.F. between the ends of the wire by the potentiometer method with that of the Clark cell. Let C be the value of the current, R the resistance employed, E the E.M.F. of the Clark. The potential difference between the ends of the wire is CR, and if R_1, R_2 be the potentiometer readings, as described in § 80, or in (2) above, we have

$$\frac{CR}{E} = \frac{R_1}{R_2}; \quad \therefore C = \frac{E}{R} \cdot \frac{R_1}{R_2}.$$

The resistance R should be so chosen that the E.M.F. CR may be comparable with that of the Clark. Moreover, since the current heats the wire, and the resistance changes with temperature, the size and material of the wire should be such as to make this change inappreciable.

In some cases it is more convenient and simpler to use the current to be measured as the main current of the potentiometer. In this case the current is passed through the potentiometer from A to B, the positive pole of the cell is connected to A, and the negative pole through the galvanometer to a point P on the wire such that no current passes

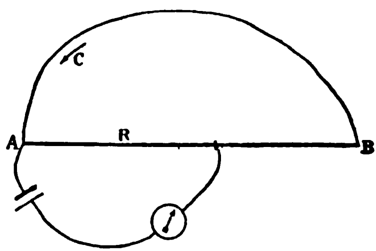
through the galvanometer. When this is the case, if R be the resistance of A P, C the current, and E the E.M.F. of the Clark cell, we have

$$E = CR;$$

$$\therefore C = \frac{E}{R}$$

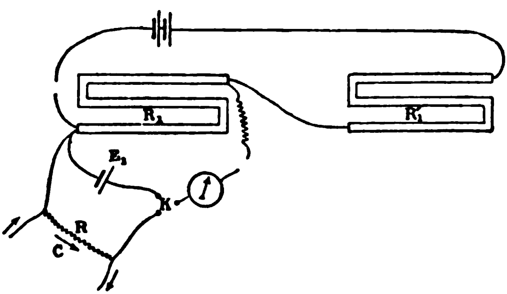
Figs. xlv and xlvi shew the connections. The stretched wire may, of course, be used instead of the resistance boxes in fig. xlvi.

FIG. xlv



The converse of this method is the one employed for determining absolutely the E.M.F. of a Clark or any other cell, for if in the above equation C and R are known absolutely, E is given in absolute units. C can be measured in various ways, e.g. as in § 71,

FIG. xlvi.



by the use of a tangent galvanometer, though this would probably not be the best method to adopt, or as in the next section, by the electrolysis of silver.

X. The Silver Voltameter.

We have discussed in § 72 the method of determining the reduction factor of a galvanometer, and thereby mea-

suring a current by the use of the copper voltameter. Now it is found that copper sulphate acts upon copper immersed in it, and this action, to some extent, depends on the current passing between the metal and the solution. In consequence, Faraday's fundamental law connecting the mass of copper deposited and the current is not exactly obeyed, and if great accuracy be required a correction has to be made. This correction is avoided by the use of the silver voltameter, in which silver is deposited on a platinum bowl, from a solution of pure nitrate of silver in water.

According to the experiments of Lord Rayleigh and Professor Kohlrausch, the value of the electro-chemical equivalent of silver in C.G.S. units—that is, the mass of silver deposited by the C.G.S. electromagnetic unit of current flowing for one second—is ·01118 grammes. For the measurement of current in ampères this number will require dividing by 10.

Method of Making a Measurement.

In employing the silver voltameter to measure currents of about 1 ampère the following arrangements should be adopted. The kathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10 cm. in diameter, and from 4 to 5 cm. in depth.

The anode should be a plate of pure silver, some 30 sq. cm. in area, and 2 or 3 mm. in thickness.

This is supported horizontally in the liquid near the top of the solution by a platinum wire, passed through holes in the plate at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling on to the kathode, the anode should be wrapped round with pure filter-paper, secured at the back with sealing-wax.

The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of salt to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an

effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

The platinum bowl is washed with nitric acid and distilled water, dried by heat, and then left to cool in a desiccator. When thoroughly dry it is weighed carefully.

It is nearly filled with the solution, and connected to the rest of the circuit by being placed on a clean copper support, to which a binding-screw is attached. This copper support must be insulated.

The anode is then immersed in the solution, so as to be well covered by it, and supported in that position; the connexions with the rest of the circuit are made.

Contact is made at the key, and the time of contact noted. The current is allowed to pass for not less than half an hour, and the time at which contact is broken is observed. Care must be taken that the clock used is keeping correct time during this interval.

The solution is now removed from the bowl, and the deposit is washed with distilled water, and left to soak for at least six hours. It is then rinsed successively with distilled water and alcohol, and dried in a hot-air bath at a temperature of about 160° C. After cooling in a desiccator it is weighed again. The gain in weight gives the silver deposited.

To find the current in ampères, this weight, expressed in grammes, must be divided by the number of seconds during which the current has been passed, and by ·001118.

The result will be the time average of the current, if during the interval the current has varied.

In determining by this method the reduction factor of an instrument, the current should be kept as nearly constant as possible, and the readings of the instrument taken at frequent observed intervals of time. These observations give a curve, from which the reading corresponding to the mean current (time average of the current) can be found. The current, as calculated by the voltameter, corresponds to this reading.

CHAPTER XXI.

GALVANOMETRIC MEASUREMENT OF A QUANTITY OF ELECTRICITY AND OF THE CAPACITY OF A CONDENSER.

WE have seen that if two points be maintained steadily at different potentials, and connected by a conductor, a current of electricity flows along the conductor and will produce a steady deflexion in a galvanometer, if there be one in the circuit. If, however, the difference of potential between the points be not maintained, the flow of electricity lasts for an exceedingly short time, sufficient merely for the equalisation of the potential throughout the conductor. A quantity of electricity passes through the galvanometer, but the time of transit is too short to allow it to be measured as a current in the ordinary way. The needle is suddenly deflected from its position of equilibrium, but swings back again through it directly, and after a few oscillations, comes to rest in the same position as before; and it is necessary for our purpose to obtain from theoretical considerations the relation between the quantity of electricity which has passed through the galvanometer and the throw of the needle.

On the Relation between the Quantity of Electricity which passes through a Galvanometer and the Initial Angular Velocity produced in the Needle.

Let K be the moment of inertia of the needle (p. 166), and suppose that it begins to move with an angular velocity ω. Then, as shewn in § D, p. 166, the moment of momentum of the needle is $K\omega$, and the kinetic energy $\frac{1}{2} K \omega^2$.

Now, by the second law of motion, the change of moment of momentum is equal to the moment of the impulse produced by the passage of the electricity,[1] and, by the principle

[1] The dynamics of the motion of the needle are also considered in discussion of the Ballistic pendulum, § F, p. 174.

of the conservation of energy, the kinetic energy is equal to the work which is done against the earth's horizontal force in reducing the needle to instantaneous rest at the extremity of its first swing. Let M be the magnetic moment of the galvanometer needle, G the galvanometer constant, Q the total quantity of electricity which passes, and β the angle through which the magnet is deflected. The moment of the force produced on the needle by a current γ is $MG\gamma$, and if this current flow for a time, τ, the impulse is $MG\gamma\tau$; but $\gamma\tau$ is the total quantity of electricity which flows through, and this has been denoted by Q.

Thus the impulse is MGQ, and if the time of transit be so short that we may assume that all the electricity has passed through the coils before the needle has appreciably moved from its position of rest—in practice with a suitable galvanometer this condition is satisfied—this impulse is equal to the moment of momentum, or $K\omega$.

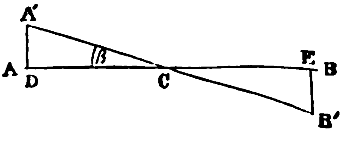
Thus

$$K\omega = MGQ. \quad . \qquad (1)$$

On the Work done in turning the Magnetic Needle through a given Angle.

Suppose first that the magnet consists of two poles, each of strength m, at a distance $2l$ apart. Let ACB (fig. 77) be the position of equilibrium of the magnet, $A'CB'$ the position of instantaneous rest, and let the angle $BCB' = \beta$.

FIG. 77.



Draw $A'D$, $B'E$ at right angles to ACB

Then the work done against the earth's magnetic field H, during the displacement, is $mH(AD + BE)$.

Now,

$$AD = BE = CA - CD = l(1 - \cos\beta).$$

Hence the work done

$$= 2\,m\,l\,\mathrm{H}\,(1 - \cos\beta).$$

The whole magnet may be considered as made up of a series of such magnetic poles, and if we indicate by Σ the result of the operation of adding together the effects on all the separate poles, the total work will be

$$\mathrm{H}\,(1 - \cos\beta)\,\Sigma\,(2\,m\,l).$$

From the definition of the magnetic moment (p. 442), it can readily be shewn that

$$\mathrm{M} = \Sigma\,(2\,m\,l).$$

Hence the total work will be

$$\mathrm{M}\,\mathrm{H}\,(1 - \cos\beta).$$

And this work is equal to the kinetic energy produced by the impulse, that is to $\frac{1}{2}\,\mathrm{K}\,\omega^2$.

So that

$$\tfrac{1}{2}\,\mathrm{K}\,\omega^2 = \mathrm{M}\,\mathrm{H}\,(1 - \cos\beta).$$

Thus from (1)

$$\frac{\mathrm{M}\,\mathrm{G}\,\mathrm{Q}}{\mathrm{K}} = \omega = \sqrt{\left\{\frac{2\,\mathrm{M}\,\mathrm{H}\,(1 - \cos\beta)}{\mathrm{K}}\right\}}. \quad . \quad . \quad (2)$$

Thus

$$\mathrm{Q} = \frac{2\sin\frac{1}{2}\beta}{\mathrm{G}}\sqrt{\left(\frac{\mathrm{H}\,\mathrm{K}}{\mathrm{M}}\right)}.$$

But if T be the time of a complete oscillation of the needle, and if we suppose that there is no appreciable damping, i.e. that the amplitude of any swing of the needle differs but very slightly in magnitude from that of the preceding, then since the couple acting on the magnet when displaced through a small angle θ is, approximately, $\mathrm{M}\,\mathrm{H}\,\theta$,

$$\mathrm{T} = 2\,\pi\sqrt{\left(\frac{\mathrm{K}}{\mathrm{M}\,\mathrm{H}}\right)} \quad . \quad . \quad (\text{p. } 166).$$

Hence substituting for K/M we find from (2)

$$\mathrm{Q} = \frac{\mathrm{H}}{\mathrm{G}}\,\frac{\mathrm{T}}{\pi}\sin\tfrac{1}{2}\beta. \quad . \quad . \quad (3)$$

If the consecutive swings decrease appreciably, then it follows, from the complete mathematical investigation (Maxwell, 'Electricity and Magnetism,' § 749), that we must replace $\sin \frac{1}{2}\beta$ in the above formula by $(1 + \frac{1}{2}\lambda) \sin \frac{1}{2}\beta$, where λ is quantity known as the logarithmic decrement, and depends on the ratio of the amplitudes of the consecutive vibrations in the following manner :—

If c_1 be the amplitude of the first and c_n that of the n^{th} vibration when the magnet, after being disturbed, is allowed to swing freely, then (Maxwell, 'Electricity and Magnetism,' § 736)

$$\lambda = \frac{1}{n-1} \log_e \left(\frac{c_1}{c_n}\right).$$

Thus we get finally

$$Q = \frac{H}{G}\frac{T}{\pi}(1 + \tfrac{1}{2}\lambda) \sin \tfrac{1}{2}\beta \quad . \quad . \quad . \quad (4)$$

We have used the symbol H for the intensity of the field in which the magnet hangs, though that field need not necessarily be produced by the action of the earth's magnetism alone ; we may replace H/G by k, the reduction factor of the galvanometer under the given conditions. Then, if k be known for the galvanometer used, and T, β and λ be determined by observation, we have all the quantities requisite to determine the quantity of electricity which has passed through. A galvanometer adapted for such a measurement is known as a ballistic galvanometer. In such a one, the time of swing should be long and the damping small. These requisites are best attained by the use of a heavy needle, supported by a long torsionless fibre of silk. For accurate work the deflexions should be observed by the use of a scale and telescope, as described in § 23.

We shall in the following sections describe some experiments in which we require to use the above formula to obtain the results desired.

On Electrical Accumulators or Condensers.

Consider an insulated conductor in the form of a plate, which is connected with one pole of a battery; let the other pole, suppose for clearness the negative one, be put to earth, it will be at zero potential. The plate will have a charge of positive electricity on it depending on its form, and its potential will be equal to the E.M.F. of the battery.

Take another plate, connected with the earth, and bring it into the neighbourhood of the first plate. This second plate will be at potential zero, and its presence will tend to lower the potential of the first plate, and thus will produce a flow of positive electricity from the battery to the first plate, sufficient to raise its potential again to that of the positive pole of the battery. The quantity of electricity which thus flows in will depend on the form and relative position of the two plates, and the nature of the insulating medium which separates them. The flow of electricity will last but an exceedingly short time; and, if allowed to pass through a ballistic galvanometer, will produce a sudden throw of the needle of the nature described on p. 582. If β be the angle through which the needle is deflected, then, as we have seen, the quantity of electricity which passes is proportional to $\sin \frac{1}{2}\beta$.

It is not necessary to connect the negative pole of the battery and the second plate of the condenser to earth; it will be sufficient if they be in electrical communication with each other; in either case the difference of potential between the plates will be equal to the E.M.F. of the battery.

Neither is it necessary that the two plates of the condenser should be capable of being separated; the effects will be exactly the same if we suppose one plate to be in connection with the negative pole of the battery, and then make contact by means of a key between the second plate and the positive pole. The condenser can be discharged

by putting its two plates in metallic connection by means of a wire.

Moreover it can be shewn that if there be a quantity Q of positive electricity on the one plate of the condenser, there will be a quantity $-Q$ on the other. (See Maxwell's 'Elementary Electricity,' p. 72.) By the charge of the condenser is meant the quantity of electricity on the positive plate.

DEFINITION OF THE CAPACITY OF A CONDENSER.—It is found by experiment that the charge required to produce a certain difference of potential between the plates of a condenser bears a constant ratio to the difference of potential. This constant ratio is called the capacity of the condenser.

Thus if the charge be Q, the difference of potential between the plates V, and the capacity C, we have, from the above definition,

$$C = \frac{Q}{V}, \text{ or } Q = C\,V.$$

The capacity, as has been said, depends on the geometrical form of the condenser and the nature of the insulating medium. If the condenser take the form of two large flat plates, separated by a short interval, the capacity is approximately proportional to the area of the plates directly, and to the distance between them inversely.

Condensers of large capacity are frequently made of a large number of sheets of tinfoil, separated from each other by thin sheets of mica. The alternate sheets 1, 3, 5, &c., are connected together and form one plate; the other set of alternate sheets, 2, 4, 6, &c., being connected together to form the other plate. Sheets of paraffined paper are sometimes used instead of mica.

DEFINITION OF THE UNIT OF CAPACITY.—The unit of capacity is the capacity of a condenser, in which unit charge produces unit difference of potential between the plates.

The C.G.S. unit thus obtained is, however, found to be far too great for practical purposes, and for these the 'farad' has been adopted as the practical unit of capacity. The farad is the capacity of a condenser in which a charge of one coulomb—that is, the charge produced by an ampère of current flowing for one second—is required to produce between the plates of the condenser a difference of potential of 1 volt.

Since the quantity of electricity conveyed by an ampère in one second is 10^{-1} C.G.S. units and 1 volt$=10^8$ C.G.S. units, we have

$$1 \text{ farad} = \frac{1}{10 \times 10^8} \text{ C.G.S. units.}$$
$$= 10^{-9} \text{ C.G.S. units.}$$

Even this capacity, 1 farad, is very large, and it is found more convenient in practice to measure capacities in terms of the millionth part of a farad or a microfarad.

$$\text{Thus 1 microfarad} = \frac{1}{10^{15}} \text{C.G.S. units.}$$

On the Form of Galvanometer suitable for the Comparison of Capacities.

The capacities of two condensers are compared most easily by comparing the quantities of electricity required to charge them to the same difference of potential, being directly proportional to these quantities.

Now the quantity of electricity required to charge a condenser to a given difference of potential will not depend on the resistance of the conductor through which the charge passes. The same total quantity will pass through the wire whatever be its resistance; the time required to charge the condenser will be greater if the resistance be greater, but, even if the resistance be many thousand ohms, the time of charging will be extremely small.

The effect produced on the galvanometer needle by a given quantity of electricity will be proportional to the num-

ber of turns of the wire of the galvanometer; thus for the present purpose the galvanometer should have a very large number of turns. This, of course, increases its resistance; but, then, this increase does not produce any evil effect. A galvanometer of five or six thousand ohms may conveniently be used. The time of swing of the needle should be considerable; a period of from two to three seconds will give fair results.

For the comparison of two capacities the damping does not matter greatly; it will affect all the throws in the same manner. If, however, it be required to express the capacity of a given condenser in absolute measure, it will be necessary to use a galvanometer in which λ can be measured with accuracy. The time of swing, too, since it requires to be accurately measured, should be greater than that mentioned above.

81. Comparison of the Capacities of two Condensers.

(1) *Approximate Method of Comparison.*

Charge the two condensers alternately with the same battery through the same galvanometer, and observe the throws.

Let c_1, c_2 be the two capacities, β_1, β_2 the corresponding throws, the mean of several being taken in each case.

Then since the differences of potential to which the condensers are charged are the same for the two, we have (pp. 469, 471).

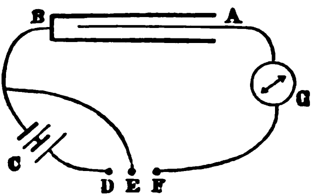
$$c_1 : c_2 = \sin \tfrac{1}{2}\beta_1 : \sin \tfrac{1}{2}\beta_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

For making contact a Morse Key is convenient.

In this apparatus there are three binding screws D, E, F (fig. 78) attached to a plate of ebonite, or other good insulating material, above which is a brass lever. F is in connection with the fulcrum of the lever, E with a metal stud under one end, and D with a similar stud under the other. A spring keeps the front end of the lever in contact with t

stud connected to E, so that E and F are, for this position of the lever, in electrical communication. On depressing the other end of the lever this contact is broken, and the end depressed is brought into contact with the stud connected with D. Thus E is insulated, and D and F put into communication.

FIG. 78.



In fig. 78, A and B are the two poles of the condenser, G is the galvanometer, and C the battery. One pole of the battery is connected with B, the other pole with D ; A is connected with the galvanometer G, and F with the other pole of the galvanometer, while B is also in connection with E. In the normal position of the key one pole of the battery, connected with D, is insulated and the two poles of the condenser B and A are in connection through E and F. Let the spot of light come to rest on the galvanometer scale, and observe its position. Depress the key, thus making contact between D and F, and observe the throw produced. The spot will swing back through the zero to nearly the same distance on the other side. As it returns towards the zero, and just before it passes it for the second time, moving in the direction of the first throw, release the key. This insulates D and discharges the condenser through the galvanometer, the electricity tends to produce a throw in the direction opposite to that in which the spot is moving, which checks the needle, reducing it nearly to rest. Wait a little until it comes to rest, and then repeat the observation. Let the mean of the throws thus found be δ_1.

Replace the first condenser by the second and make a second similar observation ; let the mean of the throws measured as before along the scale be δ_2.

To eliminate the effect of alteration in the E.M.F. of the battery repeat the observations for the first condenser, and let the mean of the throws be δ_1'. Now δ_1 and δ_1' should, if

the battery has been fairly constant, differ extremely little; the mean $\frac{1}{2}(\delta_1+\delta_1')$ should be taken for the throw.

Let D be the distance between the scale and the galvanometer mirror. Then, as we have seen (§ 71)

$$\delta = \text{D} \tan 2\beta$$

and

$$\beta = \tfrac{1}{2} \tan^{-1}\left(\frac{\delta}{\text{D}}\right);$$

so that

$$\sin \tfrac{1}{2}\beta = \sin\left\{\tfrac{1}{4} \tan^{-1}\left(\frac{\delta}{\text{D}}\right)\right\} \quad . \quad . \quad (2)$$

And if the ratio δ/D be small we may put $\frac{1}{4}\frac{\delta}{\text{D}}$ for $\sin\left\{\frac{1}{4} \tan^{-1}\left(\frac{\delta}{\text{D}}\right)\right\}$ (see p. 45).

Hence we find from (1) and (2)

$$\text{C}_1 : \text{C}_2 = \delta_1 : \delta_2.$$

With most condensers a phenomenon known as electric absorption occurs. The electricity appears to be absorbed by the insulating medium, and continues to flow in for some time: it is therefore better, in this case, to put the galvanometer between E and B. By depressing the key for an instant the condenser is charged, but in such a way that only the discharge passes through the galvanometer; or, if preferred, the galvanometer can be put between C and D, and only the charge measured; or, finally, the wires connected to D and E may be interchanged, the galvanometer being preferably between B and D; when in the normal position of the key, the condenser is charged, and a discharge, sudden or prolonged, is sent through the galvanometer on depressing the key. By these various arrangements the effects of alterations in the length of the time of charge or discharge can be tested. They all have the disadvantage that there is no ready means of checking the swing of the needle, and time is taken up in waiting for it to come to rest.

This may be obviated by a judicious use of a magnet held in the hand of the observer, and reversed in time with the galvanometer needle, or still better by having near the galvanometer a coil of wire in connection with a second battery and a key. On making contact with the key at suitable times the current in the coil produces electro-magnetic effects, by means of which the needle may gradually be stopped.

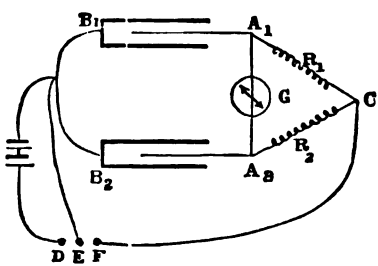
(2) *Null Method of Comparing Capacities.*

The method just given has the defects common to most methods which turn mainly on *measuring* a galvanometer deflexion.

The method which we now proceed to describe resembles closely the Wheatstone bridge method of measuring resistance.

Two condensers are substituted for two adjacent arms of the bridge; the galvanometer is put in the circuit which connects the condensers. Fig. 79 shews the arrangement of the apparatus. A_1 B_1, A_2 B_2, are the two condensers; B_1 B_2 are in connection with each other and with one pole of the battery; A_1, A_2 are connected through resistances R_1, R_2 respectively, to the point C, which is also in connection with F, one of the electrodes of the Morse key.[1] The second pole of the battery is connected with D on the Morse key, while E, the middle electrode of the key, is connected to B_1 and B_2. In the normal position of the key the plates of the condenser are connected through E and F. On depressing the key the contact between E and F is broken, and contact is made between D and F, and the condensers are thus charged.

FIG. 79.



[1] F is in connection with the axle of the key.

In general it will be found that on thus making contact the galvanometer needle is suddenly deflected. We shall shew, however, that if the condition $C_1R_1 = C_2R_2$ hold, C_1, C_2 being the two capacities, there will not be any current through the galvanometer, the needle will be undisturbed (see below). To compare the two capacities, then, the resistances R_1 R_2 must be adjusted until there is no effect produced in the galvanometer, by making or breaking contact, and when this is the case we have

$$C_1/C_2 = R_2/R_1,$$

and R_1, R_2 being known, we obtain the ratio C_1/C_2. In performing the experiment it is best to choose some large integral value, say 2000 ohms for R_1, and adjust R_2 only.

We proceed to establish the formula

$$C_1 R_1 = C_2 R_2.$$

No current will flow from A_1 to A_2 if the potential of these two points be always the same. Let V_0 be the constant potential of the pole of the battery in contact with B_1 and B_2, V_1 that of the other pole. Let V be the common potential of A_1 and A_2 at any moment during the charging, and consider the electricity which flows into the two condensers during a very short interval τ. The potential at C is V_1, and at A_1 and A_2 it is V at the beginning of the interval. The current along CA_1 will be then $(V_1-V)/R_1$, and along CA_2, $(V_1-V)/R_2$; and if the time τ be sufficiently small, the quantity which flows into the two condensers will be respectively $(V_1-V)\tau/R_1$ and $(V_1-V)\tau/R_2$. The inflow of this electricity will produce an increase in the potential of the plates A_1 and A_2; and since, if one plate of a condenser be at a constant potential, the change in the potential of the other plate is equal to the increase of the charge divided by the capacity, we have for the increase of the potential at A_1 and A_2 during the interval τ, when τ is very small, the expressions $(V_1-V)\tau/C_1R_1$ and $(V_1-V)\tau/C_2R_2$ respectively.

By the hypothesis A_1 and A_2 are at the same potential at the beginning of the interval τ, if the two expressions just found for the increment of the two potentials be equal, then the plates will be at the same potential throughout the interval.

The condition required is

$$\frac{(V_1-V)\tau}{C_1 R_1}=\frac{(V_1-V)\tau}{C_2 R_2},$$

and this clearly reduces to

$$C_1 R_1 = C_2 R_2.$$

Thus, if $C_1 R_1 = C_2 R_2$ the plates A_1, A_2 will always be at the same potential, and in consequence no effect will be produced on the galvanometer.

The complete discussion of the problem ('Philosophical Magazine,' May 1881) shews that the total quantity of electricity which flows through the galvanometer during the charging is

$$(V_1-V_0)(R_1 C_1 - R_2 C_2)/(G+R_1+R_2)$$

where G is the resistance of the galvanometer. It follows also that the error in the result, when using a given galvanometer, will be least when the resistances R_1 and R_2 are as large as possible; and that if we have a galvanometer with a given channel, and wish to fill the channel with wire so that the galvanometer may be most sensitive, we should make

$$G = R_1 + R_2.$$

The effects of electric absorption sometimes produce difficulty when great accuracy is being aimed at. They may be partially avoided by making contact only for a very short interval of time. For a fuller discussion of the sources of error reference may be made to the paper mentioned above.

Experiments.—Compaıe the capacities of the two condensers, (1) approximately; (2) by the null method last described.

Enter results thus :—

Condensers A and B.

(1) δ_1 (mean of 3 observations) 223 scale divisions.
δ_2 (mean of 6 observations) 156 " "
δ_1' (mean of 3 observations) 225 " "

$$\therefore \frac{C_1}{C_2} = \frac{224}{156} = 1{\cdot}44.$$

(2)
$$R_1 = 2000 \text{ ohms.}$$
$$R_2 = 2874 \text{ "}$$
$$\therefore \frac{C_1}{C_2} = \frac{2874}{2000} = 1{\cdot}437.$$

82. Measurement of the Capacity of a Condenser.

The methods just described enable us to compare the capacities of two condensers—that is, to determine the capacity of one in terms of that of a standard; just as Poggendorff's method (§ 80) enables us to determine the E.M.F. of a battery in terms of that of a standard. We have seen, however, in section 74 how to express in absolute measure the E.M.F. between two points; we proceed to describe how to express in absolute measure the capacity of a condenser.

Charge the condenser with a battery of E.M.F., E through a galvanometer, and let β be the throw of the needle, k the reduction factor of the galvanometer, T the time of swing, λ the logarithmic decrement, C the capacity of the condenser, and Q the quantity in the charge.

Then

$$C = \frac{Q}{E} = \frac{k\,T\,(1 + \frac{1}{2}\lambda)\sin\frac{1}{2}\beta}{\pi\,E}$$

by formula (4) of p. 585.

Shunt the galvanometer with $1/(n-1)$th of its own resistance G, so that $1/n$th only of the current passes through the galvanometer; let B be the resistance of the battery; pass a current from the battery through a large resistance R and

the galvanometer thus shunted, and let i be the current, θ the deflexion observed. Then we have

$$\frac{E}{R+B+\frac{G}{n}} = i = nk \tan\theta$$

for 1/nth of the current only traverses the galanometer, and produces the deflexion θ;

$$\therefore \frac{E}{k} = n\left(R + B + \frac{G}{n}\right)\tan\theta,$$

and

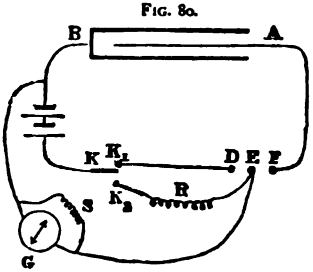
$$C = \frac{T(1 + \frac{1}{2}\lambda)\sin\frac{1}{2}\beta}{\pi . n\left(R + B + \frac{G}{n}\right)\tan\theta}.$$

The quantities on the right-hand side of this equation can all be observed, and we have thus enough data to find C.

To express C in absolute measure R, B, and G must be expressed in absolute units.

In practice B will be small compared with R, and may generally be neglected; n will be large, probably 100, so that an approximate knowledge of G will suffice. T may be observed, if it be sufficiently large, by the method of transits (§ 20), or more simply by noting the time of a large number of oscillations.

FIG. 80.



The method assumes that the value of E is the same in the two parts of the experiment. A constant battery should therefore be used, and the apparatus should be arranged so that a series of alternate observations of β and θ may be rapidly taken. Fig. 80 shews how this may be attained. One plate B of the condenser is

connected to one pole of the battery and to the galvanometer; the other plate A is connected to the electrode F of the Morse key. The other pole of the galvanometer is connected to the electrode E, so that in the normal position of the key the two plates are in connection through the galvanometer and the key E F.

The second pole of the battery is connected to one electrode K of a switch, and the electrode D of the Morse key is connected with another electrode K_1 of the switch. The centre electrode E of the key is connected through the resistance R to the third electrode K_2 of the switch. S is the shunt. With the switch in one position contact is made between K and K_1; on depressing the key the condenser is charged, the galvanometer being out of circuit, and on releasing the key the condenser is discharged through the galvanometer. Note the zero point and the extremity of the throw, and thus obtain a value δ for the throw, in scale divisions.

Shunt the galvanometer, and move the switch connection across to K_2. A steady current runs through the resistance R and the shunted galvanometer; let the deflexion in scale divisions be d; reverse the connections, and repeat the observations several times. The damping apparatus described in the previous section will be found of use. By measuring approximately the distance D between the scale and needle we can find tan θ and sin $\frac{1}{2}\beta$ in terms of d and δ. An approximate value only is required of D.

Experiment.—Determine absolutely the capacity of the given condenser.

Enter the results thus:—

D = 1230 scale divisions.
δ = 254·5 " " mean of 4 observations.
d = 266 " " " 2 "

Whence $$\frac{\beta}{2} = 2^\circ\ 55'{\cdot}35$$
$$\theta = 6^\circ\ 6'{\cdot}1.$$

Observations for λ:

$$n = 15; \quad c_1 = 220; \quad c_n = 60;$$
$$n = 10; \quad c_1 = 210; \quad c_n = 94.$$

Mean value of λ ·091.

Observations for T:

20 double vibrations take 64·5 seconds (same value for each of three observations).

$$T = 3{\cdot}225 \text{ seconds};$$

$$R = 5000 \text{ ohms} = 5 \times 10^{12} \text{ C.G.S. units};$$
$$G = 5600 \quad " \quad = 5{\cdot}6 \times 10^{12} \text{ C.G.S. units};$$

Battery 1 Daniell cell of negligible resistance

$$n = 100.$$

Whence $\qquad C = 1{\cdot}012$ microfarad.

CHAPTER XXII.

ELECTROMAGNETIC INDUCTION.

Whenever the number of lines of magnetic force which pass through the circuit of a closed conductor is varying, a current is produced in the conductor. The current is said to be due to electromagnetic induction. The electromotive force corresponding to the current is measured by the rate of decrease of the number of lines of magnetic induction which thread the circuit of the current; the measure of the current produced is obtained by dividing the electromotive force by the resistance.

To establish these results from theory the following propositions are needed :—

(1) The magnetic action of a circuit carrying a current is equivalent to that of a 'magnetic shell,' whose edge is bounded by the circuit and whose strength is a measure of the current.

(2) The potential of such a shell or circuit at any point is measured by the solid angle which the shell subtends at that point multiplied by the strength of the shell.

(3) The potential energy of a magnetic pole near such a shell, or of a shell when near a pole, is equal to the number of lines of magnetic induction due to the pole which thread the circuit in the positive direction, taken with a negative sign, multiplied by the current in the circuit.

(4) Hence the potential energy of a circuit carrying a current in any magnetic field is negative and numerically equal to the product of the strength of the current, and the number of lines of magnetic induction due to the field which thread the circuit in the positive direction.[1]

Suppose now that a circuit carrying a constant current i is caused to move in an invariable magnetic field, and that in time δt the number of lines of induction threading it increases from N to $\mathrm{N} + \delta\mathrm{N}$. If the circuit be free to move, it will do so under the electromagnetic forces acting, for it tends to take up a position in which the potential energy is as small as possible, i.e. in which N is as large as possible. The potential energy is decreased by an amount $i\delta\mathrm{N}$, and this amount of work can be done by the circuit against external forces—for example, in lifting a weight. The energy required for it is supplied by the battery, which is thus called on for an amount of work measured by $i\delta\mathrm{N}$. In the same time δt an amount of energy given by $\mathrm{R}i^2\delta t$ is expended in heating the wire. Thus the battery must supply an amount of energy equal to $i\delta\mathrm{N} + \mathrm{R}i^2\delta t$.

[1] For proofs of these and other connected propositions the reader is referred to S. P. Thompson, *Electricity and Magnetism*, ch. v.; or to Searle, *Determination of Currents in Absolute Electromagnetic Measure*; or Emtage, *Electricity and Magnetism*, pp. 92, 155.

But the work done by the battery in time δt is $\mathrm{E}i\delta t$, E being its electromotive force,

$$\therefore \ \mathrm{E}i\delta t = i\delta \mathrm{N} + \mathrm{R}i^2\delta t\,;$$

$$\therefore \ \mathrm{E} - \frac{\delta \mathrm{N}}{\delta t} = \mathrm{R}i,$$

$$i = \frac{1}{\mathrm{R}}\left(\mathrm{E} - \frac{\delta \mathrm{N}}{\delta t}\right).$$

Thus the effective E.M.F. is less by the amount $d\mathrm{N}/dt$ than it would be if the magnetic induction did not vary; there is an electromotive force of induction opposing E measured by the rate of increase of N, or an E.M.F. in the same direction as E measured by the rate of decrease of N.

It is immaterial how the variation in N be produced; it may arise from the motion of the circuit in a magnetic field, or from the motion of magnets near the circuit, or, again, from changes in electric currents circulating in neighbouring wires. If the magnetic induction through the coil arise from a current in a neighbouring wire, it will be proportional to the strength of this current so long as there is no iron or other magnetic material near; in this case, if i' be the current in the wire, the value of N may be written $\mathrm{M}i'$, where M is the number of lines of induction through the circuit produced by unit current.

The quantity M is known as the coefficient of mutual induction between the coils; it can be calculated in some simple cases from their form and relative position. The mutual potential energy of the coils is given by $-\mathrm{M}ii'$.

Again, if a circuit carry a current, lines of magnetic induction from the current pass through the circuit. If the current be increased, say, by reducing the resistance, the induction through the circuit is increased; the effect is equivalent to an electromotive force tending to check the rise of the current. This is the electromotive force of self-induction. If L measures the induction through a circuit due to unit current in itself, and i the current, then the rate

of decrease of induction is $-d(\text{L}\,i)/d\,t$, and this is the electromotive force of self-induction. L is the coefficient of self-induction, and is constant if there is no iron near.

When the current is being started, in consequence of this opposing electromotive force more energy is supplied by the battery than is used as heat in the wire, for the energy supplied in time $\delta\,t$, during which the current is i, is $\text{E}\,i\,\delta\,t$. Now

$$\text{E} = \text{R}\,i + \frac{d\text{N}}{d\,t}.$$

Thus work is done by the battery at the rate of

$$\text{R}\,i^2 + i\frac{d\text{N}}{d\,t}.$$

Let us consider the case of two circuits, in which the currents are i, i', and the electromotive forces E, E$'$. Let N, N$'$ be the number of lines of magnetic induction through the two circuits respectively, L, L$'$ their coefficients of self-induction, M the coefficient of mutual induction.

Then N is made up of the lines due to the circuit itself, together with the lines due to the second circuit. Thus

$$\begin{aligned}
\text{N} &= \text{L}\,i + \text{M}\,i', \\
\text{N}' &= \text{M}\,i + \text{L}'\,i', \\
\text{E} &= \text{R}\,i + \frac{d\text{N}}{d\,t}, \\
&= \text{R}\,i + \text{L}\frac{d\,i}{d\,t} + \text{M}\frac{d\,i'}{d\,t}, \\
\text{E}' &= \text{R}'\,i' + \frac{d\text{N}'}{d\,t}, \\
&= \text{R}'\,i' + \text{M}\frac{d\,i}{d\,t} + \text{L}'\frac{d\,i'}{d\,t}.
\end{aligned}$$

The solution of these two equations gives us the values of i, i', when E, E$'$ and the other quantities are known.

Again, the total work done by the two batteries in time $\delta\,t$ is

$$(\text{E}\,i + \text{E}'\,i')\,\delta\,t;$$

and from the above two equations we get

$$(\text{E}i + \text{E}'i')\,\delta t = (\text{R}i^2 + \text{R}'i'^2)\,\delta t + \left\{\text{L}i\frac{di}{dt} + \text{L}'i'\frac{di'}{dt} + \text{M}i\frac{di'}{dt} + i'\frac{di}{dt}\right\}\delta t.$$

Hencé, taking the sum of this from the instant at which the currents start to a time t, at which they have reached their steady values i, i', we have

Total work done in t seconds

$$= \Sigma(\text{R}i^2 + \text{R}'i'^2)\delta t + \tfrac{1}{2}\text{L}i^2 + \text{M}ii' + \tfrac{1}{2}\text{L}'i'^2.$$

The first two terms give the amount of work which has been spent as heat, the last three express what Maxwell has called the electrokinetic energy of the system.

The space in the neighbourhood of a wire carrying a current is permeated with lines of magnetic induction; to produce these lines energy is required, and the amount of this energy in the case under consideration is given by

$$\tfrac{1}{2}\text{L}i^2 + \text{M}ii' + \tfrac{1}{2}\text{L}'i'^2.$$

Thus we may define L either as the number of lines of induction which pass through each turn of a coil due to unit current in itself, or as twice the energy due to unit current which is used in producing lines of magnetic induction in the space surrounding the coil when at a distance from other conductors.

We have denoted by N the number[1] of lines of magnetic induction traversing a circuit. In the case in which the magnetic induction is due to a current the magnetising force will be proportional to the current, and this statement will, if there be no magnetic material in the neighbourhood, be true for the magnetic induction. But since in a magnetic material such as iron the magnetic induction is not proportional to the magnetising force, it follows that in the case of a circuit containing iron N is not proportional

[1] In calculating N the number of turns of wire in the circuit must be considered.

to the current, and the expressions we have deduced for the electrokinetic energy no longer hold.

The relation between the magnetic induction and the current producing it has been discussed in § V, and we have seen that it depends on the past history of the iron as well as on the magnetising force to which it is at a given time subject. We cannot therefore express N in terms of the currents directly, at any rate in any simple manner, and we cannot therefore solve the problem as completely as in the case in which there is no magnetic material present. The discovery of electromagnetic induction was made by Faraday, and independently to a great extent by Henry, in America.

The values of the coefficient of self-induction of a coil or of mutual induction between two coils can be approximately calculated in some cases. Thus, in the case of a long uniform solenoid of length l cm. containing n turns of wire, we know that except near the ends the magnetic force is uniform, and is equal to $4n\pi i/l$. Let A be the area bounded by a single turn of the wire near the centre of the solenoid. Then the number of lines of induction through A is $4n\pi i\text{A}/l$. Near the ends of the solenoid the force ceases to be uniform, and this expression does not hold, but if the solenoid is long compared with its radius, we may take it as an approximate value. Thus, since there are n turns of wire, through each of which $4n\pi\text{A}/l$ lines of induction pass per unit current, the coefficient of self-induction is n times this quantity, and we have approximately

$$\text{L} = \frac{4\,n^2\,\pi\,\text{A}}{l}.$$

If each coil be a circle of radius r, then $\text{A} = \pi r^2$, and

$$\text{L} = \frac{4n^2\pi^2 r^2}{l}.$$

It will be noticed that on this system of units a co-

efficient of self-induction has the dimensions of a *line*—that is, it is properly measured as so many centimetres. Thus, for example, let $l = 25$ cm., $r = 2\cdot 5$ cm., $n = 100$. Then we find as the approximate value for L,

$$\text{L} = 98{,}700 \text{ centimetres.}$$

Again, suppose we have a second coil whose area is A′ and number of turns n' placed within the first, the axes of the two being parallel. Then through each square centimetre of each turn of this coil, when unit current is flowing in the outer coil, $4n\pi/l$ lines of induction pass. The total number of lines of induction through the second coil, or the coefficient of mutual induction, is thus approximately

$$\text{M} = \frac{4nn'\pi \text{A}'}{l}.$$

We proceed to describe various measurements in illus tration of the laws of electromagnetic induction.

Y. Experimental Laws of Electromagnetic Induction.

Consider two circuits containing coils of wire; let one of them, the primary, be connected with a key to a battery, and suppose that i is the primary current. When contact is first made, i is zero. It rises very rapidly to its steady value, but during the time of change an electromotive force of induction is produced in the second coil, the secondary circuit. Let the ends of this coil be connected to a galvanometer, and let the resistance in circuit be R; let M be the coefficient of mutual induction between the two coils. Then the induced E.M.F. is $-\text{M}\dfrac{di}{dt}$, and the induced current due to the mutual induction is $-\text{M}\dfrac{di}{dt}/\text{R}$.

Now this current lasts for so short a time that we cannot measure its value at a given instant with the galvanometer. The effect on the needle will be impulsive, and if the needle

be not too light the whole of the current will have passed before the needle has moved appreciably from its equilibrium position.

The total quantity of electricity, however, can be measured; if this be Q, then we have, with the notation of § 82, p. 585,

$$Q = \frac{HT}{G\pi}(1 + \tfrac{1}{2}\lambda)\sin\tfrac{1}{2}\beta \quad . \quad . \quad . \qquad (1)$$

But the total quantity of electricity passing is Mi/R. Thus we get

$$\frac{Mi}{R} = \frac{HT}{G\pi}(1 + \tfrac{1}{2}\lambda)\sin\tfrac{1}{2}\beta \, . \quad . \quad . \qquad (2)$$

Now $\frac{HT}{G\pi}(1 + \frac{1}{2}\lambda)$ is a constant for a given galvanometer; and if we are using a mirror galvanometer, then $\sin\frac{1}{2}\beta$ is very approximately proportional to δ, the deflexion of the spot of light. We may therefore write

$$\frac{Mi}{R} = k\delta \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

where k is a quantity depending only on the galvanometer.

Thus, according to the above theory, the first throw of the galvanometer is proportional to the primary current, inversely proportional to the resistance in the secondary, and directly proportional to a quantity M, which, we shall shew, depends only on the relative position of the two circuits.

We wish to verify this law, and, in the first place, to prove that

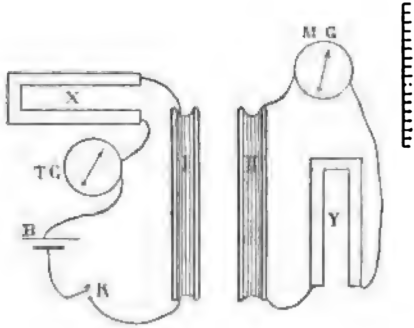
(1) *The quantity of electricity traversing the secondary is directly proportional to the primary current.*

For this purpose connect up, as the primary circuit (fig. xlvii), a battery B, of fairly constant E.M.F. (a Daniell's cell will do), a tangent galvanometer TG, a resistance box X, a key K, and the primary coil I. The latter may conve-

niently be a coil of insulated wire of some few hundred turns, about 15 to 20 cm. in diameter.

The secondary circuit consists of a similar coil I I, as the secondary, a mirror galvanometer M G, and a resistance box Y, connected together. The resistances of the mirror galvanometer and of the secondary coil will be required. If they are not known, measure them as described in § 77.

FIG. xlvii.



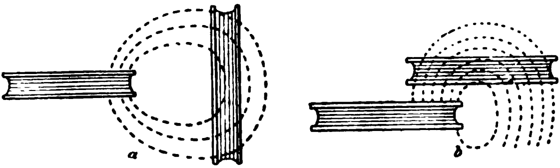
A damping circuit, by aid of which the motion of the galvanometer needle may be stopped, should be arranged as described in § 81.

Bring the spot of light to rest, the primary circuit being broken, then make the primary circuit, and leave it made. In general, a sudden throw of the spot will be observed. It is, however, possible to find an infinite number of positions for the secondary coil in which there is no throw. These are called conjugate positions; and the theory tells us that when the coils are in conjugate positions, on the whole, no lines of force from the primary cut the secondary —i.e. the number traversing it in the positive direction is equal to that which traverses it in the negative direction. Thus, if the axes of the two coils be at right angles, and the axis of one coil passes through the centre of the other, as shewn in fig. xlviii (*a*), the positions are conjugate; or, again, if the coils be as in fig. xlviii (*b*), the same is true.

Find, experimentally, a number of conjugate positions for the two coils.

Place the two coils in a position in which there is an induction throw observed. Break the secondary circuit, and leave it broken. Make the battery circuit, and leave it made. The spot of light should not be disturbed from

FIG. xlviii (*a*). FIG. xlviii (*b*).



its equilibrium position; if it be, the magnetic field produced by the primary coil is sufficiently strong to affect the galvanometer, and the coils must be moved into a position in which no such effect is produced. This can be done either by putting them sufficiently far away, or by arranging them so that the plane of the primary is horizontal, and at the same level as the galvanometer needle. In this position the lines of magnetic force due to the coil are vertical at the galvanometer, and do not affect the magnet.

When there is no direct effect of the primary on the galvanometer, make the secondary circuit again. On making the primary an induction throw, δ, is observed on the scale of the secondary galvanometer.

Observe this induction throw, and wait until the needle of the primary galvanometer has come to rest. Observe the deflexion; let it be θ. The current in the primary circuit is proportional to tan θ. Break the primary circuit. A second induction throw, δ', will be observed; and it will be found that δ' is equal in amount, opposite in direction, to δ. The quantity of electricity which traverses the secondary on breaking the primary is equal in amount, opposite in direction, to that which passes on making.

Repeat the observations a few times, and take the means; let them be δ_1, δ_1', and θ_1.

Alter the resistance in the primary by taking plugs out of or inserting them in the box X.

This will alter the primary current, and another set of observations, δ_2, δ_2', and θ_2 can be made. Again change the plugs, and continue in the same way.

Now the current in the primary is proportional to $\tan\theta$; write down, therefore, the values of $\tan\theta_1$, $\tan\theta_2$, &c., and plot a curve, taking these values as abscissæ and the corresponding throws δ_1, δ_2, &c., as ordinates. It will be found that the curve is a straight line through the origin. Thus the throw is proportional to the primary current, which is the first law we wished to verify. In arranging the experiment, the resistances and E.M.F. in the primary should be such as will produce convenient deflexions of the galvanometers. We may, for example, take as the values of θ, 30°, 40°, 45°, 50°, 55°, and 60°; the secondary galvanometer must be adjusted so that, while the smallest current to be observed gives a measurable throw, that given by the largest current may still be on the scale.

(2) *To shew that the quantity of electricity traversing the secondary is inversely proportional to the whole resistance of the secondary.*

For this purpose we make a similar series of observations to the last, with this difference. The resistance Y in the secondary circuit is varied instead of that in the primary. The whole resistance R of the secondary circuit is made up of S the resistance of the coil, G the galvanometer resistance, and Y taken from the box. Thus we have

$$R = G + S + Y.$$

By varying Y we vary R.

Observe, then, the throws δ_1, δ_2, &c., corresponding to a series of values R_1, R_2, &c. of R.

Write down the reciprocals of R_1, R_2, &c., and plot a

curve, taking $1/R_1$, $1/R_2$, &c., as abscissæ, and the throws δ_1, δ_2, &c., as ordinates. This curve will be found to be a straight line. Hence, when the relative position of the coils and the primary current remain constant, the quantity varies inversely as the resistance of the secondary.

We may, of course, prove the results just obtained without drawing the curves, by finding in the first case the series of values for $\delta/\tan\theta$, which should be all the same, and in the second the series of products $R.\delta$, these again should be equal.

(3) *To shew that the quantity of electricity traversing the secondary depends on the mutual position of the two circuits.*

Let the two coils occupy a position we will call A. Observe the throw produced by breaking the primary. Suddenly remove one of the coils from the A position to a conjugate position. A throw will be observed, and that throw will be exactly the same as was obtained by breaking the primary. Let it be α. For this experiment any conjugate position will do theoretically. It is not easy, however, to move the coil suddenly to an exactly defined position. It is best, therefore, to carry off the primary to a considerable distance from the secondary, where the magnetic field it produces is negligible. After repeating the experiment a few times, replace the coils in the A position, and move one of them suddenly from this into another position B, in which they are not conjugate. Let the throw observed be γ. With the coils in the B position observe the throw produced either by breaking the primary or by removing one coil to a considerable distance. Let it be β.

We shall find that γ is the difference between α and β, so that, supposing $\alpha > \beta$,

$$\gamma = \alpha - \beta.$$

Now, our first series of observations has shewn that

both α and β are proportional to i, and inversely proportional to R. We may therefore write

$$\alpha = \frac{M_1 i}{R}, \quad \beta = \frac{M_2 i}{R},$$

where M_1 and M_2 do not involve the primary current or the resistance of the secondary.

Thus we have

$$\gamma = (M_1 - M_2)\frac{i}{R}.$$

But $M_1 - M_2$ is the change produced in some quantity M by shifting the coils from the A position to the R. This shift has produced a change in the relative position of the coils, and we see, therefore, that when the primary current and the resistance of the secondary are kept constant the quantity of electricity in the induction current is proportional to the change which takes place in some quantity depending on the geometrical relations of the two coils. This quantity is M, the coefficient of mutual induction of the coils. To prove experimentally that M is measured by the number of lines of magnetic induction due to the one coil which thread the other, we should have to take a case in which we can determine by calculation this number, and then obtain M by direct measurement. We shall see shortly how this last measurement may be made, and then compare our results.

(4) *To examine the effect of the medium near the coils.*

In the above experiments we have supposed that, except the bobbins on which the coils are wound, there is only air in their immediate neighbourhood. We may repeat the experiments by placing pieces of various materials—wood, glass, brass, &c.—between the coils or in their immediate neighbourhood. We shall find that so long as there is no magnetic material near the results are unchanged. If, however, we bring iron between the coils, the induction

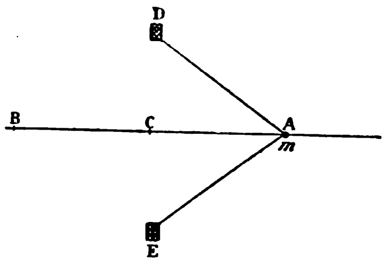
effects are enormously increased, for the magnetic induction in iron due to a given magnetic force is very much greater than in any other material.

(5) *Induction due to the motion of a magnet.*

We have described the induction effects produced by a current in a coil of wire. Similar results take place when a magnet is moved near a coil; these may be observed in a similar way. Thus, if we push the pole of a magnet through a coil, an induction throw is produced, lasting so long as the magnet continues to move. If we pull the magnet back, an equal throw is produced in the opposite direction.

Now let DE (fig. xlix) be a section of a circular coil of wire of n turns, C being its centre, ACB its axis, A and B being two points on the axis equidistant from C, and let the angle DAC $= \alpha$. Let there be a magnet pole of strength m at A, then the number of lines of induction from m which pass through all the turns of the coil is nm multiplied by the solid angle which A subtends at the coil, or $nm.2\pi(1-\cos\alpha)$.

FIG. xlix.



If m is moved to B the same number traverse the coil, but now they pass through it in the other direction; the solid angle subtended by the positive side of the coil is now $2\pi(1+\cos\alpha)$, and the change in the number of lines of induction passing is $4nm\pi\cos\alpha$. If, then, the theory be true, the impulse of the electromotive force round the coil is $4nm\pi\cos\alpha$, and the total flow of electricity is $4m\pi\cos\alpha/R$. If β be the throw of the galvanometer needle, we have

$$\frac{4nm\pi}{R}\cos\alpha = \frac{H.T}{G.\pi}(1+\tfrac{1}{2}\lambda)\sin\tfrac{1}{2}\beta.$$

Now if we could obtain a single isolated magnetic pole all the quantities in this equation might be measured, and the theory verified by seeing if the equation reduced to an identity. In reality this is impossible; but if the coil be not very large in diameter, and we take a very long thin bar magnet, say a meter or so in length, and place one pole at a moderate distance from the coil, say 10 cm. away, and then move it through to the same distance on the other side, the distant pole will not produce any great effect, and we may, approximately at least, treat the lines of force near the one pole as though they were due to that pole only. We can, of course, only obtain an approximate value for $\cos\alpha$, for the effect of the magnet, even if we neglect that of the distant end, is not with any strictness that of a single pole.

The quantity H/G, which comes into the equation, is, of course, the same as k, the reduction factor of the galvanometer, and can be found, as in § 82, by the use of a battery of known electromotive force.

If we have a condenser of known capacity as well, we can eliminate from our expression a number of the quantities it involves. For, charge the condenser of capacity C through the galvanometer, let the throw be θ, then

$$\mathrm{CE} = \frac{\mathrm{H}}{\mathrm{G}}\,\frac{\mathrm{T}}{\pi}\,(1+\tfrac{1}{2}\lambda)\sin\tfrac{1}{2}\theta.$$

Thus, comparing this equation with the previous one, we have

$$\frac{4 n m \pi \cos\alpha}{\mathrm{RCE}} = \frac{\sin\frac{1}{2}\beta}{\sin\frac{1}{2}\theta}.$$

The quantities in this equation can be all fairly accurately measured, and we obtain a means of verifying the law that the electromotive force of induction is measured by the rate of decrease in the number of lines of magnetic force through the coil

Z. Comparison of a Coefficient of Mutual Induction with the Product of a Resistance and a Time.

A method similar to that last described will enable us to measure electrically a coefficient of mutual induction, or rather, to be more exact, to compare it with the product of a resistance and a time.

For let us recur to the equation (2), of § Y, giving the relation between the coefficient of induction of two coils and the throw of the galvanometer needle. Let k be the reduction factor of the mirror galvanometer, k' that of the tangent instrument, and let θ be the deflexion of the tangent galvanometer; then

$$i = k' \tan \theta' \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the equation

$$\frac{\text{M}\,\dot{i}}{\text{R}} = \frac{\text{H}}{\text{G}}\,\frac{\text{T}}{\pi}(1+\tfrac{1}{2}\lambda)\sin\tfrac{1}{2}\beta \quad . \quad . \quad . \quad (2)$$

becomes

$$\text{M} = \frac{\text{R}\,.\,\text{T}\,.\,(1+\tfrac{1}{2}\lambda)\,k\sin\tfrac{1}{2}\beta}{\pi\,k'\tan\theta'} \quad . \quad . \quad . \quad (3)$$

If, now, we know k and k', the other quantities on the right-hand side can all be observed, and thus M is found in terms of R, and T the time of swing.

It will be observed that the ratio k/k' only is required. This may be determined directly by another electrical experiment. It will be found that in practice, in order to get convenient deflexions, the mirror galvanometer must be many times more sensitive than the tangent; thus k/k' is small.

Shunt the mirror galvanometer; let K be its resistance and S the resistance of the shunt, and connect it in series with a battery, the tangent galvanometer, and a resistance box. Take out of the resistance box plugs to give a convenient deflexion θ'' on the tangent galvanometer, and adjust the shunt until a suitable deflexion θ is obtained ir

the sensitive galvanometer. For this purpose it may be necessary to put a large resistance in series with the mirror galvanometer, and to shunt the whole.

The current through the mirror galvanometer is then equal to

$$\frac{S}{K+S} \times \text{current through tangent galvanometer};$$

$$\therefore\ k \tan\theta = \frac{S}{K+S} k' \tan\theta'';$$

$$\therefore\ M = \frac{R\,.\,T\,.\,(1+\frac{1}{2}\lambda)\,.\,S\,.\,\tan\theta''\,.\,\sin\frac{1}{2}\beta}{\pi\,.\,(K+S)\,.\,\tan\theta'\,.\,\tan\theta}\,. \quad (4)$$

This expression will be simplified if, by means of the resistance box, we make the current in the second part of the experiment equal to the primary current in the induction experiment; we have then $\theta' = \theta''$, and thus

$$M = \frac{R\,.\,T\,.\,(1+\frac{1}{2}\lambda)\,.\,S\,.\,\sin\frac{1}{2}\beta}{\pi\,(K+S)\tan\theta.} \quad . \quad . \quad . \quad (5)$$

In this experiment a value is found for M by electrical observations. If the experiment be performed with a pair of coils for which M can be calculated by direct measurement, we have a means of verifying the statement that the coefficient of mutual induction is equal to the number of lines of induction, due to unit current in the one coil, which pass through the other.

In comparing the results with calculation, care must be taken as to the units of measurement employed. T is, of course, in seconds; λ, π, $S/(K+S)$, $\sin\frac{1}{2}\beta/\tan\theta$, are all ratios of quantities of the same kind, and are therefore pure numbers; and we have only left R, the resistance. Now R will usually be measured in ohms; but the absolute unit of resistance based on the C.G.S. system is not 1 ohm, but 10^{-9} of an ohm. If, then, we wish to get M in absolute C.G.S. units, i.e. in centimetres, we must multiply the value of R in ohms by 10^9.

The experiment described is really one of the original methods by which the absolute resistance of a wire was determined. For suppose we work with a pair of coils of which the coefficient of mutual induction can be found by calculation based on the value of their radii and their relative position, then M is known in centimetres, and we may re-write the equation thus :—

$$R = M \frac{\pi(K + S)\tan\theta}{T(1 + \frac{1}{2}\lambda) S \sin\frac{1}{2}\beta} \quad . \quad . \quad . \quad (6)$$

The various ratios on the right-hand side can be observed, and the value of R found in terms of the units of length and time.

It follows from the above equation that the dimensions of R are those of a velocity.

It will readily be understood that to obtain high accuracy in a complicated experiment like the above various precautions, which have only been just alluded to, or even passed over in silence, are necessary. It is inserted here chiefly by way of introduction to the system of absolute measurement.

Experiment.—Determine in terms of a resistance and a time the coefficient of mutual induction of two coils.

Enter the results thus :—

R = Resistance of secondary = 5840 ohms.
K = 90000 + 5600 ohms.
S = 5·62 ohms.
λ = ·044.
T = 5·27 seconds.
β (in scale divisions) = 95 (mean of 4 observations).
θ „ „ = 163 „ „
Distance from mirror to scale, in scale divisions = 1000.
$\theta' = 38^\circ$, $\theta'' = 35^\circ\ 30'$.

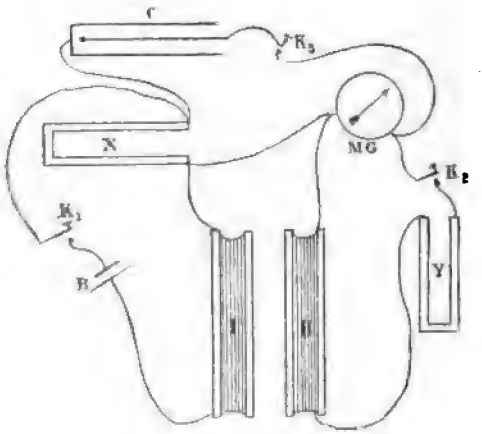
$$\frac{M}{10^9} = \frac{5840 \times 5{\cdot}62 \times 5{\cdot}27 \times 1{\cdot}022 \times 47{\cdot}5 \times {\cdot}7133}{3{\cdot}142 \times 95600 \times 163 \times {\cdot}7813}.$$

$M = {\cdot}1566 \times 10^9$ centimetres.

Γ. Comparison of a Coefficient of Mutual Induction with the Capacity of a Condenser and the Product of Two Resistances.

The method of making this measurement has been already indicated in § Y (1), fig. xlvii. In order to carry it out connect up the apparatus as in fig. l, placing a key, K_2, in the secondary circuit. Connect one set of plates of the

FIG. l.



condenser to one end of the resistance box X, the other set being connected through a key, K_3, to the mirror galvanometer M G. Connect the galvanometer to the other end of the resistance box X. Thus, when K_2 is made and K_3 broken, the galvanometer is in the secondary circuit; when K_3 is made and K_2 broken, it is in the condenser circuit. The tangent galvanometer in the primary circuit is not needed for the present experiment. To perform the experiment, make K_2 and break K_3. On making contact at K_1 we get

an induction throw in the galvanometer, and if θ_1 be the throw, we have—

$$\frac{Mi}{R}=\frac{HT}{G\pi}(1+\tfrac{1}{2}\lambda)\sin\tfrac{1}{2}\theta_1 \quad . \quad . \quad . \quad (1)$$

Now break the battery circuit; then break K_2 and make K_3. On again making the battery the condenser is charged through the galvanometer. The potential difference between the plates of the condenser will be that between the ends of the box X, or Xi. Thus the quantity of electricity which flows through the galvanometer is CXi, C being the capacity of the condenser; and if θ_2 is the throw,

$$CXi=\frac{HT}{G\pi}(1+\tfrac{1}{2}\lambda)\sin\tfrac{1}{2}\theta_2 \quad . \quad . \quad . \quad (2)$$

$$\therefore \frac{M}{CXR}=\frac{\sin\frac{1}{2}\theta_1}{\sin\frac{1}{2}\theta_2}=\frac{\delta_1}{\delta_2}. \quad . \quad . \quad . \quad (3)$$

very approximately, if δ_1, δ_2 be the two observed deflexions. Thus

$$\frac{M}{C}=XR\frac{\delta_1}{\delta_2}. \quad . \quad . \quad . \quad . \quad (4)$$

We have seen that in experiments with a condenser a high-resistance galvanometer is required. The quantity of electricity required to charge the condenser is small, and the same quantity will pass round the galvanometer, whatever be its resistance. By having a large number of turns this quantity is made to circulate round the needle a large number of times, and hence to produce a measureable effect. For measuring the induction current the galvanometer resistance need not be very large. In the above experiment we must use a galvanometer which will make the quantities δ_1 and δ_2 not very different.

Suppose that the condenser has a capacity of about 1 microfarad; a pair of coils such as those described in § Y may have a coefficient of mutual induction comparable with 10^8 centimetres, and 1 microfarad is equal to 10^{-15} C.G.S. units of capacity.

Thus the ratio M/C is of the order 10^{23}. The values of δ_1 and δ_2 should be of about the same order; thus the product of the two resistances X, R is 10^{23} in C.G.S. units, or, since 1 ohm $= 10^9$ C.G.S. units, the product XR in ohms is of the order $10^{23}/10^{18}$, or 10^5. Thus, if the resistance of the secondary circuit, including the galvanometer, is of the order 1000 ohms, then the resistance X will be comparable with 100 ohms. In order to obtain a sufficient current through this to affect the galvanometer appreciably several cells should be used.

Experiments.

Compare the coefficient of mutual induction between the given coils with the capacity of the given condenser.

Enter results thus :—

(1) R = 5840 ohms.
X = 100 ohms.
C = ·2 microfarad.
δ_1 = 127, 127, 127; mean 127.
δ_2 = 101, 100, 100; „ 100·3.
M = $·148 \times 10^9$.

(2) X = 20 ohms.
C = 1 microfarad
R = 5840 ohms.
δ_1 = 160, 160, 159; mean 159·6.
δ_2 = 122, 122, 123; „ 122·6.
M = $·152 \times 10^9$.

§ Δ. Comparison of Two Coefficients of Mutual Induction.

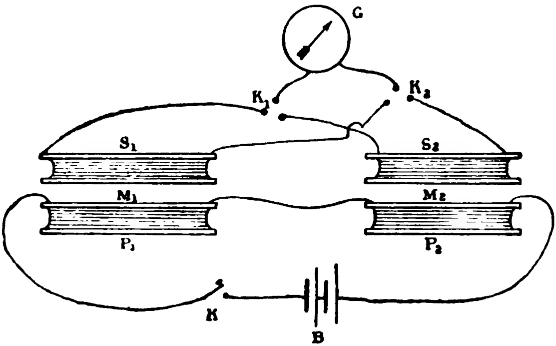
If the coefficient of mutual induction of one pair of coils be known, it may be used as a standard, and that of another pair under varying circumstances compared with it. Thus we could examine how the coefficient of a pair of coaxial circular coils changes as the distance between the coils varies. For this comparison various methods are available; we will consider two.

(1) Arrange the two pairs of coils so that the current in

the primary of one does not produce induction in the secondary of the other, and also so that neither primary affects the galvanometer directly.

Connect the two primaries in series with a battery and key (fig. li). Measure the resistance of each of the two

FIG. li.



secondaries and of the galvanometer; let the secondary resistances be S_1 and S_2, that of the galvanometer R, and let M_1, M_2 be the coefficients of induction. Connect the secondary S_1 to the galvanometer, and make contact in the primary circuit; an induction throw δ_1 will be observed. Measure this; let i be the primary current. Then if δ_1 is not too large, since the secondary resistance is $S_1 + R$, we have

$$\delta_1 \text{ is proportional to } \frac{M_1 i}{S_1 + R}.$$

Now break the connection between S_1 and the galvanometer, and connect S_2 in the same way. The resistance in the primary circuit is the same, so that if the battery be fairly constant the primary current will be the same, and we have

$$\delta_2 \text{ proportional to } \frac{M_2 i}{S_2 + R}.$$

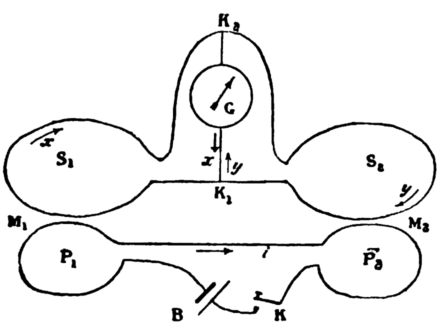
From this we obtain the result

$$\frac{M_1}{M_2} = \frac{\delta_1(S_1 + R)}{\delta_2(S_2 + R)}.$$

Fig. li shews the connections. K_1, K_2 are each two-way keys. One binding-screw of each key is in connection with the galvanometer. When the switches are to the left the coi S_1 is in circuit, when they are moved over to the right S_2 is put into circuit. Thus, to carry out the experiment, put the switches to the left. Make K, and observe the kick δ_1; then break K, put the switches K_1, K_2 to the right, make K again, and thus observe δ_2. In order to eliminate small variations in the primary current, if there are any, put S_1 again in circuit, and find a second value, δ_1', for δ_1. By taking a series of sets, and taking the means, we can compare M_1 and M_2 with some accuracy.

(2) The following plan, however, due to Maxwell, is free from some of the objections which can be made to the last, for (*a*) it is a null method—we do not require to measure two throws of the galvanometer, but only to adjust resistances till there is no throw; and (*b*) the result is independent of variations in the battery circuit. Fig. lii gives the plan of the connections; in fig. liii the method of realising this in practice is shewn.

FIG. lii.



The two primary coils P_1, P_2 are connected together in series with a battery B and a key K. The two secondaries are connected at K_1, K_2, and a galvanometer of resistance G connected between K_1 and K_2. When contact is made at K an in-

duction current x is produced in the secondary circuit S_1, which tends to flow through the galvanometer from K_2 to K_1. At the same time an induction current y is produced in the secondary circuit S_2, and the connections are so made that y would traverse the galvanometer from K_1 to K_2. Let us find the condition that the total flow through the galvanometer due to x may be equal to that due to y. The galvanometer needle will then remain at rest, and we shall find that the condition for this enables us to compare M_1 and M_2. Let L_1, L_2 be the coefficients of self-induction of the secondary circuits. Now we know from Kirchhoff's laws that if we take any circuit and multiply the current in each branch by the resistance of that branch, then the sum of the products thus formed is equal to the electromotive force round the circuit. From this it follows that if we multiply the total flow in each branch by the resistance of that branch, the sum of the products is equal to the total electromotive force. Now if δi is the rate of change of the primary current when contact is first made, and δx the rate of change of x, the electromotive force in the secondary circuit S_1 is equal to $M_1 \delta i - L_1 \delta x$, the last term is negative, for x and i are opposite in direction. This electromotive force lasts only so long as the primary current varies; when this has reached its steady value i, δi is zero. If, then, we denote by Σ the operation of finding the sum of the electromotive forces, the total value of the electromotive force is equal to $M_1 \Sigma \delta i - L_1 \Sigma \delta x$, and $\Sigma \delta i = i$, $\Sigma \delta x = 0$, for the induced current is zero at the beginning and end; therefore the total electromotive force is $M_1 i$.

Let $\bar{x}$ be the total flow in the circuit S_1, $\bar{y}$ that in S_2; the total flow through the galvanometer is $\bar{x} - \bar{y}$. Consider the circuit round S_1 and through the galvanometer. The flow round S_1 is $\bar{x}$, the resistance S_1; the flow from K_2 to K_1 is $\bar{x} - \bar{y}$, and the resistance G, while the electromotive force is $M_1 i$. Thus

$$S_1 \bar{x} + G(\bar{x} - \bar{y}) = M_1 i \quad . \quad . \qquad (1)$$

For the second circuit S_2 we have

$$S_2\bar{y} + G(\bar{y} - \bar{x}) = M_2 i \quad . \quad . \quad . \quad . \quad (2)$$

Divide the first equation by S_1, the second by S_2, and subtract; we get

$$(\bar{x} - \bar{y})\left(1 + \frac{G}{S_1} + \frac{G}{S_2}\right) = \left(\frac{M_1}{S_1} - \frac{M_2}{S_2}\right) i \quad . \quad (3)$$

This equation gives us the total flow through the galvanometer, and if this is to vanish so that the needle remains undisturbed, we must have $\bar{x} = \bar{y}$, and thus

$$\frac{M_1}{S_1} = \frac{M_2}{S_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We can secure this result by putting in series with one or both of the coils S_1, S_2 a resistance, and adjusting it until there is no throw. In this case S_1, S_2 mean, of course, the sum of the resistances of the coils and the adjustable resistances respectively.

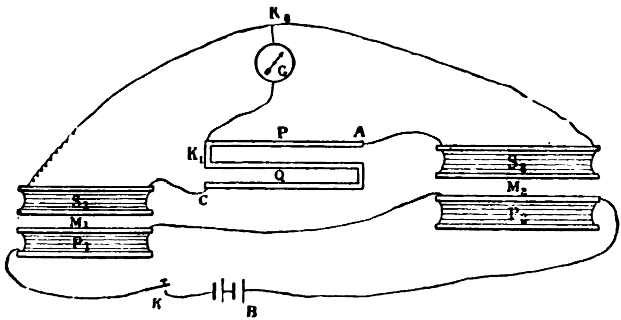
The above equation is not the condition that the current through the galvanometer at each moment should be zero. The actual value of the current involves, as we have seen, the self-induction of the secondary. While the current in the secondary is increased, the electromotive force of self-induction tends to stop it; as the current is decreasing, the electromotive force of self-induction helps it. Thus the total flow is not affected by self-induction, though the value of the current at any moment and the time for which it lasts are.

Fig. liii gives the practical details of the method. AK_1C is an ordinary resistance box, P being the resistance between A and K_1, Q that between K_1 and C. If a box of the ordinary post-office form be used, P would be a resistance of 10, 100, or 1,000 ohms, as might prove convenient; Q might be anything between 1 and 10,000. It may, of course, be more convenient to connect A to S_1, and C to S_2.

In making the measurements, the first point to attend

to is to see if the induction throws due to the two coils, respectively, are in opposite directions through the galvanometer. For this purpose break the connection between

FIG. liii.



A and S_2, make contact at K; an induction throw takes place; note its direction. Then join A and S_2, and break the connection CS_1; make contact again at K. The induction throw is now due to S_2 alone; if it is in the opposite direction to the first, the connections are right; if not, interchange the wires at the terminals of S_2. This will alter the direction through G of the induction current in S_2, and it will now oppose that due to S_1. When this is done, take a convenient resistance out of P, and adjust the resistance Q till there is no kick; then, if S_1, S_2 now mean the resistances of the secondary coils alone, which must be measured if they are not known, the total resistance on one side of $K_1 K_2$ is $S_1 + Q$, and that on the other is $S_2 + P$, so that we have

$$\frac{M_1}{M_2} = \frac{S_1 + Q}{S_2 + P},$$

and the required ratio is found.

Experiments.

Compare the coefficients of induction of the two given pairs of coils.

Enter results thus :—

(1) $S_1 = 232$, $Q = 18{\cdot}5$.
$S_2 = 10{\cdot}3$, $P = 6$.
$\frac{M_1}{M_2} = 15{\cdot}4$.

(2) $S_1 = 232$, $Q = 160{\cdot}5$
$S_2 = 10{\cdot}3$, $P = 15$.
$\frac{M_1}{M_2} = 15{\cdot}5$

§ Θ. The Earth Inductor.

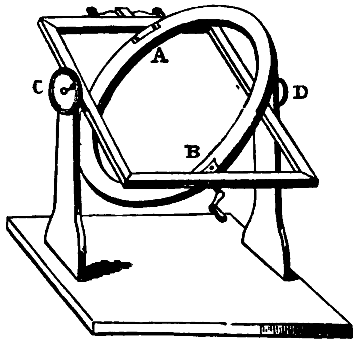
We know that a field of magnetic force exists at any point of the earth's surface. Thus, in general, lines of magnetic induction pass through any coil of wire, and by moving the coil a variation in their number is produced. This causes an electromotive force of induction round the coil, which, if the coil be closed, is the cause of an induction current. A knowledge of the strength of the induction current, of the constants of the coil and the galvanometer, and of the manner in which the coil has been moved, can be used to measure the strength of the magnetic field.

Thus, consider a circular coil of wire containing n turns. Let r be the mean radius of the coil. The area of each turn is πr^2, and the whole area is $n \pi r^2$ sq. cm. Let the coil be placed in a vertical plane, with its axis north and south, and let us call the face of the coil which is turned towards the north the positive face.

Fig. liv gives a drawing of the usual form of the instrument. The coil A B can turn round a diameter A B. This is attached to a rectangular frame, which can be turned round a horizontal axis C D at right angles to A B. A pointer, moving over a graduated circle at C, indicates the angle which the plane of the frame makes with the horizon. Suppose the axis C D to be set east and west, then, as the frame is turned, A B moves in the magnetic meridian, and can be inclined at any angle to the direction of the earth's magnetic field.

Let the axis A B be set vertical, and let H be the horizontal component of the earth's magnetic field.

FIG. liv.



In the given position H lines of induction pass through each square centimetre of the coil from the negative to the positive face.

Thus the total number traversing the coil is $n \pi r^2$ H. Turn the coil about the vertical diameter through a right angle, then the plane of the coil coincides with the magnetic meridian, and no lines of induction traverse it. Turn it through another right angle in the same direction; the positive face of the coil is now to the south, and again $n \pi r^2$ H lines of induction traverse it; but in this position they pass from the positive to the negative face. Thus the total change in the number of lines of induction passing from negative to positive is $2 n \pi r^2$ H, and this is the total electromotive force round the coil. Thus, if the ends of the coil be connected to a suitable galvanometer of reduction factor k, and if R be the resistance of the circuit, including the galvanometer, we have

$$\frac{2 n \pi r^2 \mathrm{H}}{\mathrm{R}} = \text{total flow round circuit}$$

$$= \frac{k \mathrm{T} (1 + \frac{1}{2}\lambda) \sin \frac{1}{2}\theta}{\pi} \quad . \quad . \quad . \quad (1)$$

if the coil be turned rapidly; and if the various constants involved are known, we can find H from this.

The method, however, would be a bad one for finding H, for the constants involved are numerous, and not easy to determine with accuracy. We may, however, use it with advantage for some other determinations. Thus

(1) *To determine the dip by the earth inductor.*

Arrange the experiment as already described, and observe the throw, θ_H, of the galvanometer needle. Now place the coil so that the axis about which it rotates is horizontal. The lines of induction which traverse it are those due to the vertical component of the earth's magnetic field. Let V be this component. Then on turning the coil through 180° the total change in the number of lines of induction is $2n\pi r^2 V$, and if θ_V be the throw, we have

$$\frac{2n\pi r^2 V}{R} = \frac{k(1+\frac{1}{2}\lambda)T}{\pi} \sin \tfrac{1}{2}\theta_V \quad . \quad . \quad (2)$$

Comparing this with (1), we have

$$\frac{V}{H} = \frac{\sin \frac{1}{2}\theta_V}{\sin \frac{1}{2}\theta_H} = \frac{v}{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

if v and h are the observed displacements of the spot on the scale, θ_V and θ_H being both small. But if i is the dip, we have

$$V = H \tan i\,;$$

$$\therefore \tan i = \frac{v}{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

(2) *To measure the Magnetic Induction at any point of a magnetic field by means of an induced current.*

The theory of the experiment is practically identical with that already described. With the earth inductor, however, the field is uniform over a considerable area, and the radius of the coil may, therefore, be considerable. In many cases we wish to measure the strength of a field which varies appreciably from point to point. For this purpose we take a small coil of wire, the area of which is known. Let it be A square centimetres. Place it in the field, and connect it with a suitable galvanometer; then suddenly remove it to a position in which no lines of induction traverse the coil. The throw of the galvanometer measures the total number of lines of induction through

the coil, and, assuming the induction uniform over the area, its value at each point can be found.

To determine the various constants involved the earth inductor may conveniently be used.

Thus, arrange in a single circuit the earth inductor, the small coil, and the galvanometer, the axis of the earth inductor being vertical, and the small coil in the magnetic field to be measured. This may, for example, be the field of an electromagnet. The resistance which comes into the formula will then be that of the whole circuit, but, as we shall see, its value disappears from the result. Let it be R. Let **B** be the induction per square centimetre through the coil.

Remove the coil from the field, and let the throw observed be θ_1; then the total change in the number of lines of induction is **B**A. Thus

$$\frac{\mathbf{B}A}{R} = \frac{k(1+\frac{1}{2}\lambda)T}{\pi}\sin\tfrac{1}{2}\theta_1 \quad . \quad . \quad . \quad (1)$$

Now rotate the earth inductor through 180°. Then, if θ_2 is the throw,

$$\frac{2n\pi Hr^2}{R} = \frac{k(1+\frac{1}{2}\lambda)T}{\pi}\sin\tfrac{1}{2}\theta_2 \quad . \quad . \quad (2)$$

$$\therefore \frac{\mathbf{B}A}{2n\pi Hr^2} = \frac{\sin\frac{1}{2}\theta_1}{\sin\frac{1}{2}\theta_2} = \frac{\delta_1}{\delta_2},$$

approximately, if δ_1, δ_2 are the deflexions.

$$\therefore \mathbf{B} = \frac{2n\pi Hr^2}{A}\frac{\delta_1}{\delta_2} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This method may be used to study the magnetic induction in iron or other materials, instead of that described in § U.

Details of the method will be found in the original papers of Rowland and Hopkinson, or in Ewing's 'Magnetic Induction in Iron and other Metals,' to which reference has been made already.

INDEX.

ABSORPTION, electric, 591
— of light, measurement of, 422
Acceleration due to gravity, method of measuring, 152, 163
Accumulator, 586
Acoustics, definitions, &c., 218
Air lines, spectrum of, 382
— thermometer, 265
Alluard, modification of Regnault's hygrometer, 310
Ampère, definition of, 496
Angle of a wedge of glass, measurement of, 372
— of incidence of light on a reflecting surface, 394
— of prism, method of measuring, 388
Angles, measurement of, 94
Approximation, 41
Aqueous vapour, chemical determination of density of, 301
— — tension of, 299
Areas, measurement of, 85
Arithmetical manipulation, 36
Arms of a balance, 116
Aspirator, 302
Astatic magnets, 508
Atwood's machine, 160

B.A. wire bridge, 557
— — —, Fleming's form of, 565
Balance, adjustment of, 103
—, hydrostatic, 123
—, Jolly's, 136
—, sensitiveness of the, 100, 113
—, testing adjustments of a, 114
—, theory of the, 99
Ballistic galvanometer, 585, 588
— pendulum, 174
Barometer, aneroid, 202
—, comparison of aneroid and mercury, 203
—, correction of readings, 200
—, measurement of height by the, 204
—, mercury, 198
Base line, measurement of, 73
Battery resistance, Mance's method, 553
Beam-compass, 63
Beats, 219
Binomial theorem, 42
Bi-prism, 399
Bi-quartz, 407
—, method of determining plane of polarisation by aid of, 408
Bird-call, 235
Boiling point, determination of, 247
— —, of a liquid, 250
Borda's method of weighing, 117
Boyle's law, 206
Bridge-wire, calibration of a, 566
Bunsen's photometer, 312
Buoyancy of air, correction of weighings for, 119
Burette, 93

CALIBRATION of a tube, 89
Calipers, 59
Calorimeter, description of, 274
Calorimetry, 271
Capacities, comparison of electrical, 589
—, Null method of comparing, 592
Capacity, absolute measurement of, 595
—, definition of electrical, 587
—, unit of electrical, 587
Capillarity, 210
Carey Foster's method of comparing resistances, 561
Caustic curve, 330
Circuit, primary, 604, 605
—, secondary, 604, 605
Clark cell, 572
Clifford, 4
Collimator, 305
Colour-box, 425
— top, 417
— vision, 417
— —, Lord Rayleigh's observations on, 420
— photometry, 427
Commutator, 514
Comparison of spectra, 381

Compressibility, 185
Concave mirror, focal length of, 337
Condenser, definition of capacity of, 587
Condensers, 586
Conductivity, 527
Conjugate positions, 606
Cooling, method of, 285
Cornu's prism, 414
Corrections, arithmetical calculation of, 39
Current, absolute measure of a, 457
— of electricity, 492

DALTON'S experiment, 294
— law, 299, 297
Damping, correction for, 585
Density, definition of, 121
—, determination of, by the volumenometer, 209
Deviation, position of minimum, 391
Dew point, 301
— — determination of, 307, 309
Diffraction, experiments, 404
— grating, 395
Dimensional equations, 24, 27
Dines's hygrometer, 306
Division, abbreviated form of, 38
Double weighing, 116
Drying, method of, 128
— tubes, 302

EARTH'S magnetic force, measurement of, 459, 461
Elasticity, theory of, 185
Electric battery, 491
— current, 491
— field, 487
Electrical decomposition of water, 517
— force, definition of, 488
— phenomena, 487
— potential, definition of, 489
— quantity, 487
— resistance, absolute unit of, 531
— —, definition of, 527
— — measured by Wheatstone's bridge, 549
Electricity, conductors of, 488
—, definition and explanation of terms, 381
Electro-chemical equivalents, 512
— —, comparison of, 517
Electrodes, 512
Electro-kinetic energy, 602
Electrolysis, 512
Electro-magnetic unit current, definition of, 494
Electromotive force, definition of, 489
— — measurement of, 522
— — of induction, 600, 604
— forces, methods of comparing, 541
— — compared by equal deflection method 542
Electromotive forces compared by equal resistance method, 542
— —, Poggendorff's method of comparing, 567
Equipotential surfaces, 437
Error, Mean, 34
Error, Probable, 34
Errors, 31
— calculation of effect of hypothetical, 46
—, special sources of, 48
Ether vapour, pressure of, 296
Expansion, 237
—, coefficient of, 255
—, measurement of, 256
— — — by a weight thermometer, 259
— of water, measurement of, 246
Extrapolation, 243

FARADAY'S law of electrolysis, 512
Fluid pressure, measurement of, 197
Fly-wheel, storage of energy in, 166
Focal lengths of lenses, 343
Focal lengths, 352
Focal lines, 354, 356, 371
Focussing for parallel rays, 361
Force, gravitation measure of, 12
—, comparison of magnetic and gravitational, 467
Freezing point, determination of, 247
Frequency of a note, 219
Fresnel's mirrors, 403
Fusing point of a solid, 251, 253
Funicular polygon, 182

GALVANOMETER, 495, 501
— adjustment of a, 510
—, adjustment of reflecting, 497
—, ballistic, 585
—, best arrangement for a tangent, 47
—, control magnet of a, 507, 509, 510
—, determination of reduction factor of a, 511
—, Helmholtz form, 506
—, reduction factor of, 507
—, relation between current and deflexion of a, 511
—, sensitiveness of a, 508
—, shunting a, 530
— constant, definition of, 503
— deflexion measured by reflexion of a spot of light, 498
— deflexions, methods of reading, 504
— for comparison of capacities, 588
— for strong currents, 507
— resistance, Thomson's method, 551
Gas flame, sensitive, 235
Gas-meter, 313
Gauss, law of magnetic force, 470
Glass tubes, methods of drying, 89

HARMONICS, 219
Heat, quantities of, 271
—, units of, 271
— generated by a current, 522
Height, measurement by the barometer, 204
—, measurement of, by hypsometer, 249
Homogeneous light, method of obtaining, 413
Horizon, artificial, 327
Humidity, relative, 300
Hydrometer, common, 139
—, Nicholson's, 133
Hygrometer, Dines's, 306
—, Regnault's, 309
Hygrometry, 299
Hypsometer, 247, 249
Hysteresis, 484

IMPULSE due to passage of a quantity of electricity, 583
Index of refraction, definition of, 382
— — —, measurement of, with a microscope, 383
— — — — — — with a spectrometer, 389
Induction, coefficient of mutual, 600, 603, 610
— — — comparison with capacity, 611
— — —, — resistance, 613
— — —, — two, 618
— — — self, 600, 603
— coil, spectrum of spark of an, 379
Inductor, earth, 624
— — measurement of dip by, 626
Inertia, moment of, 190
Interpolation, 41
Ions, 512

JELLETT'S prism, 414
Jolly's air thermometer, 265
Jolly's balance, 136
Joule's law, 522

KATER'S pendulum, 158
Kathetometer, 78, 141
— microscope, 77
Kilogramme standard, 11

LATENT heat of steam, 281
— — of water, 279
Latimer-Clark's potentiometer, 569
Laurent's method of measuring position of plane of polarisation, 415
Length, apparatus for measuring, 59, 63, 66, 68, 73
Lens, focal length of a long focussed, 373
—, magnifying power of, 363
Lenses, measurement of focal lengths of, &c., 343
Leyden jar, used with spectroscope and coil, 379
Lippich's method of measuring position of plane of polarisation, 414
Liquid, measurement of refractive index of, 384, 392
Logarithmic decrement, 585, 595
Lupton, 30

MAGNET and mirror, 497
Magnetic axis, 434
— declination, measurement of, 461
— field, 436
— — due to a current, 493
— —, exploration of a, 476
— —, measurement of strength of, 459
— force, laws of, 435
— —, line of, 436
— —, measurement of, 450, 459
— — due to a current, verification of law of, 500
— forces on a magnet, 441, 445, 467
— induction, 452, 474, 479
— —, measurement of, 480, 626
Magnetic meridian, 434
— moment, 442
— —, measurement of, 452, 456, 459, 461
— —, comparison of, 464
— poles, 434
— potential, 437, 439, 444
— shell, 599
— potential of, 599
— susceptibility, 482
Magnetisation by divided touch, 455
— of a steel bar, 453
Magnetism, definitions and explanations concerning, 434 &c.
Magnets, experiments with, 453
—, properties of, 433
Magnifying power of a telescope, 359
— — of a microscope, 363
— — of a lens, 363
— powers of optical instruments, 358
Mapping a spectrum, 377
Maxwell's theory of colour, 418
— vibration needle, 191
Mean of observations, 32, 34, 35
Measurement, methods of, 2, 5
—, units of, 9
Measurements, approximate, 30
— possible accuracy of, 35
Mercury, filling a barometer tube with, 294
Method of mixture, 272
Metre, standard, 11
Microscope, magnifying power of, 363
— used to measure refractive indices, 383
Microscopes, travelling, 73
— used to measure expansion, 257
Mirror telescope and scale, method of adjusting, 192

Moment of inertia, 166
— — —, determination of, 166, 195
Monochord, 229
Morse key, 589, 597
Multiplication, abbreviated form of, 38, 39
Musical note, 218

NICOL'S prism, 405
— — adjustments of, 406

OHM, 532
— practical unit of resistance, 531
Ohm's law, 526
Optical bench, 398
— measurements, general method for some, 335
Oscillation, method of observing time of, 152, 180
Oscillations, method of comparing times of, 155
—, — of weighing by, 107, 113

PARALLAX, method of avoiding error due to, 248
Parallax, optical adjustment by means of, 319, 338, 347
Parallel rays, focussing for, 361
Partial tones, 219
Pendulum, 152
— of any shape, 171
Photometer, Bunsen's, 312
—, Rumford's, 316
Photometry, theory of, 311
Physical constants, tables of, 30
— laws, quantitative, 13
— quantities, 2
Pitch, absolute, 229
— of tuning forks, comparison of, 219
Plane surface to set a ——— in a given position, 387
— surfaces, optical tests for, 367
Planimeter, 86
Polarimeters, shadow, 412
Polarisation, determination of plane of, 405
—, measurement of position of plane of, 414
—, rotation of plane of, 405
Polarised light, 405
Polarising apparatus, 405
Potential, electrical, 489
—, zero of, 491
— due to a magnet, 444
— of the earth, 490
Potentiometer, 567
Pound standard, 11
Poynting's method of measuring the position of the plane of polarisation, 416
Pressure, measurement of fluid, 197
Principal points of a lens, 346, 349
Prism, adjusting a, 377
—, adjustment of, on a spectrometer, 387
—, measurement of angle of, 388, 321
— measurement of refractive index of. 389
Projection, orthogonal, 87
Pumice stone, 302

QUANTITY, numerical measure of, 13
— of electricity, galvanometric measurement of, 582

RADIUS of curvature, optical measurement of, 337, 339, 370
Reflexion, radius of curvature measured by, 370
— of light, verification of law of, 318, 388
Refraction through a plate and prism, 328
Refractive index, definition of, 382
— —, measurement of, 329, 389
Regnault, experiments on Dalton's law. 299
Regnault's hygrometer, 309
Resistance, comparison of, 536
—, — — with mutual induction, 613
—, definition of electrical, 527
—, measurement of battery, 537, 553
—, — — galvanometer, 539, 551
—, relation between temperature and, 535, 564
—, — — work and, 527
—, specific, 535
—, standards of, 527
— boxes, 533
— measured by B.A. wire bridge, 557
— of conductors in multiple arc, 529
— — in series, 528
— — mercury, 532
Resistances compared by Carey Foster's method, 561
Resonance tube, 226
Resonator, 219
Rigidity, 185
Rumford's photometer, 316

SCALE pans, comparison of weights of, 117
— telescope and mirror, method of adjusting, 192
Screw-gauge, 66
Sensitiveness of a balance, 102, 113, 115
Sextant, adjustments of, 322, 323
Shearing strain, 185

Shunts, 530
Sine galvanometer, 495, 503
Siren, 222
—, method of using, 224
Solenoidal magnet, 434, 442, 444, 445
— —, force due to a, 445
Solid angle, 96, 611
Sound, velocity of, 226
Specific gravity, definition of, 121
— — bottle, 128
— — of a liquid, methods of measuring, 127, 132, 134, 139, 141
— — of a solid, methods of measuring, 123, 125, 128, 132, 133, 137, 209
— heat, definition of, 272
— —, method of cooling, 285
— —, — of mixture, 272
— — of a liquid, 278
— resistance, 535
— —, measurement of, 559
Spectra, comparison of, 381
Spectro-photometer, 312, 421
Spectrometer, 385
—, adjustment of, 386
Spectroscope, 377
—, adjustment of a, 377
Spectrum, mapping a, 377
—, method of obtaining a pure, 376
—, pure, 375
— of electric spark, 379
Spherometer, 68
Standard office of Board of Trade, 2, 11
Strain, 185
Strengths of absorbing solutions, comparison of, 424
Stress, 185
Strings, vibration of, 229

TABLES, use of, 40
Tangent galvanometer, 495, 503
Tap, three-way, 129
Taring, 117
Telescope, magnifying powers of, 359, 361
— mirror and scale, method of adjusting, 192
Temperature, definition of, 237
—, measurement of high, 243
— method of measuring, 237, 239
— and resistance, determination of relation between, 564
Thermometer, air, 265
—, centigrade, 239
—, construction of a, 244
—, corrections of, 240
—, Fahrenheit, 239
—, Kew corrections, 241, 242
—, — standard, 241
—, mercury, 238
—, testing a, 247
Thermometer, the weight, 259
—, wet and dry bulb, 306
Thermometry, 237
Time, measurement of, 96
— of oscillation, method of observing, 152
Torsion, determination of modulus of, 191
—, modulus of, 186, 190
Trigonometrical approximations, 45
Tuning forks, comparison of pitch of, 219

UNIT of heat, 271
— of resistance, British Association, 532
Units, absolute, 17
—, arbitrary and absolute, 10
—, change of, 24, 28
—, derived, 17
—, fundamental, 17
—, practical electrical, 22
—, table of arbitrary, 23
—, the C.G.S. system of, 21

VELOCITY of sound, 226
Verniers, 59, 62
Vibration, method of observing time of complete, 193
— frequency, determination of, 222
— needle, Maxwell's, 191
— number, 218
— of strings, 229
Volt, practical unit of E.M.F., 525
Voltameter, 512, 514, 516
— as a current meter, 515
— for decomposing water, 517
— silver, 579
Volume, measurement of, 92
Volumenometer, 205

WATCH, rating a, 97
Water, electrical decomposition of, 517
—, expansion of, 246
— equivalent, 273
— —, determination of, 276
Wave length, measurement of, 395, 399
— —, — — by spectrometer, 378
— — of a high note, 234
Weighing, method of double, 117
— — — oscillations, 107, 113
Weight thermometer, 259
Weights, 107, 113
Wheatstone Bridge, theory of, 543
Worthington's capillary multiplier, 215

YARD standard, 11
Young's modulus, 186, 187

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